# Painted cube — Up to 8 on each edge

Name: ______________________  Date: __________

A block is painted all over, then cut into little cubes. Count the ones the question asks for, and show why.

1. A solid wooden block, 6 cubes long on every side, is painted red all over its outside and then sawn into 6 × 6 × 6 little cubes.

**How many of the little cubes have paint on exactly three faces?**

**Why** Three painted faces means a corner, where three sides meet. A cube has 8 corners no matter how big it is, so the answer is 8 — the size of the block never enters into it.

8

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2. A foam cube measuring 6 units along each edge is spray-painted on all six faces, then cut into 6 × 6 × 6 unit cubes.

**How many of the unit cubes have paint on exactly one face?**

**Why**

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# Painted cube — Up to 8 on each edge

Name: ____________________ Date: ____________

A block is painted all over, then cut into little cubes. Count the ones the question asks for, and show why.

**3** A foam cube measuring 8 units along each edge is spray-painted on all six faces, then cut into $8 \times 8 \times 8$ unit cubes.

**How many of the unit cubes have paint on exactly two faces?**

**Why**

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**4** A solid wooden block, 8 cubes long on every side, is painted red all over its outside and then sawn into $8 \times 8 \times 8$ little cubes.

**How many of the little cubes have no paint on them at all?**

**Why**

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Name: ____________________ Date: ____________

A rod is pushed corner to corner through a block of unit cubes. Count the ones it passes through, and show why a + b + c is too many.

1. A solid block is built from unit cubes, 2 along one edge, 4 along the next and 7 along the last. A straight rod is pushed from one corner right through to the opposite corner.

**How many of the little cubes does it pass through?**

**Why** Corner to corner, the rod enters a new little cube at every wall it crosses — 13 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens gcd(2,4) = 2, gcd(4,7) = 1 and gcd(7,2) = 1 times, 4 in all, each an over-count of one. Taking them off: 13 − 4 = 9. All three walls meet together gcd(2,4,7) = 1 times, and that was subtracted once too often, so add it back: 9 + 1 = 10.

10

2. A 5 × 5 × 8 box is packed exactly with unit cubes. A laser is aimed from one corner to the corner diagonally opposite, passing straight through the block.

**How many of the unit cubes does it pass through?**

**Why**

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Name: ____________________ Date: ____________

A rod is pushed corner to corner through a block of unit cubes. Count the ones it passes through, and show why a + b + c is too many.

3 A solid block is built from unit cubes, 3 along one edge, 5 along the next and 8 along the last. A straight rod is pushed from one corner right through to the opposite corner.

**How many of the little cubes does it pass through?**

**Why**

___

4 A 2 × 5 × 7 box is packed exactly with unit cubes. A laser is aimed from one corner to the corner diagonally opposite, passing straight through the block.

**How many of the unit cubes does it pass through?**

**Why**

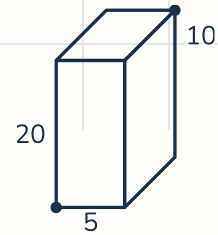
___

# Spider and fly — Edges up to 20

Name: ______________________ Date: ______________

> A bug crosses the outside of a box, corner to corner. Find the shortest path on the surface, and show the unfolding.

**1** An ant is at one corner of a $5 \times 10 \times 20$ box and wants to reach the corner diagonally opposite. It can only crawl along the outside faces — it cannot cut through the box.




**What is the shortest distance across the surface?**

**Why** The ant cannot cut through, so unfold the two faces a path crosses and it becomes a straight line — three ways to pick the two faces, three flat distances to compare. Pairing the two shorter edges puts $5 + 10 = 15$ along one side and 20 along the other. By Pythagoras, $15 \times 15 = 225$ and $20 \times 20 = 400$, together 625; and $25 \times 25 = 625$, so that path is 25. The other two unfoldings come out to squared lengths 725 and 925, both longer, and cutting straight THROUGH the box would be shorter but leaves the surface. Shortest path: 25.

25

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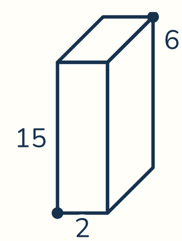
**2** A spider sits in one corner of a room $2 \times 6 \times 15$ and a fly rests in the far opposite corner. The spider must travel across the walls, floor and ceiling — never through the air.




**What is the shortest distance across the surface?**

**Why**

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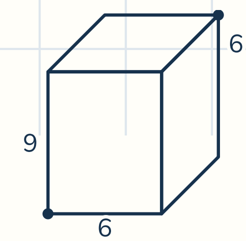
Name: ______________________ Date: ____________

A bug crosses the outside of a box, corner to corner. Find the shortest path on the surface, and show the unfolding.

**3** A spider sits in one corner of a room 6 × 6 × 9 and a fly rests in the far opposite corner. The spider must travel across the walls, floor and ceiling — never through the air.

**What is the shortest distance across the surface?**




**Why**

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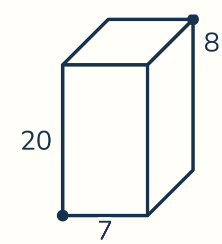
**4** A beetle starts at a corner of a 7 × 8 × 20 wooden crate and heads for the corner furthest away, keeping to the surface the whole time.

**What is the shortest distance across the surface?**




**Why**

Name: ____________________ Date: __________

Count the triangles with whole-number sides and the given perimeter, and show why the impossible ones are out.

**1** A farmer has 26 metres of fence and wants a triangular pen whose three sides are each a whole number of metres.

**How many differently shaped triangular pens can be built? (Turning or flipping a pen over does not make it a new one.)**

**Why** Sides $a \le b \le c$ with $a + b + c = 26$ only make a pen when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 24) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (2, 12, 12), (3, 11, 12), (4, 10, 12), (4, 11, 11), (5, 9, 12), ..., (8, 9, 9). That is 14 different triangles.

14

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**2** A farmer has 23 metres of fence and wants a triangular pen whose three sides are each a whole number of metres.

**How many differently shaped triangular pens can be built? (Turning or flipping a pen over does not make it a new one.)**

**Why**

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Name: ______________________ Date: ____________

Count the triangles with whole-number sides and the given perimeter, and show why the impossible ones are out.

**3** 30 matchsticks of equal length are laid end to end to make the three sides of a triangle, with none left over and none broken.

**How many different triangles can be made? (The same triangle turned around counts once.)**

**Why**

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**4** 15 matchsticks of equal length are laid end to end to make the three sides of a triangle, with none left over and none broken.

**How many different triangles can be made? (The same triangle turned around counts once.)**

**Why**

___

Name: ____________________ Date: __________

A row of lockers, and every student flips a share of them. Say how many end open, and why.

**1** A school corridor has 30 lockers in a row, numbered 1 to 30, all shut. Student 1 opens every locker. Student 2 changes every second locker (2, 4, 6, ...). Student 3 changes every third, and so on up to student 30 — each student k changes every locker whose number k divides.

**In the end, how many lockers are left open?**

**Why** Each locker numbered L is changed once for each divisor of L. Divisors come in pairs (d with L ÷ d), an even count, so a locker ends shut unless a divisor pairs with itself — that is, unless L is a perfect square. The lockers left open are the perfect squares up to 30: 1, 4, 9, 16, 25. The largest is $5 \times 5 = 25$, so there are 5 of them and 5 lockers stay open.

5

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**2** A hall has 59 lamps in a line, numbered 1 to 59, all off. Switch 1 flips every lamp. Switch 2 flips every second lamp, switch 3 every third, and so on up to switch 59 — switch k flips every lamp whose number k divides.

**In the end, how many lamps are left on?**

**Why**

___

Name: ____________________ Date: __________

A row of lockers, and every student flips a share of them. Say how many end open, and why.

3 A long hallway has 73 doors, numbered 1 to 73, all closed. Person 1 opens every door. Person 2 changes every second door, person 3 every third, and so on to person 73 — person k changes every door whose number k divides.

**In the end, how many doors are left open?**

**Why**

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4 A long hallway has 92 doors, numbered 1 to 92, all closed. Person 1 opens every door. Person 2 changes every second door, person 3 every third, and so on to person 92 — person k changes every door whose number k divides.

**In the end, how many doors are left open?**

**Why**

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# Cycles — Digits and days

Name: ____________________ Date: __________

The number is far too big to write. Find what repeats, and count carefully.

**1** A number is made by multiplying 128 threes together. It is far too big for any calculator to show.

**What is its last digit?**

**Why** Powers of 3 end in 3, 9, 7, 1, then start again — a cycle of 4. 128 ÷ 4 = 32 exactly, so the 128th lands on the LAST of the four, not the first. Its last digit is 1.

1

**2** Today is Monday. A comet will be overhead in 45 days' time.

**What day of the week will that be?**

**Why**

# Cycles — Digits and days

Name: ____________________ Date: __________

The number is far too big to write. Find what repeats, and count carefully.

**3** Today is Thursday. A comet will be overhead in 454 days' time.

**What day of the week will that be?**

**Why**

______________

**4** A number is made by multiplying 400 eights together. It is far too big for any calculator to show.

**What is its last digit?**

**Why**

______________

**If they are stuck**

- Where must a little cube sit to touch that many faces? Peel the painted shell off and look at what is left.
- Count a + b + c first. Then ask where the rod crosses two walls at once — those got counted twice.
- The bug stays on the surface. What happens to a bent path when you unfold the two faces it crosses flat?
- Fix the longest side first, then ask which shorter pairs can actually reach across it.
- How many times does each one get changed? Ask when that count is ODD.
- It cannot be computed, so look at the small cases. Do they settle down?

## The painted cube

1. Three painted faces means a corner, where three sides meet. A cube has 8 corners no matter how big it is, so the answer is 8 — the size of the block never enters into it.

2. One painted face means a cube in the middle of a face, off every edge. Each face's middle is 4 × 4 = 16, and a cube has 6 faces: 6 × 16 = 96.

3. Two painted faces means a cube along an edge, between the two corners. Taking the corners off each edge leaves 6 along it, and a cube has 12 edges: 12 × 6 = 72.

4. No paint at all means a cube deep inside, never on the surface. Peel one layer off all six sides and the cube left inside is 6 on each edge: 6 × 6 = 36, then 36 × 6 = 216 cubes hidden in the middle.

## The rod through the box

1. Corner to corner, the rod enters a new little cube at every wall it crosses — 13 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens gcd(2,4) = 2, gcd(4,7) = 1 and gcd(7,2) = 1 times, 4 in all, each an over-count of one. Taking them off: 13 − 4 = 9. All three walls meet together gcd(2,4,7) = 1 times, and that was subtracted once too often, so add it back: 9 + 1 = 10.

2. Corner to corner, the rod enters a new unit cube at every wall it crosses — 18 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens gcd(5,5) = 5, gcd(5,8) = 1 and gcd(8,5) = 1 times, 7 in all, each an over-count of one. Taking them off: 18 − 7 = 11. All three walls meet together gcd(5,5,8) = 1 times, and that was subtracted once too often, so add it back: 11 + 1 = 12.

3. Corner to corner, the rod enters a new little cube at every wall it crosses — 16 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens gcd(3,5) = 1, gcd(5,8) = 1 and gcd(8,3) = 1 times, 3 in all, each an over-count of one. Taking them off: 16 − 3 = 13. All three walls meet together gcd(3,5,8) = 1 times, and that was subtracted once too often, so add it back: 13 + 1 = 14.

4. Corner to corner, the rod enters a new unit cube at every wall it crosses — 14 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens gcd(2,5) = 1, gcd(5,7) = 1 and gcd(7,2) = 1 times, 3 in all, each an over-count of one. Taking them off: 14 − 3 = 11. All three walls meet together gcd(2,5,7) = 1 times, and that was subtracted once too often, so add it back: 11 + 1 = 12.

# Power Pack · Space puzzles — Answer key

**If they are stuck**

- Where must a little cube sit to touch that many faces? Peel the painted shell off and look at what is left.
- Count a + b + c first. Then ask where the rod crosses two walls at once — those got counted twice.
- The bug stays on the surface. What happens to a bent path when you unfold the two faces it crosses flat?
- Fix the longest side first, then ask which shorter pairs can actually reach across it.
- How many times does each one get changed? Ask when that count is ODD.
- It cannot be computed, so look at the small cases. Do they settle down?

### The spider and the fly

1. The ant cannot cut through, so unfold the two faces a path crosses and it becomes a straight line — three ways to pick the two faces, three flat distances to compare. Pairing the two shorter edges puts 5 + 10 = 15 along one side and 20 along the other. By Pythagoras, 15 × 15 = 225 and 20 × 20 = 400, together 625; and 25 × 25 = 625, so that path is 25. The other two unfoldings come out to squared lengths 725 and 925, both longer, and cutting straight THROUGH the box would be shorter but leaves the surface. Shortest path: 25.

2. The spider cannot cut through, so unfold the two faces a path crosses and it becomes a straight line — three ways to pick the two faces, three flat distances to compare. Pairing the two shorter edges puts 2 + 6 = 8 along one side and 15 along the other. By Pythagoras, 8 × 8 = 64 and 15 × 15 = 225, together 289; and 17 × 17 = 289, so that path is 17. The other two unfoldings come out to squared lengths 325 and 445, both longer, and cutting straight THROUGH the room would be shorter but leaves the surface. Shortest path: 17.

3. The spider cannot cut through, so unfold the two faces a path crosses and it becomes a straight line — three ways to pick the two faces, three flat distances to compare. Pairing the two shorter edges puts 6 + 6 = 12 along one side and 9 along the other. By Pythagoras, 12 × 12 = 144 and 9 × 9 = 81, together 225; and 15 × 15 = 225, so that path is 15. The other two unfoldings come out to squared lengths 261 and 261, both longer, and cutting straight THROUGH the room would be shorter but leaves the surface. Shortest path: 15.

4. The beetle cannot cut through, so unfold the two faces a path crosses and it becomes a straight line — three ways to pick the two faces, three flat distances to compare. Pairing the two shorter edges puts 7 + 8 = 15 along one side and 20 along the other. By Pythagoras, 15 × 15 = 225 and 20 × 20 = 400, together 625; and 25 × 25 = 625, so that path is 25. The other two unfoldings come out to squared lengths 793 and 833, both longer, and cutting straight THROUGH the crate would be shorter but leaves the surface. Shortest path: 25.

# Power Pack · Space puzzles — Answer key

**If they are stuck**

- Where must a little cube sit to touch that many faces? Peel the painted shell off and look at what is left.
- Count $a + b + c$ first. Then ask where the rod crosses two walls at once — those got counted twice.
- The bug stays on the surface. What happens to a bent path when you unfold the two faces it crosses flat?
- Fix the longest side first, then ask which shorter pairs can actually reach across it.
- How many times does each one get changed? Ask when that count is ODD.
- It cannot be computed, so look at the small cases. Do they settle down?

### Integer triangles

1. Sides $a \le b \le c$ with $a + b + c = 26$ only make a pen when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 24) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (2, 12, 12), (3, 11, 12), (4, 10, 12), (4, 11, 11), (5, 9, 12), ..., (8, 9, 9). That is 14 different triangles.

2. Sides $a \le b \le c$ with $a + b + c = 23$ only make a pen when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 21) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 11, 11), (2, 10, 11), (3, 9, 11), (3, 10, 10), (4, 8, 11), ..., (7, 8, 8). That is 14 different triangles.

3. Sides $a \le b \le c$ with $a + b + c = 30$ only make a triangle when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 28) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (2, 14, 14), (3, 13, 14), (4, 12, 14), (4, 13, 13), (5, 11, 14), ..., (10, 10, 10). That is 19 different triangles.

4. Sides $a \le b \le c$ with $a + b + c = 15$ only make a triangle when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 13) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 7, 7), (2, 6, 7), (3, 5, 7), (3, 6, 6), (4, 4, 7), ..., (5, 5, 5). That is 7 different triangles.

### The hundred lockers

1. Each locker numbered $L$ is changed once for each divisor of $L$. Divisors come in pairs ($d$ with $L \div d$), an even count, so a locker ends shut unless a divisor pairs with itself — that is, unless $L$ is a perfect square. The lockers left open are the perfect squares up to 30: 1, 4, 9, 16, 25. The largest is $5 \times 5 = 25$, so there are 5 of them and 5 lockers stay open.

2. Each lamp numbered $L$ is changed once for each divisor of $L$. Divisors come in pairs ($d$ with $L \div d$), an even count, so a lamp ends off unless a divisor pairs with itself — that is, unless $L$ is a perfect square. The lamps left on are the perfect squares up to 59: 1, 4, 9, 16, ..., 49. The largest is $7 \times 7 = 49$, so there are 7 of them and 7 lamps stay on.

3. Each door numbered $L$ is changed once for each divisor of $L$. Divisors come in pairs ($d$ with $L \div d$), an even count, so a door ends closed unless a divisor pairs with itself — that is, unless $L$ is a perfect square. The doors left open are the perfect squares up to 73: 1, 4, 9, 16, ..., 64. The largest is $8 \times 8 = 64$, so there are 8 of them and 8 doors stay open.

4. Each door numbered $L$ is changed once for each divisor of $L$. Divisors come in pairs ($d$ with $L \div d$), an even count, so a door ends closed unless a divisor pairs with itself — that is, unless $L$ is a perfect square. The doors left open are the perfect squares up to 92: 1, 4, 9, 16, ..., 81. The largest is $9 \times 9 = 81$, so there are 9 of them and 9 doors stay open.

# Power Pack · Space puzzles — Answer key

> **If they are stuck**
>
> - Where must a little cube sit to touch that many faces? Peel the painted shell off and look at what is left.
> - Count a + b + c first. Then ask where the rod crosses two walls at once — those got counted twice.
> - The bug stays on the surface. What happens to a bent path when you unfold the two faces it crosses flat?
> - Fix the longest side first, then ask which shorter pairs can actually reach across it.
> - How many times does each one get changed? Ask when that count is ODD.
> - It cannot be computed, so look at the small cases. Do they settle down?

### Numbers too big to compute

1. Powers of 3 end in 3, 9, 7, 1, then start again — a cycle of 4. 128 ÷ 4 = 32 exactly, so the 128th lands on the LAST of the four, not the first. Its last digit is 1.

2. Days of the week repeat every 7, so only the leftover matters. 6 whole weeks is 6 × 7 = 42 days, and 45 − 42 = 3 left over. So the comet arrives 3 days after Monday: Thursday.

3. Days of the week repeat every 7, so only the leftover matters. 64 whole weeks is 64 × 7 = 448 days, and 454 − 448 = 6 left over. So the comet arrives 6 days after Thursday: Wednesday.

4. Powers of 8 end in 8, 4, 2, 6, then start again — a cycle of 4. 400 ÷ 4 = 100 exactly, so the 400th lands on the LAST of the four, not the first. Its last digit is 6.