GCF and LCM puzzles — Answers to 70

Name: _____ Date: _____

Find the two numbers. Show how you worked them out.

1 Two numbers have a greatest common factor of 6 and a least common multiple of 252.

Both numbers are less than 50. What are the two numbers?

Why Both numbers are multiples of 6, so they are 6×m and 6×n. Their LCM is 6 × m × n, so m × n = 252 ÷ 6 = 42. m and n share no factor, or the GCF would be more than 6. The coprime pairs multiplying to 42 give 6 and 252, 12 and 126, 18 and 84, 36 and 42. Of those, only 36 and 42 are under 50. More than one way works here: the GCF times the LCM is the two numbers multiplied, 6 × 252 = 1512, so the same pairs turn up as the factor pairs of 1512.

36 and 42

2 Two numbers have a greatest common factor of 9 and a least common multiple of 108.

Both numbers are greater than 10. What are the two numbers?

Why

___ and ___

GCF and LCM puzzles — Answers to 70

Name: _____ Date: _____

Find the two numbers. Show how you worked them out.

3 Two numbers have a greatest common factor of 2 and a least common multiple of 70.

Both numbers have two digits. What are the two numbers?

Why

___ and ___

4 Two numbers have a greatest common factor of 10 and a least common multiple of 120.

Both numbers are greater than 10. What are the two numbers?

Why

___ and ___

Who am I? — Up to 4 clues

Name: _____ Date: _____

Use every clue. Show what each one rules out.

1 I am a four-digit number. The ones digit is even. The thousands digit is 8 more than the ones digit. Its four digits add up to 14. The first two digits add to 13.

What number am I?

Why Start with all 9000 four-digit numbers. The ones digit is even → 4500 left; The thousands digit is 8 more than the ones digit → 100 left; Its four digits add up to 14 → 7 left; The first two digits add to 13 → 1 left. That one is 8510.

8510

2 I am a four-digit number. The ones digit is even. No digit is bigger than the thousands digit. Its four digits multiply to 3888. The first two digits add to 17.

What number am I?

Why

_____

Who am I? — Up to 4 clues

Name: _____ Date: _____

Use every clue. Show what each one rules out.

3 I am a four-digit number. The first two digits add to 1. The ones digit is odd. Its four digits add up to 18.

What number am I?

Why

_____

4 I am a four-digit number. Its four digits multiply to 1008. The ones digit is even. The first two digits add to 12.

What number am I?

Why

_____

Remainder puzzles — 3 conditions

Name: _____ Date: _____

Sieve for it. Show the numbers you checked.

1 A basket of eggs is counted out in groups. Counted in 4s there are 2 left over. Counted in 7s there are 4 left over.

What is the smallest number of eggs the basket could hold?

Why Start with the numbers that leave 2 when divided by 4: 2, 6, 10, 14, 18. They go up in 4s. Check those against "leaves 4 when divided by 7". The first that passes every check is 18 eggs.

18

2 Some marbles are shared out in different sized bags. In bags of 4, 2 are left over. In bags of 5, 1 is left over. In bags of 7, 1 is left over.

What is the smallest number of marbles there could be?

Why

Remainder puzzles — 3 conditions

Name: _____ Date: _____

Sieve for it. Show the numbers you checked.

3 Some marbles are shared out in different sized bags. In bags of 5, 3 are left over. In bags of 8, 5 are left over.

What is the smallest number of marbles there could be?

Why

_____

4 Some marbles are shared out in different sized bags. In bags of 5, 4 are left over. In bags of 7, 2 are left over. In bags of 9, 2 are left over.

What is the smallest number of marbles there could be?

Why

_____

Smudged sums — Totals to 70

Name: _____ Date: _____

Two digits are covered in each sum. Work out what they were, and show how.

1 A corner of Sam's workbook page is torn away, so a digit of each number is missing: 3■ + ■8 = 61.

What were the two numbers?

Why Start with the ones: something and 8 has to end in 1, and 3 + 8 = 11 — so the covered digit is 3, and a ten goes across to the next column. The tens column has that ten in it: 3 + 1 = 4, and 4 + 2 = 6, so the other covered digit is 2. The numbers are 33 and 28. Read the tens column on its own and it asks for 3 instead, giving 38 — and 33 + 38 = 71, ten too many.

33, 28

2 Mia leaned on her page before the ink was dry, and a digit of each number has smudged: 1■ + ■7 = 33.

What were the two numbers?

Why

Smudged sums — Totals to 70

Name: _____ Date: _____

Two digits are covered in each sum. Work out what they were, and show how.

3 This sum was photocopied badly and two digits came out blank: 1■ + ■8 = 52.

What were the two numbers?

Why

_____

4 Mia leaned on her page before the ink was dry, and a digit of each number has smudged: 4■ + ■9 = 62.

What were the two numbers?

Why

_____

Square gaps — Numbers up to 30

Name: _____ Date: _____

Two facts each time, and neither is enough alone. Show how you narrowed it.

1 Two square patches of floor are laid with the same tiles. The bigger patch takes 168 more tiles than the smaller, and both patches are between 9 and 19 tiles along each side.

How many tiles long is each patch?

Why A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 168. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $2 \times 84 \rightarrow 41$ and 43; $4 \times 42 \rightarrow 19$ and 23; $6 \times 28 \rightarrow 11$ and 17; $12 \times 14 \rightarrow 1$ and 13. Only 11 and 17 are both between 9 and 19. Check: $17 \times 17 = 289$, $11 \times 11 = 121$, and $289 - 121 = 168$.

11 and 17

2 Two whole numbers are both between 8 and 14. Squaring the bigger one gives 88 more than squaring the smaller one.

What are the two numbers?

Why

_____ and _____

Name: _____ Date: _____

Two facts each time, and neither is enough alone. Show how you narrowed it.

3 Two square gardens sit side by side. The bigger one covers 493 square meters more than the smaller one, and both have sides that are a whole number of meters between 4 and 26.

How long is each side?

Why

_____ and _____

4 A carpenter has two square boards. The larger has 208 square centimeters more surface than the smaller, and each side is a whole number of centimeters between 6 and 18.

How long is the side of each board?

Why

_____ and _____

Off-by-one puzzles — 3 kinds

Name: _____ Date: _____

Decide whether you want the things or the gaps. Show your working.

1 A path is 104 meters long. A tree is planted every 8 meters, with one at each end.

How many trees are there?

Why 104 ÷ 8 = 13, but that counts the GAPS between the trees. There is a tree at both ends, so there is always one more tree than gap: 13 + 1 = 14 trees.

14

2 A path is 126 meters long. A tree is planted every 9 meters, with one at each end.

How many trees are there?

Why

Off-by-one puzzles — 3 kinds

Name: _____ Date: _____

Decide whether you want the things or the gaps. Show your working.

3 A log is sawn into 13 equal pieces.

How many cuts does that take?

Why

4 A fence is 88 meters long. A post goes every 8 meters, starting 8 meters in from the left-hand end and finishing at the right-hand end.

How many posts are there?

Why

Power Pack · Number puzzles — Answer key

If they are stuck

- Their GCF divides both numbers. What does that let you write each of them as?
- Which clue rules out the most numbers? Use that one first.
- Write out the numbers that fit the FIRST condition. What do they go up in?
- One of the two columns can be settled on its own. Which, and what does settling it change?
- What does $a^2 - b^2$ factor into? Read that backwards.
- Draw four of them in a row and count the gaps. Is the answer the things or the spaces?

GCF and LCM

1. Both numbers are multiples of 6, so they are 6×m and 6×n. Their LCM is 6 × m × n, so m × n = 252 ÷ 6 = 42. m and n share no factor, or the GCF would be more than 6. The coprime pairs multiplying to 42 give 6 and 252, 12 and 126, 18 and 84, 36 and 42. Of those, only 36 and 42 are under 50. More than one way works here: the GCF times the LCM is the two numbers multiplied, 6 × 252 = 1512, so the same pairs turn up as the factor pairs of 1512.

2. Both numbers are multiples of 9, so they are 9×m and 9×n. Their LCM is 9 × m × n, so m × n = 108 ÷ 9 = 12. m and n share no factor, or the GCF would be more than 9. The coprime pairs multiplying to 12 give 9 and 108, 27 and 36. Of those, only 27 and 36 are over 10. More than one way works here: the GCF times the LCM is the two numbers multiplied, 9 × 108 = 972, so the same pairs turn up as the factor pairs of 972.

3. Both numbers are multiples of 2, so they are 2×m and 2×n. Their LCM is 2 × m × n, so m × n = 70 ÷ 2 = 35. m and n share no factor, or the GCF would be more than 2. The coprime pairs multiplying to 35 give 2 and 70, 10 and 14. Of those, only 10 and 14 have two digits. More than one way works here: the GCF times the LCM is the two numbers multiplied, 2 × 70 = 140, so the same pairs turn up as the factor pairs of 140.

4. Both numbers are multiples of 10, so they are 10×m and 10×n. Their LCM is 10 × m × n, so m × n = 120 ÷ 10 = 12. m and n share no factor, or the GCF would be more than 10. The coprime pairs multiplying to 12 give 10 and 120, 30 and 40. Of those, only 30 and 40 are over 10. More than one way works here: the GCF times the LCM is the two numbers multiplied, 10 × 120 = 1200, so the same pairs turn up as the factor pairs of 1200.

Four-digit numbers

1. Start with all 9000 four-digit numbers. The ones digit is even → 4500 left; The thousands digit is 8 more than the ones digit → 100 left; Its four digits add up to 14 → 7 left; The first two digits add to 13 → 1 left. That one is 8510.

2. Start with all 9000 four-digit numbers. The ones digit is even → 4500 left; No digit is bigger than the thousands digit → 1594 left; Its four digits multiply to 3888 → 4 left; The first two digits add to 17 → 1 left. That one is 9896.

3. Start with all 9000 four-digit numbers. The first two digits add to 1 → 100 left; The ones digit is odd → 50 left; Its four digits add up to 18 → 1 left. That one is 1089.

4. Start with all 9000 four-digit numbers. Its four digits multiply to 1008 → 72 left; The ones digit is even → 39 left; The first two digits add to 12 → 1 left. That one is 6674.

Power Pack · Number puzzles — Answer key

If they are stuck

- Their GCF divides both numbers. What does that let you write each of them as?
- Which clue rules out the most numbers? Use that one first.
- Write out the numbers that fit the FIRST condition. What do they go up in?
- One of the two columns can be settled on its own. Which, and what does settling it change?
- What does $a^2 - b^2$ factor into? Read that backwards.
- Draw four of them in a row and count the gaps. Is the answer the things or the spaces?

A number from its remainders

1. Start with the numbers that leave 2 when divided by 4: 2, 6, 10, 14, 18. They go up in 4s. Check those against "leaves 4 when divided by 7". The first that passes every check is 18 eggs.

2. Start with the numbers that leave 2 when divided by 4: 2, 6, 10, 14, 18, 22, and so on. They go up in 4s. Check those against "leaves 1 when divided by 5" — the survivors go up in 20s; check those against "leaves 1 when divided by 7". The first that passes every check is 106 marbles.

3. Start with the numbers that leave 3 when divided by 5: 3, 8, 13. They go up in 5s. Check those against "leaves 5 when divided by 8". The first that passes every check is 13 marbles.

4. Start with the numbers that leave 4 when divided by 5: 4, 9, 14, 19, 24, 29, and so on. They go up in 5s. Check those against "leaves 2 when divided by 7" — the survivors go up in 35s; check those against "leaves 2 when divided by 9". The first that passes every check is 254 marbles.

The smudged sum

1. Start with the ones: something and 8 has to end in 1, and 3 + 8 = 11 — so the covered digit is 3, and a ten goes across to the next column. The tens column has that ten in it: 3 + 1 = 4, and 4 + 2 = 6, so the other covered digit is 2. The numbers are 33 and 28. Read the tens column on its own and it asks for 3 instead, giving 38 — and 33 + 38 = 71, ten too many.

2. Start with the ones: something and 7 has to end in 3, and 6 + 7 = 13 — so the covered digit is 6, and a ten goes across to the next column. The tens column has that ten in it: 1 + 1 = 2, and 2 + 1 = 3, so the other covered digit is 1. The numbers are 16 and 17. Read the tens column on its own and it asks for 2 instead, giving 27 — and 16 + 27 = 43, ten too many.

3. Start with the ones: something and 8 has to end in 2, and 4 + 8 = 12 — so the covered digit is 4, and a ten goes across to the next column. The tens column has that ten in it: 1 + 1 = 2, and 2 + 3 = 5, so the other covered digit is 3. The numbers are 14 and 38. Read the tens column on its own and it asks for 4 instead, giving 48 — and 14 + 48 = 62, ten too many.

4. Start with the ones: something and 9 has to end in 2, and 3 + 9 = 12 — so the covered digit is 3, and a ten goes across to the next column. The tens column has that ten in it: 4 + 1 = 5, and 5 + 1 = 6, so the other covered digit is 1. The numbers are 43 and 19. Read the tens column on its own and it asks for 2 instead, giving 29 — and 43 + 29 = 72, ten too many.

Power Pack · Number puzzles — Answer key

If they are stuck

- Their GCF divides both numbers. What does that let you write each of them as?
- Which clue rules out the most numbers? Use that one first.
- Write out the numbers that fit the FIRST condition. What do they go up in?
- One of the two columns can be settled on its own. Which, and what does settling it change?
- What does $a^2 - b^2$ factor into? Read that backwards.
- Draw four of them in a row and count the gaps. Is the answer the things or the spaces?

The gap between two squares

1. A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 168. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $2 \times 84 \rightarrow 41$ and 43; $4 \times 42 \rightarrow 19$ and 23; $6 \times 28 \rightarrow 11$ and 17; $12 \times 14 \rightarrow 1$ and 13. Only 11 and 17 are both between 9 and 19. Check: $17 \times 17 = 289$, $11 \times 11 = 121$, and $289 - 121 = 168$.

2. A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 88. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $2 \times 44 \rightarrow 21$ and 23; $4 \times 22 \rightarrow 9$ and 13. Only 9 and 13 are both between 8 and 14. Check: $13 \times 13 = 169$, $9 \times 9 = 81$, and $169 - 81 = 88$.

3. A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 493. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $1 \times 493 \rightarrow 246$ and 247; $17 \times 29 \rightarrow 6$ and 23. Only 6 and 23 are both between 4 and 26. Check: $23 \times 23 = 529$, $6 \times 6 = 36$, and $529 - 36 = 493$.

4. A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 208. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $2 \times 104 \rightarrow 51$ and 53; $4 \times 52 \rightarrow 24$ and 28; $8 \times 26 \rightarrow 9$ and 17. Only 9 and 17 are both between 6 and 18. Check: $17 \times 17 = 289$, $9 \times 9 = 81$, and $289 - 81 = 208$.

Things and the gaps between

1. $104 \div 8 = 13$, but that counts the GAPS between the trees. There is a tree at both ends, so there is always one more tree than gap: $13 + 1 = 14$ trees.

2. $126 \div 9 = 14$, but that counts the GAPS between the trees. There is a tree at both ends, so there is always one more tree than gap: $14 + 1 = 15$ trees.

3. Picture the log: the 13 pieces have cuts BETWEEN them, not at the ends. Four pieces need three cuts, five need four — always one fewer. So 13 pieces take $13 - 1 = 12$ cuts.

4. $88 \div 8 = 11$ gaps. This time there is no post at the left-hand end, so the posts and the gaps come out equal: 11 posts. Not every one of these questions adds one.