Balance puzzles — Up to 70 coins

Name: ______________________ Date: ____________

Show what you would weigh each time, and how you know nothing shorter works.

1. 26 coins look identical. One is fake and slightly lighter than the rest.
What is the fewest weighings that are certain to find the fake?

Weighings Coins still suspect after each weighing: 26 → 9 → 3, then weigh two of the last three — the lighter pan or the one left off. That is 3 weighings.

3

2. 7 coins look identical. One is fake and slightly lighter than the rest.
What is the fewest weighings that are certain to find the fake?

Weighings

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3. 47 coins look identical. One is fake and slightly heavier than the rest.
What is the fewest weighings that are certain to find the fake?

Weighings

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Name: ______________________ Date: ____________

Show what you would weigh each time, and how you know nothing shorter works.

4 66 coins look identical. One is fake and slightly lighter than the rest.

What is the fewest weighings that are certain to find the fake?

Weighings

Arithmetic — Who is lying

Name: ______________________ Date: ____________

Work out how many of the islanders are liars, from what each of them says.

1. On an island, every person is either a truth-teller, who only says true things, or a liar, who only says false things. Ben says, "Ana and I are of different kinds." Ana says, "Cara is a liar." Cara says, "Ben is a liar."

   How many of the three are liars?

   Why Start with Ben's statement, because a "same kind / different kind" claim gives the OTHER person away whoever makes it. If Ben tells the truth, then Ben and Ana really are of different kinds, so Ana is a liar. If Ben is lying, they are really the same kind, and since Ben would be a liar, Ana is a liar too. Either way, Ana is a liar. Now Ana says "Cara is a liar". We know Ana is a liar, so that is a lie — which makes Cara a truth-teller. Then Cara says "Ben is a liar". As a truth-teller, that is true, so Ben is a liar. So Ana is a liar, Ben is a liar and Cara is a truth-teller — 2 of them are liars.

   2

Name: ______________________ Date: ______________

Work out how many of the islanders are liars, from what each of them says.

2 On an island, every person is either a truth-teller, who only says true things, or a liar, who only says false things. Ben says, "Ana and I are the same kind." Ana says, "Cara is a liar." Cara says, "Ben always tells the truth."

How many of the three are liars?

Why

Name: ____________________ Date: __________

Work out how many of the islanders are liars, from what each of them says.

3 On an island, every person is either a truth-teller, who only says true things, or a liar, who only says false things. Elsa says, "Dev and I are the same kind." Dev says, "Faye is a liar." Faye says, "Elsa is a liar."

How many of the three are liars?

Why

Name: ____________________ Date: __________

Work out how many of the islanders are liars, from what each of them says.

4 On an island, every person is either a truth-teller, who only says true things, or a liar, who only says false things. Priya says, "Omar and I are of different kinds." Omar says, "Quinn always tells the truth." Quinn says, "Priya always tells the truth."

How many of the three are liars?

Why

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Name: ____________________ Date: ____________

Say how you know your number is always enough, however the picking goes.

1. A drawer holds 6 red socks and 10 blue socks, all mixed up. The light is off, so you cannot see what you are taking.

How many socks must you take to be sure of a matching pair?

Why There are 2 colors here (6 red and 10 blue). The worst that can happen is taking one of each — 2 socks and still no pair. The next one has to match something, so 2 + 1 = 3. Notice the counts never came into it.

3

2. A box holds 10 red marbles and 5 blue marbles, and you are picking without looking.

How many must you take to be sure of two red ones?

Why

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Name: ______________________ Date: ____________

Say how you know your number is always enough, however the picking goes.

3. A box holds 9 red marbles and 9 blue marbles, and you are picking without looking.

How many must you take to be sure of two red ones?

Why

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4. A bag holds 8 red beads and 7 blue beads. You reach in without looking.

How many beads must you take to be sure two of them match?

Why

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Name: ____________________ Date: ____________

Find what every move leaves alone. Show why the order cannot matter.

1. The numbers 3 to 13 are written on a board. Rub out any two of them and write their sum minus 1 in their place. Keep going until one is left.

What is the last number?

Why Each move rubs out two numbers and writes one in their place, so the count falls by 1 each time: 11 numbers means 10 moves. Each move also drops the TOTAL by exactly 1, so $10 \times 1 = 10$ is lost altogether. The numbers add to 88, so whatever order anyone uses, the last number is $88 - 10 = 78$.

78

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2. The numbers 4 to 10 are written on a board. Rub out any two of them and write their sum plus 2 in their place. Keep going until one is left.

What is the last number?

Why

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Name: ______________________ Date: ____________

Find what every move leaves alone. Show why the order cannot matter.

3. The numbers 2 to 7 are written on cards. Two cards are taken away and replaced by one showing their sum plus 2. Repeat until one card is left.

What number is on the last card?

Why

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4. The numbers 1 to 8 are written on cards. Two cards are taken away and replaced by one showing their sum minus 3. Repeat until one card is left.

What number is on the last card?

Why

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Possible or never — Up to 14 things

Name: ____________________ Date: __________

Some of these cannot be done. Say which, and say how you know.

1 13 cups stand on a tray and 2 of them are the right way up. You may turn over exactly 4 of them at a time, never more and never fewer.

What is the fewest turns that leaves them all the right way up? Write NEVER if it cannot be done.

Why Every turn touches 4 of the 13, and whatever state those 4 were in, the number standing the right way up changes by an EVEN amount — 4, or 2, and so on down to −4. So it keeps its odd-or-even-ness forever. It starts at 2 and would have to reach 13, and 13 − 2 = 11 is odd. No number of turns can ever get there.

NEVER

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2 11 cups stand on a tray and 5 of them are the right way up. You may turn over exactly 2 of them at a time, never more and never fewer.

What is the fewest turns that leaves them all the right way up? Write NEVER if it cannot be done.

Why

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Name: ____________________ Date: ____________

Some of these cannot be done. Say which, and say how you know.

3. 7 cups stand on a tray and 6 of them are the right way up. You may turn over exactly 4 of them at a time, never more and never fewer.

What is the fewest turns that leaves them all the right way up? Write NEVER if it cannot be done.

Why

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4. 9 cups stand on a tray and 6 of them are the right way up. You may turn over exactly 2 of them at a time, never more and never fewer.

What is the fewest turns that leaves them all upside down? Write NEVER if it cannot be done.

Why

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Win or go second — Piles up to 24

Name: ____________________ Date: ____________

Take turns taking counters. Say how to force a win, or that you cannot.

1. 15 pebbles are heaped on the ground. Two players take turns, and on your turn you take between 1 and 4 of them. Whoever takes the very last pebble wins. You go first.

Can you be sure to win? If so, how many should you take on your first turn? Write SECOND if going first cannot win.

Why 15 is already a multiple of 5 — 15 ÷ 5 = 3 whole groups of 5. Whatever you take first, from 1 up to 4, the other player takes the rest of that group, bringing the pile back to a multiple of 5. You are always the one facing a multiple, so however you play it is the other player who takes the last. Going first cannot win here: the answer is SECOND.

SECOND

Win or go second — Piles up to 24

Name: ______________________ Date: ______________

Take turns taking counters. Say how to force a win, or that you cannot.

2. A stack of 24 counters sits between the two players. Two players take turns, and on your turn you take between 1 and 2 of them. Whoever takes the very last counter wins. You go first.

Can you be sure to win? If so, how many should you take on your first turn? Write SECOND if going first cannot win.

Why

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Win or go second — Piles up to 24

Name: ______________________ Date: ____________

Take turns taking counters. Say how to force a win, or that you cannot.

3. 8 pebbles are heaped on the ground. Two players take turns, and on your turn you take between 1 and 2 of them. Whoever takes the very last pebble wins. You go first.

Can you be sure to win? If so, how many should you take on your first turn? Write SECOND if going first cannot win.

Why

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Name: ______________________________ Date: ______________

Take turns taking counters. Say how to force a win, or that you cannot.

4 A stack of 21 counters sits between the two players. Two players take turns, and on your turn you take between 1 and 4 of them. Whoever takes the very last counter wins. You go first.

Can you be sure to win? If so, how many should you take on your first turn? Write SECOND if going first cannot win.

Why

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Power Pack · Reasoning puzzles — Answer key

If they are stuck

- A balance can say three things, not two. What is the third?
- A 'same kind' or 'different kind' claim gives the OTHER person away whoever says it. Start there.
- Do not count the cases — ask what the unluckiest possible run looks like.
- Play two moves. The numbers all change — is there anything that does not?
- Trying it will not settle this. Ask what ONE move does to the count.
- Play the smallest piles by hand. Which sizes lose for the player who must move, and what do they share?

The fake coin

1. Coins still suspect after each weighing: 26 → 9 → 3, then weigh two of the last three — the lighter pan or the one left off. That is 3 weighings.

2. Coins still suspect after each weighing: 7 → 3, then weigh two of the last three — the lighter pan or the one left off. That is 2 weighings.

3. Coins still suspect after each weighing: 47 → 16 → 6 → 2, then weigh the last two — the heavier one is fake. That is 4 weighings.

4. Coins still suspect after each weighing: 66 → 22 → 8 → 3, then weigh two of the last three — the lighter pan or the one left off. That is 4 weighings.

Truth-tellers and liars

1. Start with Ben's statement, because a "same kind / different kind" claim gives the OTHER person away whoever makes it. If Ben tells the truth, then Ben and Ana really are of different kinds, so Ana is a liar. If Ben is lying, they are really the same kind, and since Ben would be a liar, Ana is a liar too. Either way, Ana is a liar. Now Ana says "Cara is a liar". We know Ana is a liar, so that is a lie — which makes Cara a truth-teller. Then Cara says "Ben is a liar". As a truth-teller, that is true, so Ben is a liar. So Ana is a liar, Ben is a liar and Cara is a truth-teller — 2 of them are liars.

2. Start with Ben's statement, because a "same kind / different kind" claim gives the OTHER person away whoever makes it. If Ben tells the truth, then Ben and Ana really are the same kind, so Ana is a truth-teller. If Ben is lying, then they are really of different kinds, and since Ben would be a liar, Ana is a truth-teller once more. Either way, Ana is a truth-teller. Now Ana says "Cara is a liar". We know Ana is a truth-teller, so that is true — which makes Cara a liar. Then Cara says "Ben always tells the truth". As a liar, that is false, so Ben is a liar. So Ana is a truth-teller, Ben is a liar and Cara is a liar — 2 of them are liars.

3. Start with Elsa's statement, because a "same kind / different kind" claim gives the OTHER person away whoever makes it. If Elsa tells the truth, then Elsa and Dev really are the same kind, so Dev is a truth-teller. If Elsa is lying, then they are really of different kinds, and since Elsa would be a liar, Dev is a truth-teller once more. Either way, Dev is a truth-teller. Now Dev says "Faye is a liar". We know Dev is a truth-teller, so that is true — which makes Faye a liar. Then Faye says "Elsa is a liar". As a liar, that is false, so Elsa is a truth-teller. So Dev is a truth-teller, Elsa is a truth-teller and Faye is a liar — 1 of them is a liar.

4. Start with Priya's statement, because a "same kind / different kind" claim gives the OTHER person away whoever makes it. If Priya tells the truth, then Priya and Omar really are of different kinds, so Omar is a liar. If Priya is lying, they are really the same kind, and since Priya would be a liar, Omar is a liar too. Either way, Omar is a liar. Now Omar says "Quinn always tells the truth". We know Omar is a liar, so that is a lie — which makes Quinn a liar. Then Quinn says "Priya always tells the truth". As a liar, that is false, so Priya is a liar. So Omar is a liar, Priya is a liar and Quinn is a liar — 3 of them are liars.

Power Pack · Reasoning puzzles — Answer key

If they are stuck

- A balance can say three things, not two. What is the third?
- A 'same kind' or 'different kind' claim gives the OTHER person away whoever says it. Start there.
- Do not count the cases — ask what the unluckiest possible run looks like.
- Play two moves. The numbers all change — is there anything that does not?
- Trying it will not settle this. Ask what ONE move does to the count.
- Play the smallest piles by hand. Which sizes lose for the player who must move, and what do they share?

Socks in the dark

1. There are 2 colors here (6 red and 10 blue). The worst that can happen is taking one of each — 2 socks and still no pair. The next one has to match something, so 2 + 1 = 3. Notice the counts never came into it.

2. The box holds 10 red marbles and 5 blue marbles. The worst that can happen is taking every other one first: all 5 of the others, and not a single red yet. Then two more must both be red, so 5 + 2 = 7.

3. The box holds 9 red marbles and 9 blue marbles. The worst that can happen is taking every other one first: all 9 of the others, and not a single red yet. Then two more must both be red, so 9 + 2 = 11.

4. There are 2 colors here (8 red and 7 blue). The worst that can happen is taking one of each — 2 beads and still no pair. The next one has to match something, so 2 + 1 = 3. Notice the counts never came into it.

The number that cannot change

1. Each move rubs out two numbers and writes one in their place, so the count falls by 1 each time: 11 numbers means 10 moves. Each move also drops the TOTAL by exactly 1, so 10 × 1 = 10 is lost altogether. The numbers add to 88, so whatever order anyone uses, the last number is 88 − 10 = 78.

2. Each move rubs out two numbers and writes one in their place, so the count falls by 1 each time: 7 numbers means 6 moves. Each move also raises the TOTAL by exactly 2, so 6 × 2 = 12 is gained altogether. The numbers add to 49, so whatever order anyone uses, the last number is 49 + 12 = 61.

3. Each move takes away two cards and puts one back, so the count falls by 1 each time: 6 numbers means 5 moves. Each move also raises the TOTAL by exactly 2, so 5 × 2 = 10 is gained altogether. The numbers add to 27, so whatever order anyone uses, the number on the last card is 27 + 10 = 37.

4. Each move takes away two cards and puts one back, so the count falls by 1 each time: 8 numbers means 7 moves. Each move also drops the TOTAL by exactly 3, so 7 × 3 = 21 is lost altogether. The numbers add to 36, so whatever order anyone uses, the number on the last card is 36 − 21 = 15.

Power Pack · Reasoning puzzles — Answer key

If they are stuck

- A balance can say three things, not two. What is the third?
- A 'same kind' or 'different kind' claim gives the OTHER person away whoever says it. Start there.
- Do not count the cases — ask what the unluckiest possible run looks like.
- Play two moves. The numbers all change — is there anything that does not?
- Trying it will not settle this. Ask what ONE move does to the count.
- Play the smallest piles by hand. Which sizes lose for the player who must move, and what do they share?

Can it be done at all?

1. Every turn touches 4 of the 13, and whatever state those 4 were in, the number standing the right way up changes by an EVEN amount — 4, or 2, and so on down to −4. So it keeps its odd-or-even-ness forever. It starts at 2 and would have to reach 13, and 13 − 2 = 11 is odd. No number of turns can ever get there.

2. There are 11 to work with. One turn changes the number standing the right way up by at most 2, and there are 11 − 5 = 6 to gain, so fewer than 3 turns cannot possibly do it. And 3 can, counting the number standing the right way up as you go: 5 → 7 → 9 → 11.

3. Every turn touches 4 of the 7, and whatever state those 4 were in, the number standing the right way up changes by an EVEN amount — 4, or 2, and so on down to −4. So it keeps its odd-or-even-ness forever. It starts at 6 and would have to reach 7, and 7 − 6 = 1 is odd. No number of turns can ever get there.

4. There are 9 to work with. One turn changes the number standing the right way up by at most 2, and there are 6 − 0 = 6 to lose, so fewer than 3 turns cannot possibly do it. And 3 can, counting the number standing the right way up as you go: 6 → 4 → 2 → 0.

The taking game

1. 15 is already a multiple of 5 — 15 ÷ 5 = 3 whole groups of 5. Whatever you take first, from 1 up to 4, the other player takes the rest of that group, bringing the pile back to a multiple of 5. You are always the one facing a multiple, so however you play it is the other player who takes the last. Going first cannot win here: the answer is SECOND.

2. 24 is already a multiple of 3 — 24 ÷ 3 = 8 whole groups of 3. Whatever you take first, from 1 up to 2, the other player takes the rest of that group, bringing the pile back to a multiple of 3. You are always the one facing a multiple, so however you play it is the other player who takes the last. Going first cannot win here: the answer is SECOND.

3. The losing pile sizes are the multiples of 3: from one of them, whatever the other player takes — 1 up to 2 — you take the rest of a group of 3, and the pile drops straight to the next multiple. So aim to always hand them a multiple of 3. Here 8 − 2 = 6, and 6 is a multiple of 3, so take 2 pebbles first. After that, when they take t you take 3 − t; every round removes 3 between you, and you take the last.

4. The losing pile sizes are the multiples of 5: from one of them, whatever the other player takes — 1 up to 4 — you take the rest of a group of 5, and the pile drops straight to the next multiple. So aim to always hand them a multiple of 5. Here 21 − 1 = 20, and 20 is a multiple of 5, so take 1 counter first. After that, when they take t you take 5 − t; every round removes 5 between you, and you take the last.