Series — Nth-term test

Name: _____ Date: _____

Find $\lim a_n$. Not 0: the series diverges. 0: the test says nothing.

1. $\sum_{n=1}^{\infty} \frac{-n-3}{n^2+3}$

$\lim_{n\to\infty} \frac{-n-3}{n^2+3} = \lim_{n\to\infty} \frac{-1/n - 3/n^2}{1+3/n^2}$

$= \frac{0+0}{1+0} = 0$

$\lim a_n = 0$: the test says nothing

2. $\sum_{n=1}^{\infty} \frac{-n}{2n+1}$

3. $\sum_{n=1}^{\infty} \frac{2n}{n^2+2}$

4. $\sum_{n=1}^{\infty} \frac{-n^2+3}{2n^2-1}$

5. $\sum_{n=1}^{\infty} \frac{-n}{2n^2+2}$

6. $\sum_{n=1}^{\infty} \frac{-n+1}{2n+3}$

Series — Geometric and telescoping

Name: _____ Date: _____

Does each series converge? If it does, find its sum.

1. $\sum_{n=1}^{\infty} \frac{4}{(n+2)(n+4)}$

$\frac{4}{(n+2)(n+4)} = 2\left(\frac{1}{n+2} - \frac{1}{n+4}\right)$

$S_n = 2\left(\frac{1}{3} + \frac{1}{4} - \frac{1}{n+3} - \frac{1}{n+4}\right)$

$S = \lim_{n\to\infty} S_n = 2\left(\frac{1}{3} + \frac{1}{4}\right) = \frac{7}{6}$

2. $\sum_{n=0}^{\infty} -3 \cdot 4^n$

3. $\sum_{n=1}^{\infty} -2\left(-\frac{1}{4}\right)^{n-1}$

4. $\sum_{n=1}^{\infty} -2(-4)^{n-1}$

5. $\sum_{n=1}^{\infty} -\left(-\frac{1}{4}\right)^{n-1}$

6. $\sum_{n=1}^{\infty} \frac{4}{(n+1)(n+3)}$

Series — P-series and comparison

Name: _____ Date: _____

Does each series converge? Compare it with a p-series, $\sum 1/n^p$.

1. $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}$

   A number times a p-series: $\frac{3}{n\sqrt{n}} = 3 \cdot \frac{1}{n\sqrt{n}}$

   $p = \frac{3}{2} > 1$, so the series converges

2. $\sum_{n=1}^{\infty} \frac{n+1}{2n^2+1}$

3. $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}$

4. $\sum_{n=1}^{\infty} \frac{2}{n^2+2}$

5. $\sum_{n=1}^{\infty} \frac{1}{n}$

6. $\sum_{n=1}^{\infty} \frac{n+1}{2n^3+1}$

Series — Ratio test

Name: _____ Date: _____

Does each series converge? Use the ratio test: $L = \lim |a_{n+1} \div a_n|$.

1. $\sum_{n=1}^{\infty} \frac{n^2}{4^n}$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{4^{n+1}} \cdot \frac{4^n}{n^2}$$
$$= \frac{(n+1)^2}{n^2} \cdot \frac{1}{4}$$
$$L = \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 1 \cdot \frac{1}{4} = \frac{1}{4}$$

$L < 1$, so the series converges

2. $\sum_{n=1}^{\infty} \frac{n}{3^n}$

3. $\sum_{n=1}^{\infty} \frac{3^n}{n}$

4. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

5. $\sum_{n=1}^{\infty} \frac{4^n}{n^2}$

6. $\sum_{n=1}^{\infty} \frac{n!}{3^n}$

Series — Alternating

Name: _____ Date: _____

Does each series converge absolutely, converge conditionally, or diverge?

1. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2\sqrt{n}}$

   $\sum |a_n| = \sum \frac{1}{n^2\sqrt{n}}$: $p = \frac{5}{2} > 1$, converges

   The series converges absolutely

2. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n}$

3. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{\sqrt{n}}$

4. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+3}$

5. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{2n+1}$

6. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n\sqrt{n}}$

Power Pack · Series — Answer key

The nth-term test

1. $\infty\sum n = 1\frac{-n-3}{n^2+3}$ → the test says nothing
2. $\infty\sum n = 1\frac{-n}{2n+1}$ → diverges
3. $\infty\sum n = 1\frac{2n}{n^2+2}$ → the test says nothing
4. $\infty\sum n = 1\frac{-n^2+3}{2n^2-1}$ → diverges
5. $\infty\sum n = 1\frac{-n}{2n^2+2}$ → the test says nothing
6. $\infty\sum n = 1\frac{-n+1}{2n+3}$ → diverges

Geometric and telescoping

1. $\infty\sum n = 1\frac{4}{(n+2)(n+4)}$ → converges, $S = \frac{7}{6}$
2. $\infty\sum n = 0-3\cdot 4^n$ → diverges
3. $\infty\sum n = 1-2(-\frac{1}{4})^{n-1}$ → converges, $S = \frac{-8}{5}$
4. $\infty\sum n = 1-2(-4)^{n-1}$ → diverges
5. $\infty\sum n = 1-(-\frac{1}{4})^{n-1}$ → converges, $S = \frac{-4}{5}$
6. $\infty\sum n = 1\frac{4}{(n+1)(n+3)}$ → converges, $S = \frac{5}{3}$

p-series and comparison

1. $\infty\sum n = 1\frac{3}{n\sqrt{n}}$ → converges
2. $\infty\sum n = 1\frac{n+1}{2n^2+1}$ → diverges
3. $\infty\sum n = 1\frac{2}{\sqrt{n}}$ → diverges
4. $\infty\sum n = 1\frac{2}{n^2+2}$ → converges
5. $\infty\sum n = 1\frac{1}{n}$ → diverges
6. $\infty\sum n = 1\frac{n+1}{2n^3+1}$ → converges

The ratio test

1. $\infty\sum n = 1\frac{n^2}{4^n}$ → converges
2. $\infty\sum n = 1\frac{n}{3^n}$ → converges
3. $\infty\sum n = 1\frac{3^n}{n}$ → diverges
4. $\infty\sum n = 1\frac{n^2}{2^n}$ → converges
5. $\infty\sum n = 1\frac{4^n}{n^2}$ → diverges
6. $\infty\sum n = 1\frac{n!}{3^n}$ → diverges

Power Pack · Series — Answer key

Alternating series

1. $\infty\sum n = 1(-1)^{n+1}\frac{1}{n^2\sqrt{n}}$ → converges absolutely

2. $\infty\sum n = 1(-1)^{n+1}\frac{2}{n}$ → converges conditionally

3. $\infty\sum n = 1(-1)^{n+1}\frac{3}{\sqrt{n}}$ → converges conditionally

4. $\infty\sum n = 1(-1)^{n+1}\frac{n}{n+3}$ → diverges

5. $\infty\sum n = 1(-1)^{n+1}\frac{2n}{2n+1}$ → diverges

6. $\infty\sum n = 1(-1)^{n+1}\frac{3}{n\sqrt{n}}$ → converges absolutely