Grade 8 Geometry Workbook

7 lessons · printable · answer key at the back

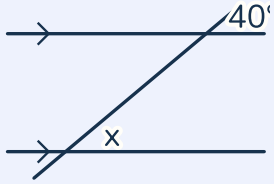
Name _____ Date _____

Contents

1-1

Angles in Parallel Lines

WORKED EXAMPLE



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1. Angles in parallel lines

The two lines are parallel. Find x.

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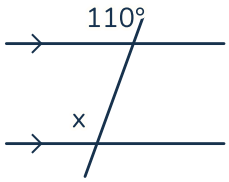

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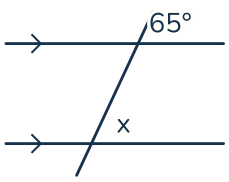

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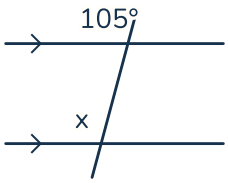

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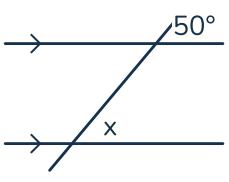

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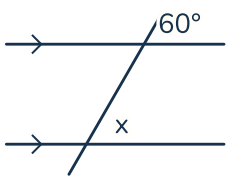

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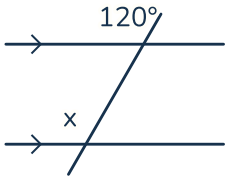

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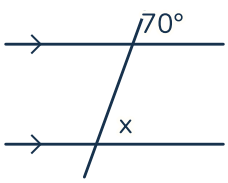

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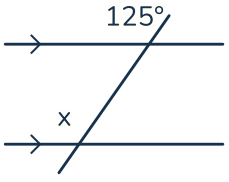

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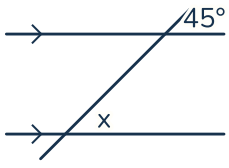

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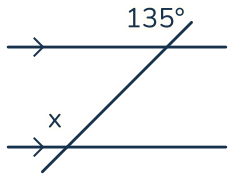

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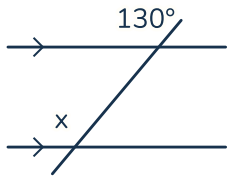

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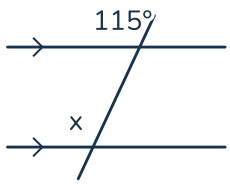

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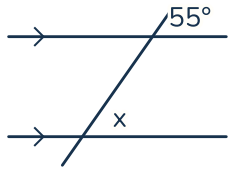

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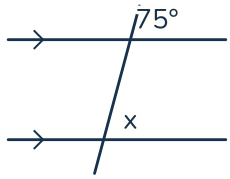


2. Rate of change from a table

Each table is a straight line. Find what each one asks for.

15

x	9	10	11	12
y	43	42	41	40

rate → ___

16

x	1	2	3	4
y	15	17	19	21

rate → ___

17

x	4	8	12	16
y	56	100	144	188

rate → ___

18

x	6	7	8	9
y	93	102	111	120

rate → ___

19

x	7	8	9	10
y	27	23	19	15

rate → ___

20

x	5	8	11	14
y	29	44	59	74

rate → ___

21

x	3	4	5	6
y	33	40	47	54

rate → ___

22

x	2	3	4	5
y	27	22	17	12

rate → ___

23

x	9	10	11	12
y	25	22	19	16

rate → ___

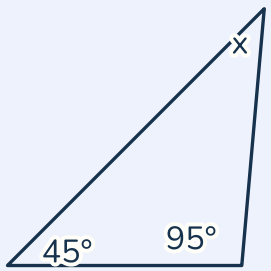
24

x	6	11	16	21
y	46	71	96	121

rate → ___

Angles in a Triangle

WORKED EXAMPLE

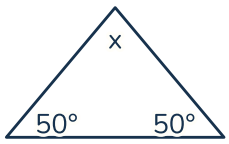


40 degrees

3. The third angle of a triangle

Find the angle marked x. Give each answer in degrees.

1



_____ degrees

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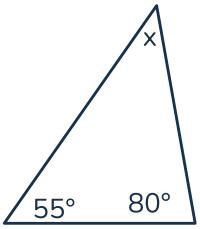

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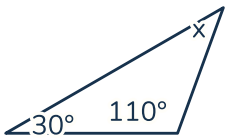

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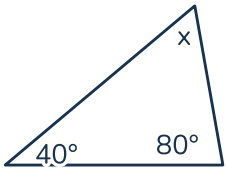

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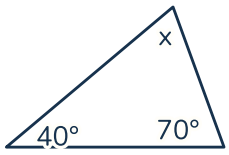

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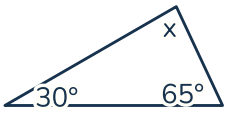

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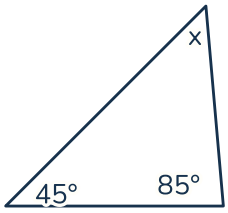

_____ degrees

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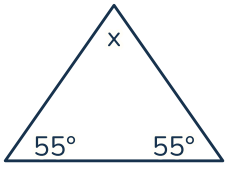

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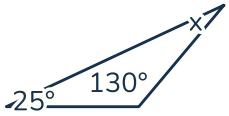

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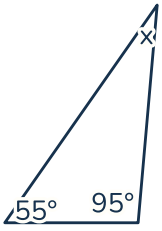

_____ degrees

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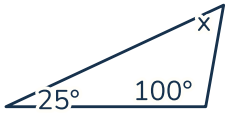
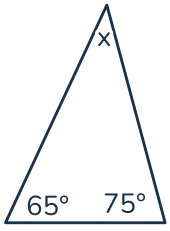


_____ degrees

12



_____ degrees

4. Which rate is greater?

Which one is greater, A or B? Work out each rate first. Circle one.

13 A **y = 2x, y pages and x days**
B **42 pages in 7 days**
A / B

14 A **84 km in 7 hours**
B **y = 6x, y km and x hours**
A / B

15 A **y = 11x, y km and x hours**
B **28 km in 7 hours**
A / B

4. Which rate is greater? (continued)

Which one is greater, A or B? Work out each rate first. Circle one.

16 A 56 dollars in 7 weeks
B y = 14x, y dollars and x weeks
A / B

17 A y = 9x, y km and x hours
B 64 km in 8 hours
A / B

18 A y = 11x, y words and x minutes
B 20 words in 5 minutes
A / B

19 A 40 meters in 5 seconds
B y = 11x, y meters and x seconds
A / B

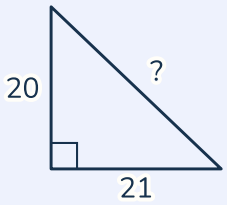
20 A 88 km in 8 hours
B y = 8x, y km and x hours
A / B

21 A y = 2x, y words and x minutes
B 60 words in 5 minutes
A / B

22 A y = 8x, y km and x hours
B 12 km in 6 hours
A / B

Pythagoras

WORKED EXAMPLE

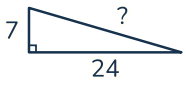


29 cm

5. Pythagorean theorem

Find the missing side. All lengths are in centimeters.

1



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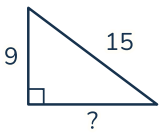

_____ cm

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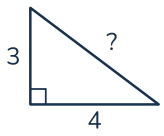

_____ cm

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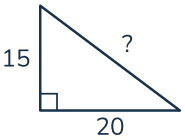

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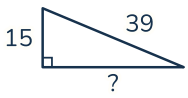

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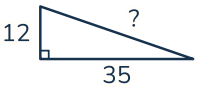

_____ cm

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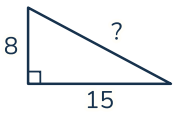

_____ cm

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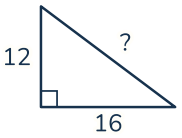

_____ cm

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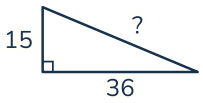

_____ cm

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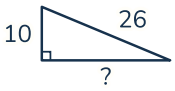

_____ cm

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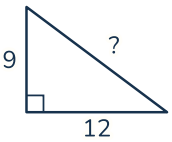

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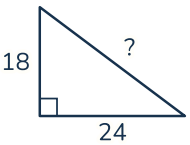

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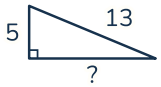

_____ cm

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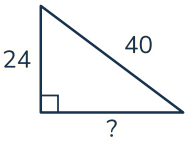


_____ cm

6. Linear or nonlinear

Is each equation linear or nonlinear? Circle one.

15 $y = 24x - 16$ Linear / Nonlinear

16 $y = 4x^2 + 19$ Linear / Nonlinear

17 $y = x/15 + 17$ Linear / Nonlinear

18 $y = x/14 + 6$ Linear / Nonlinear

19 $y = 79/x$ Linear / Nonlinear

20 $y = 8x^2 + 2$ Linear / Nonlinear

21 $y = 24x + 34$ Linear / Nonlinear

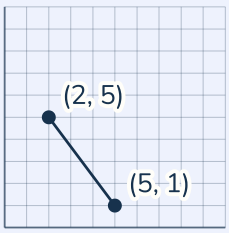
22 $y = x/3 + 20$ Linear / Nonlinear

23 $y = 2x^2 + 3$ Linear / Nonlinear

24 $y = 2/x$ Linear / Nonlinear

Distance Between Points

WORKED EXAMPLE

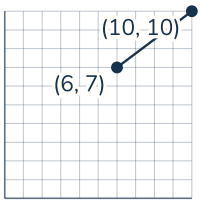


5 units

7. Distance between two points

Find the distance between the two points.

1



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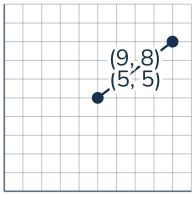

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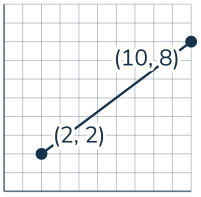

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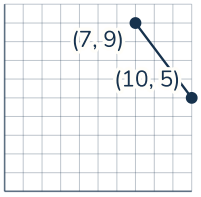

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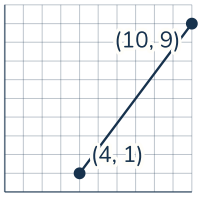

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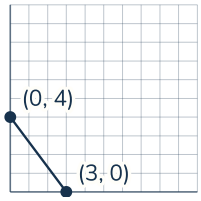

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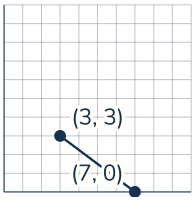

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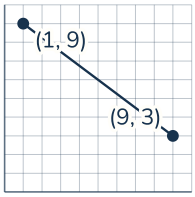

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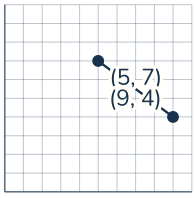
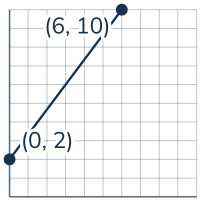


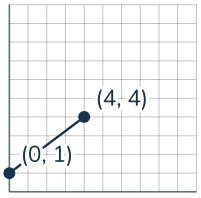
_____ units

10



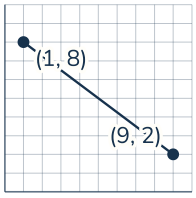
_____ units

11



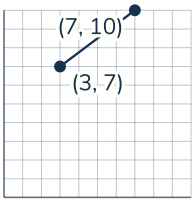
_____ units

12



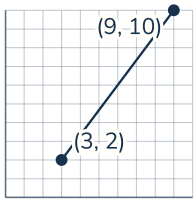
_____ units

13



_____ units

14



_____ units

8. Is it a function?

Does each table show a function? Show how you know.

15

x	13	1	16	12
y	60	31	34	6

16

x	17	21	23	15
y	40	56	56	23

17

x	16	15	15	0
y	48	44	42	59

18

x	9	13	19	9
y	38	4	24	34

19

x	12	3	17	5
y	58	43	58	13

20

x	6	6	16	20
y	43	40	56	25

21

x	17	7	12	15
y	59	59	52	32

22

x	14	24	18	18
y	49	60	36	19

23

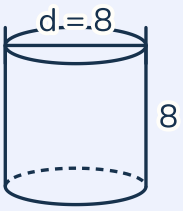
x	3	4	2	10
y	37	24	24	30

24

x	7	11	7	23
y	8	54	42	45

Volume of Cylinders

WORKED EXAMPLE

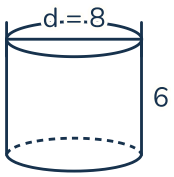


401.92

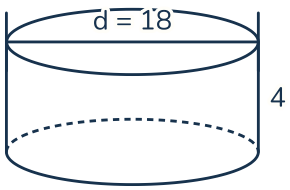
9. Volume of a cylinder

Find the volume of each cylinder. Use π = 3.14.

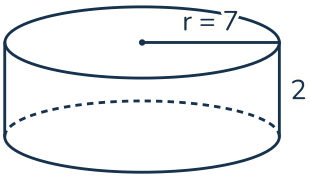
1



2



3



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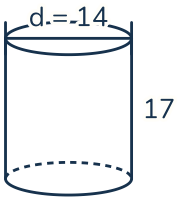
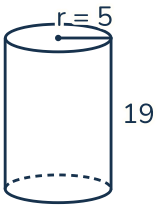


5



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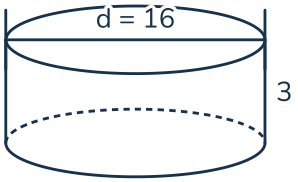

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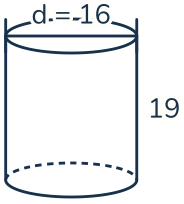


10. Laws of exponents

Write each product as one power. Give the exponent n.

8 $12^8 \times 12^4 = 12^n$, n = ___

9 $6^9 \times 6^{10} = 6^n$, n = ___

10 $12^4 \times 12^2 = 12^n$, n = ___

11 $9^8 \times 9^5 = 9^n$, n = ___

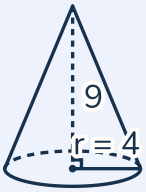
12 $12^{12} \times 12^6 = 12^n$, n = ___

13 $3^2 \times 3^{14} = 3^n$, n = ___

14 $2^9 \times 2^9 = 2^n$, n = ___

Volume of Cones

WORKED EXAMPLE

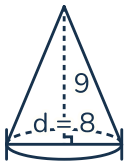


150.72

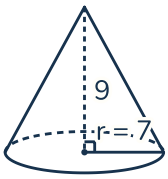
11. Volume of a cone

Find the volume of each cone. Use π = 3.14.

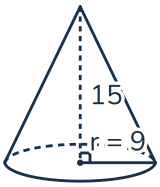
1



2



3



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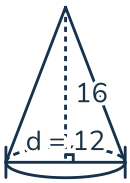

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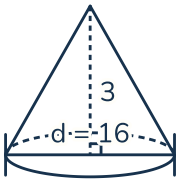

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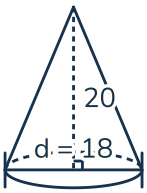

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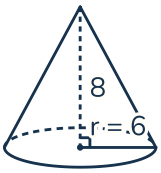


12. Square and cube roots

Find each root. Check the symbol before you answer.

8 $\sqrt{225} =$ ___

9 $\sqrt{484} =$ ___

10 $\sqrt[3]{125} =$ ___

11 $\sqrt{441} =$ ___

12 $\sqrt{49} =$ ___

13 $\sqrt{841} =$ ___

14 $\sqrt{676} =$ ___

Volume of Spheres

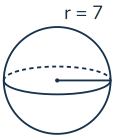
WORKED EXAMPLE

523.33

13. Volume of a sphere

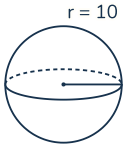
Find the volume of each sphere. Use π = 3.14. Round to the hundredth.

1



2

3



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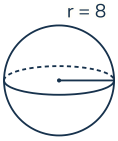

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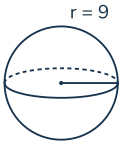

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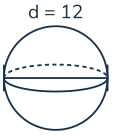

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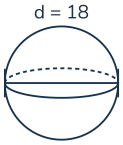

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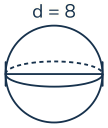

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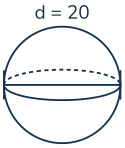

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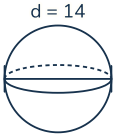

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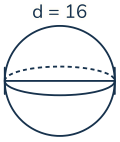
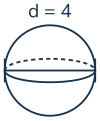


12

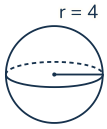
13

14



15

16



14. Between which whole numbers

Each root lies between two whole numbers. Write the smaller one.

17 $\sqrt{227} > \_\_\_$

18 $\sqrt{128} > \_\_\_$

19 $\sqrt{450} > \_\_\_$

20 $\sqrt{291} > \_\_\_$

21 $\sqrt{373} > \_\_\_$

22 $\sqrt{286} > \_\_\_$

23 $\sqrt{487} > \_\_\_$

24 $\sqrt{108} > \_\_\_$

25 $\sqrt{394} > \_\_\_$

26 $\sqrt{191} > \_\_\_$

Answer Key (1 of 3)

Angles in Parallel Lines

Angles in parallel lines

1. corresponding, 110°: 110°
2. corresponding, 65°: 65°
3. corresponding, 105°: 105°
4. corresponding, 50°: 50°
5. corresponding, 60°: 60°
6. corresponding, 120°: 120°
7. corresponding, 70°: 70°
8. corresponding, 125°: 125°
9. corresponding, 45°: 45°
10. corresponding, 135°: 135°
11. corresponding, 130°: 130°
12. corresponding, 115°: 115°
13. corresponding, 55°: 55°
14. corresponding, 75°: 75°

Rate of change from a table

15. x: 9, 10, 11, 12 y: 43, 42, 41, 40 → −1 (y moves −1 while x moves 1, and −1 ÷ 1 = −1)
16. x: 1, 2, 3, 4 y: 15, 17, 19, 21 → 2 (y moves 2 while x moves 1, and 2 ÷ 1 = 2)
17. x: 4, 8, 12, 16 y: 56, 100, 144, 188 → 11 (y moves 44 while x moves 4, and 44 ÷ 4 = 11)
18. x: 6, 7, 8, 9 y: 93, 102, 111, 120 → 9 (y moves 9 while x moves 1, and 9 ÷ 1 = 9)
19. x: 7, 8, 9, 10 y: 27, 23, 19, 15 → −4 (y moves −4 while x moves 1, and −4 ÷ 1 = −4)
20. x: 5, 8, 11, 14 y: 29, 44, 59, 74 → 5 (y moves 15 while x moves 3, and 15 ÷ 3 = 5)
21. x: 3, 4, 5, 6 y: 33, 40, 47, 54 → 7 (y moves 7 while x moves 1, and 7 ÷ 1 = 7)
22. x: 2, 3, 4, 5 y: 27, 22, 17, 12 → −5 (y moves −5 while x moves 1, and −5 ÷ 1 = −5)
23. x: 9, 10, 11, 12 y: 25, 22, 19, 16 → −3 (y moves −3 while x moves 1, and −3 ÷ 1 = −3)
24. x: 6, 11, 16, 21 y: 46, 71, 96, 121 → 5 (y moves 25 while x moves 5, and 25 ÷ 5 = 5)

Angles in a Triangle

The third angle of a triangle

1. 50° + 50° = 100°: 80°
2. 55° + 80° = 135°: 45°
3. 30° + 110° = 140°: 40°
4. 40° + 80° = 120°: 60°
5. 40° + 70° = 110°: 70°
6. 30° + 65° = 95°: 85°
7. 45° + 85° = 130°: 50°
8. 55° + 55° = 110°: 70°
9. 25° + 130° = 155°: 25°
10. 55° + 95° = 150°: 30°

Angles in a Triangle (continued)

The third angle of a triangle (continued)

11. 25° + 100° = 125°: 55°
12. 65° + 75° = 140°: 40°

Which rate is greater?

13. A y = 2x, y pages and x days B 42 pages in 7 days → B (A is 2 and B is 6 per day)
14. A 84 km in 7 hours B y = 6x, y km and x hours → A (A is 12 and B is 6 per hour)
15. A y = 11x, y km and x hours B 28 km in 7 hours → A (A is 11 and B is 4 per hour)
16. A 56 dollars in 7 weeks B y = 14x, y dollars and x weeks → B (A is 8 and B is 14 per week)
17. A y = 9x, y km and x hours B 64 km in 8 hours → A (A is 9 and B is 8 per hour)
18. A y = 11x, y words and x minutes B 20 words in 5 minutes → A (A is 11 and B is 4 per minute)
19. A 40 meters in 5 seconds B y = 11x, y meters and x seconds → B (A is 8 and B is 11 per second)
20. A 88 km in 8 hours B y = 8x, y km and x hours → A (A is 11 and B is 8 per hour)
21. A y = 2x, y words and x minutes B 60 words in 5 minutes → B (A is 2 and B is 12 per minute)
22. A y = 8x, y km and x hours B 12 km in 6 hours → A (A is 8 and B is 2 per hour)

Pythagoras

Pythagorean theorem

1. 7, 24, 25 — hypotenuse: 25 cm
2. 9, 12, 15 — leg: 12 cm
3. 3, 4, 5 — hypotenuse: 5 cm
4. 15, 20, 25 — hypotenuse: 25 cm
5. 15, 36, 39 — leg: 36 cm
6. 12, 35, 37 — hypotenuse: 37 cm
7. 8, 15, 17 — hypotenuse: 17 cm
8. 12, 16, 20 — hypotenuse: 20 cm
9. 15, 36, 39 — hypotenuse: 39 cm
10. 10, 24, 26 — leg: 24 cm
11. 9, 12, 15 — hypotenuse: 15 cm
12. 18, 24, 30 — hypotenuse: 30 cm
13. 5, 12, 13 — leg: 12 cm
14. 24, 32, 40 — leg: 32 cm

Linear or nonlinear

15. y = 24x − 16: Linear — x is only multiplied by a number and added to, so equal steps in x give equal steps in y
16. $y = 4x^2 + 19$: Nonlinear — x is squared, so the steps in y grow as x grows
17. y = x/15 + 17: Linear — dividing by 15 is multiplying by a number, so the steps in y stay equal

Answer Key (2 of 3)

Pythagoras (continued)

Linear or nonlinear (continued)

18. $y = x/14 + 6$: Linear — dividing by 14 is multiplying by a number, so the steps in y stay equal
19. $y = 79/x$: Nonlinear — x is underneath, so doubling x halves y instead of adding the same amount again
20. $y = 8x^2 + 2$: Nonlinear — x is squared, so the steps in y grow as x grows
21. $y = 24x + 34$: Linear — x is only multiplied by a number and added to, so equal steps in x give equal steps in y
22. $y = x/3 + 20$: Linear — dividing by 3 is multiplying by a number, so the steps in y stay equal
23. $y = 2x^2 + 3$: Nonlinear — x is squared, so the steps in y grow as x grows
24. $y = 2/x$: Nonlinear — x is underneath, so doubling x halves y instead of adding the same amount again

Distance Between Points

Distance between two points

1. (6, 7) to (10, 10): 5 units
2. (5, 5) to (9, 8): 5 units
3. (2, 2) to (10, 8): 10 units
4. (7, 9) to (10, 5): 5 units
5. (4, 1) to (10, 9): 10 units
6. (0, 4) to (3, 0): 5 units
7. (3, 3) to (7, 0): 5 units
8. (1, 9) to (9, 3): 10 units
9. (5, 7) to (9, 4): 5 units
10. (0, 2) to (6, 10): 10 units
11. (0, 1) to (4, 4): 5 units
12. (1, 8) to (9, 2): 10 units
13. (3, 7) to (7, 10): 5 units
14. (3, 2) to (9, 10): 10 units

Is it a function?

15. (13, 60) (1, 31) (16, 34) (12, 6): Yes — every input appears once, so no input can go to two places
16. (17, 40) (21, 56) (23, 56) (15, 23): Yes — every input appears once — 56 turns up twice, but that is an output, and two inputs may land on the same one
17. (16, 48) (15, 44) (15, 42) (0, 59): No — the input 15 goes to 44 and also to 42, and one input may only go to one place
18. (9, 38) (13, 4) (19, 24) (9, 34): No — the input 9 goes to 38 and also to 34, and one input may only go to one place
19. (12, 58) (3, 43) (17, 58) (5, 13): Yes — every input appears once — 58 turns up twice, but that is an output, and two inputs may land on the same one
20. (6, 43) (6, 40) (16, 56) (20, 25): No — the input 6 goes to 43 and also to 40, and one input may only go to one place

Distance Between Points (continued)

Is it a function? (continued)

21. (17, 59) (7, 59) (12, 52) (15, 32): Yes — every input appears once — 59 turns up twice, but that is an output, and two inputs may land on the same one
22. (14, 49) (24, 60) (18, 36) (18, 19): No — the input 18 goes to 36 and also to 19, and one input may only go to one place
23. (3, 37) (4, 24) (2, 24) (10, 30): Yes — every input appears once — 24 turns up twice, but that is an output, and two inputs may land on the same one
24. (7, 8) (11, 54) (7, 42) (23, 45): No — the input 7 goes to 8 and also to 42, and one input may only go to one place

Volume of Cylinders

Volume of a cylinder

1. d = 8, h = 6: 301.44 cm^3
2. d = 18, h = 4: 1017.36 cm^3
3. r = 7, h = 2: 307.72 cm^3
4. d = 14, h = 17: 2615.62 cm^3
5. r = 5, h = 19: 1491.5 cm^3
6. d = 16, h = 3: 602.88 cm^3
7. d = 16, h = 19: 3818.24 cm^3

Laws of exponents

8. $12^8 \times 12^4 = 12^{12}$
9. $6^9 \times 6^{10} = 6^{19}$
10. $12^4 \times 12^2 = 12^6$
11. $9^8 \times 9^5 = 9^{13}$
12. $12^{12} \times 12^6 = 12^{18}$
13. $3^2 \times 3^{14} = 3^{16}$
14. $2^9 \times 2^9 = 2^{18}$

Volume of Cones

Volume of a cone

1. d = 8, h = 9: 150.72 cm^3
2. r = 7, h = 9: 461.58 cm^3
3. r = 9, h = 15: 1271.7 cm^3
4. d = 12, h = 16: 602.88 cm^3
5. d = 16, h = 3: 200.96 cm^3
6. d = 18, h = 20: 1695.6 cm^3
7. r = 6, h = 8: 301.44 cm^3

Square and cube roots

8. $\sqrt{225} = 15$
9. $\sqrt{484} = 22$
10. $\sqrt[3]{125} = 5$
11. $\sqrt{441} = 21$
12. $\sqrt{49} = 7$
13. $\sqrt{841} = 29$
14. $\sqrt{676} = 26$

Answer Key (3 of 3)

Volume of Spheres

Volume of a sphere

1. r = 7: 1436.03 cm³
2. d = 6: 113.04 cm³
3. r = 10: 4186.67 cm³
4. r = 8: 2143.57 cm³
5. r = 9: 3052.08 cm³
6. d = 12: 904.32 cm³
7. d = 18: 3052.08 cm³
8. d = 8: 267.95 cm³
9. d = 20: 4186.67 cm³
10. d = 14: 1436.03 cm³
11. d = 16: 2143.57 cm³
12. r = 2: 33.49 cm³
13. r = 6: 904.32 cm³
14. d = 4: 33.49 cm³
15. r = 3: 113.04 cm³
16. r = 4: 267.95 cm³

Between which whole numbers

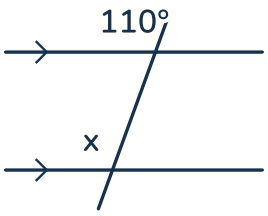
17. √227 is between 15 and 16: 15
18. √128 is between 11 and 12: 11
19. √450 is between 21 and 22: 21
20. √291 is between 17 and 18: 17
21. √373 is between 19 and 20: 19
22. √286 is between 16 and 17: 16
23. √487 is between 22 and 23: 22
24. √108 is between 10 and 11: 10
25. √394 is between 19 and 20: 19
26. √191 is between 13 and 14: 13

Solutions (1 of 16)

Angles in Parallel Lines

Angles in parallel lines

1.



___

$110° = x$

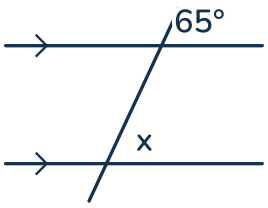
$x = 110°$

Start with the arrows: they say the two lines are parallel. Without that, none of the angle facts apply and this picture would not have an answer.

Now find where the two marked angles sit. They are in the same corner at each crossing — both above their line and both on the same side of the slanted one. That makes them corresponding angles.

Corresponding angles are equal, so x is the same as the angle already marked. x = 110°. Nothing to work out — but only once you have seen which pair it is.

2.



___

$65° = x$

$x = 65°$

Start with the arrows: they say the two lines are parallel. Without that, none of the angle facts apply and this picture would not have an answer.

Now find where the two marked angles sit. They are in the same corner at each crossing — both above their line and both on the same side of the slanted one. That makes them corresponding angles.

Corresponding angles are equal, so x is the same as the angle already marked. x = 65°. Nothing to work out — but only once you have seen which pair it is.

Solutions (2 of 16)

Angles in Parallel Lines (continued)

Rate of change from a table

15.

x	9	10	11	12
y	43	42	41	40

rate → ___

rate of change = −1

Rate of change means how much y moves for every ONE x. So it is a division, and the two things being divided are both differences: how far y went, over how far x went.

Take both rows of steps. x moves 1, 1, 1 — the same each time — and y moves −1, −1, −1. They are steady, so this table really does have one rate rather than a different one in every gap.

Written out, that is one step of each: y moved −1 while x moved 1. Those two numbers are the whole of the division below.

Now divide: −1 ÷ 1 = −1. Dividing by one changes nothing, which is exactly why it is worth writing down — on a table whose x steps by more, it will not.

So the rate of change is −1: y moves −1 while x moves 1, and −1 ÷ 1 = −1.

Solutions (3 of 16)

Angles in Parallel Lines (continued)

Rate of change from a table (continued)

16.

x	1	2	3	4
y	15	17	19	21

rate → ___

rate of change = 2

Rate of change means how much y moves for every ONE x. So it is a division, and the two things being divided are both differences: how far y went, over how far x went.

Take both rows of steps. x moves 1, 1, 1 — the same each time — and y moves 2, 2, 2. They are steady, so this table really does have one rate rather than a different one in every gap.

Written out, that is one step of each: y moved 2 while x moved 1. Those two numbers are the whole of the division below.

Now divide: 2 ÷ 1 = 2. Dividing by one changes nothing, which is exactly why it is worth writing down — on a table whose x steps by more, it will not.

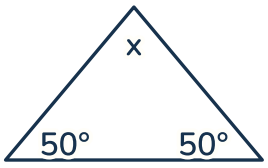
So the rate of change is 2: y moves 2 while x moves 1, and 2 ÷ 1 = 2.

Solutions (4 of 16)

Angles in a Triangle

The third angle of a triangle

1.



___ degrees

$$50 + 50 = 100$$

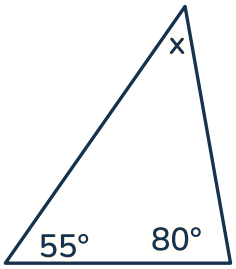
$$180 - 100 = \mathbf{80°}$$

Every triangle's three angles add to 180 — and that is not a number to take on trust. Tear the three corners off any triangle you like and they fit together into a straight line, every time.

Two of them are given: 50 and 50, which is 100 between them.

So the third takes up what is left of the 180: 180 – 100 = 80°.

2.



___ degrees

$$55 + 80 = 135$$

$$180 - 135 = \mathbf{45°}$$

Every triangle's three angles add to 180 — and that is not a number to take on trust. Tear the three corners off any triangle you like and they fit together into a straight line, every time.

Two of them are given: 55 and 80, which is 135 between them.

So the third takes up what is left of the 180: 180 – 135 = 45°.

Solutions (5 of 16)

Angles in a Triangle (continued)

Which rate is greater?

13. **A y = 2x, y pages and x days**
B 42 pages in 7 days

A / B

A 2 **B 6**

greater: **B**

These two are written in different ways, so there is nothing to compare yet — the numbers on the page are not the same kind of number. Turn each one into a rate PER ONE day first, and do not look at which looks bigger until you have.

A: the number in front of x, which is already a rate per one day: 2. So A is 2 pages per day.

B: 42 ÷ 7 = 6. So B is 6 pages per day.

Now they can be compared: A is 2 and B is 6 per day, so B is greater.

14. **A 84 km in 7 hours**
B y = 6x, y km and x hours

A / B

A 12 B 6

greater: **A**

These two are written in different ways, so there is nothing to compare yet — the numbers on the page are not the same kind of number. Turn each one into a rate PER ONE hour first, and do not look at which looks bigger until you have.

A: 84 ÷ 7 = 12. So A is 12 km per hour.

B: the number in front of x, which is already a rate per one hour: 6. So B is 6 km per hour.

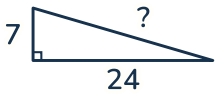
Now they can be compared: A is 12 and B is 6 per hour, so A is greater.

Solutions (6 of 16)

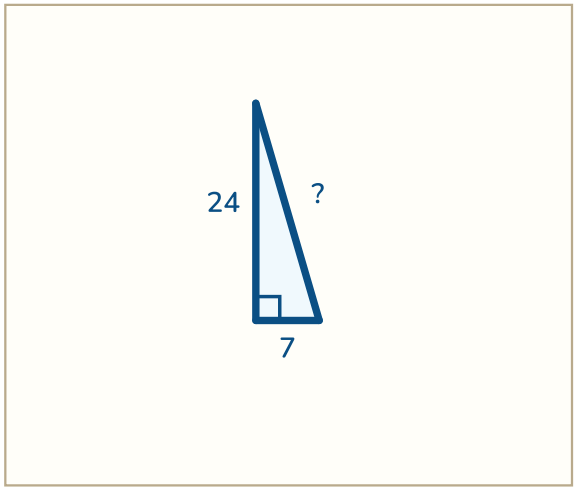
Pythagoras

Pythagorean theorem

1.



___ cm



The hypotenuse is the side OPPOSITE the right angle — never one of the two that make the corner. It is also always the longest, so you can check yourself twice.

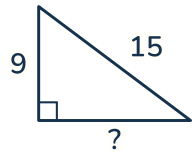
Here the hypotenuse is the missing one, so the two given sides are both legs: 7 and 24.

Square the two you have: $7^2 = 49$ and $24^2 = 576$.

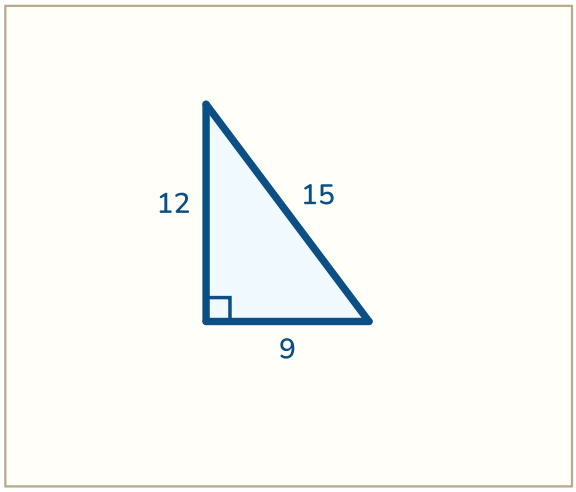
The two squares ADD to the square on the hypotenuse: 49 + 576 = 625.

That is the square, not the side. Take the square root: $\sqrt{625} = 25$ cm.

2.



___ cm



The hypotenuse is the side OPPOSITE the right angle — never one of the two that make the corner. It is also always the longest, so you can check yourself twice.

Here the hypotenuse is given — 15 cm — and a LEG is missing. That is the case where the sum is a subtraction, not an addition.

Square the two you have: $15^2 = 225$ and $9^2 = 81$.

The leg's square is what is LEFT when you take the other leg's square off the hypotenuse's: 225 – 81 = 144.

That is the square, not the side. Take the square root: $\sqrt{144} = 12$ cm.

Solutions (7 of 16)

Pythagoras (continued)

Linear or nonlinear

15. **$y = 24x - 16$** **Linear** / **Nonlinear**

Linear or nonlinear? **Linear**

Linear means the graph is a straight line, and one thing decides it: what is being DONE to x. Multiplied by a number and added to — that is a line. Anything else is not.

Look at what happens to x: it is multiplied by a number, and a number is added or taken away. Nothing else is done to it.

That is exactly the mx + b shape — x is only multiplied by a number and added to, so equal steps in x give equal steps in y. It is a line.

16. **$y = 4x^2 + 19$** **Linear** / **Nonlinear**

Linear or nonlinear?

Nonlinear

Linear means the graph is a straight line, and one thing decides it: what is being DONE to x. Multiplied by a number and added to — that is a line. Anything else is not.

Look at what x is carrying: it is squared. Squaring is not multiplying by a fixed number — going from x = 1 to 2 to 3 gives 1, 4, 9, and those gaps are 3 then 5.

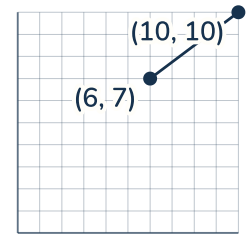
So equal steps in x do NOT give equal steps in y: x is squared, so the steps in y grow as x grows. The graph curves, so it is not a line.

Solutions (8 of 16)

Distance Between Points

Distance between two points

1.



___ units

$4^2 + 3^2 = 25$

distance = **5 units**

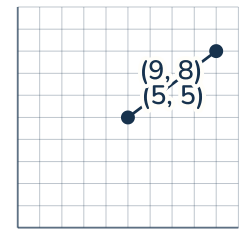
There is no new formula here. Draw the horizontal and vertical steps between the two points and they make a right triangle — the line you were given is its longest side.

Across: 10 − 6 = 4, so that side is 4 long. Up: 10 − 7 = 3, so that side is 3. A length has no sign, so take both as positive.

Now it is the theorem you already know: $4^2 + 3^2 = 16 + 9 = 25$.

The distance is the square root of that: $\sqrt{25} = 5$. So the two points are 5 units apart.

2.



___ units

$4^2 + 3^2 = 25$

distance = **5 units**

There is no new formula here. Draw the horizontal and vertical steps between the two points and they make a right triangle — the line you were given is its longest side.

Across: 9 − 5 = 4, so that side is 4 long. Up: 8 − 5 = 3, so that side is 3. A length has no sign, so take both as positive.

Now it is the theorem you already know: $4^2 + 3^2 = 16 + 9 = 25$.

The distance is the square root of that: $\sqrt{25} = 5$. So the two points are 5 units apart.

Distance Between Points (continued)

Is it a function?

15.

x	13	1	16	12
y	60	31	34	6

inputs 13, 1, 16, 12 — no repeat

A function

A function has one rule: every input gets exactly ONE output. So there is only one column to check — the inputs, which are the top row of the table.

Pull the inputs out on their own and look at them together: 13, 1, 16, 12. Nothing about the outputs matters yet.

Now the check: every input appears once, so no input can go to two places.

Every input has exactly one output, so this relation is a function.

16.

x	17	21	23	15
y	40	56	56	23

inputs 17, 21, 23, 15 — no repeat

A function

A function has one rule: every input gets exactly ONE output. So there is only one column to check — the inputs, which are the top row of the table.

Pull the inputs out on their own and look at them together: 17, 21, 23, 15. Nothing about the outputs matters yet.

Now the check: every input appears once — 56 turns up twice, but that is an output, and two inputs may land on the same one.

Two different inputs are ALLOWED to land on the same output — the way both 2 and −2 square to 4, or every hour of a flat day has the same temperature. It is the input column that may not repeat, and it is the only one.

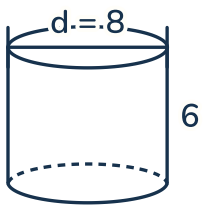
Every input has exactly one output, so this relation is a function.

Solutions (10 of 16)

Volume of Cylinders

Volume of a cylinder

1.



base: 50.24

50.24 × 6 = **301.44**

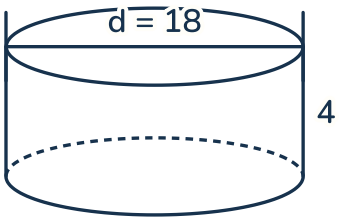
The 8 cm goes right across, so it is the diameter. Halve it first: the radius is 4. Every one of these formulas SQUARES the radius, so getting this wrong is out by a factor of four, not two.

Same idea as a box: the area of the base times how tall it is. Here the base is a circle, and that is the only difference.

The circle's area is π × r × r = 3.14 × 4 × 4 = 50.24 square cm.

Times the height: 50.24 × 6 = 301.44 cubic cm.

2.



base: 254.34

254.34 × 4 = **1017.36**

The 18 cm goes right across, so it is the diameter. Halve it first: the radius is 9. Every one of these formulas SQUARES the radius, so getting this wrong is out by a factor of four, not two.

Same idea as a box: the area of the base times how tall it is. Here the base is a circle, and that is the only difference.

The circle's area is π × r × r = 3.14 × 9 × 9 = 254.34 square cm.

Times the height: 254.34 × 4 = 1017.36 cubic cm.

Solutions (11 of 16)

Volume of Cylinders (continued)

Laws of exponents

8. **$12^8 \times 12^4 = 12^n$, n = ___**

$$12^8 \times 12^4 = 12^{12}$$
$$n = 12$$

12^8 means 8 12s multiplied together, and 12^4 means 4 12s multiplied together. The question multiplies those two lots together.

So altogether that is 8 12s and 4 more — 8 + 4 = 12 of them.

12 12s multiplied together is 12^{12}. So n = 12. That is the shortcut: multiplying powers of the same base ADDS the exponents.

9. **$6^9 \times 6^{10} = 6^n$, n = ___**

$$6^9 \times 6^{10} = 6^{19}$$
$$n = 19$$

6^9 means 9 6s multiplied together, and 6^{10} means 10 6s multiplied together. The question multiplies those two lots together.

So altogether that is 9 6s and 10 more — 9 + 10 = 19 of them.

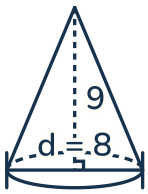
19 6s multiplied together is 6^{19}. So n = 19. That is the shortcut: multiplying powers of the same base ADDS the exponents.

Solutions (12 of 16)

Volume of Cones

Volume of a cone

1.



cylinder: 452.16

3 cones fill 1 cylinder

452.16 ÷ 3 =

150.72 cubic cm

The 8 cm goes right across, so it is the diameter. Halve it first: the radius is 4. Every one of these formulas SQUARES the radius, so getting this wrong is out by a factor of four, not two.

Same idea as a box: the area of the base times how tall it is. Here the base is a circle, and that is the only difference.

The circle's area is π × r × r = 3.14 × 4 × 4 = 50.24 square cm.

Times the height: 50.24 × 9 = 452.16 cubic cm. That is the CYLINDER it fits inside, not the cone yet.

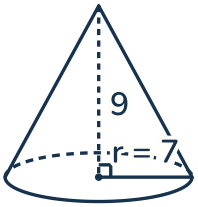
Now the part that gets forgotten. Exactly THREE cones fill the cylinder they sit inside — it is a fact about cones, not a number bolted onto a formula. So take a third: 452.16 ÷ 3 = 150.72 cubic cm.

Solutions (13 of 16)

Volume of Cones (continued)

Volume of a cone (continued)

2.



_____

cylinder: 1384.74

3 cones fill 1 cylinder

1384.74 ÷ 3 =

461.58 cubic cm

Same idea as a box: the area of the base times how tall it is. Here the base is a circle, and that is the only difference.

The circle's area is π × r × r = 3.14 × 7 × 7 = 153.86 square cm.

Times the height: 153.86 × 9 = 1384.74 cubic cm. That is the CYLINDER it fits inside, not the cone yet.

Now the part that gets forgotten. Exactly THREE cones fill the cylinder they sit inside — it is a fact about cones, not a number bolted onto a formula. So take a third: 1384.74 ÷ 3 = 461.58 cubic cm.

Square and cube roots

8. $\sqrt{225}$ = ___

$15 \times 15 = 225$

$\sqrt{225} = 15$

√ asks a question: which number times itself gives 225? It is not an instruction to halve 225 — that is the mistake this symbol collects.

Bracket it first. $10^2 = 100$, which is under 225, and $20^2 = 400$, which is over. So the answer is between 10 and 20.

Now the last digit. 225 ends in 5, and the only digit whose square ends in 5 is 5. Between 10 and 20 that leaves 15.

Check it: 15 × 15 = 225. So √225 = 15.

Solutions (14 of 16)

Volume of Cones (continued)

Square and cube roots (continued)

9. $\sqrt{484} =$ ___

$22 \times 22 = 484$

$\sqrt{484} = \mathbf{22}$

√ asks a question: which number times itself gives 484? It is not an instruction to halve 484 — that is the mistake this symbol collects.

Bracket it first. $20^2 = 400$, which is under 484, and $30^2 = 900$, which is over. So the answer is between 20 and 30.

Now the last digit. 484 ends in 4, and the only digits whose square ends in 4 are 2 and 8. Between 20 and 30 that leaves 22 or 28.

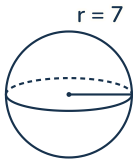
Two left, and the bracket picks between them: 484 is nearer 400 than 900, so take the smaller one — 22.

Check it: $22 \times 22 = 484$. So $\sqrt{484} = 22$.

Volume of Spheres

Volume of a sphere

1.



_____

radius: 7 cm

$7 \times 7 \times 7 = \mathbf{343}$

$4/3 \times 3.14 \times 343 \approx$

1436.03

1436.03 cm³

The 7 cm marked here runs from the center to the surface, so it is already the radius — which is what the formula wants.

Cube the radius on a line of its own. r^3 means $r \times r \times r$, not $r \times 3$: $7 \times 7 \times 7 = 343$.

Then four-thirds of that, times π: $4/3 \times 3.14 \times 343$ comes to 1436.0267..., which is 1436.03 to two decimal places. A third rarely lands on a whole number of hundredths, so this kind of answer is usually rounded.

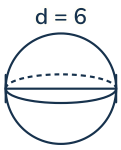
So the volume is 1436.03 cubic cm. Cubic, not square — a volume fills a space, and the unit is the quickest way to notice you have answered a surface-area question instead.

Solutions (15 of 16)

Volume of Spheres (continued)

Volume of a sphere (continued)

2.



_____

diameter: 6 cm

6 ÷ 2 = **3**

3 × 3 × 3 = **27**

4/3 × 3.14 × 27 =

113.04

113.04 cm³

The 6 cm marked here goes right ACROSS the sphere — that is the diameter. The formula wants the radius, and using the diameter instead makes the answer EIGHT times too big, because it is cubed.

Half of 6 is 3, so r = 3 cm.

Cube the radius on a line of its own. r^3 means r × r × r, not r × 3: 3 × 3 × 3 = 27.

Then four-thirds of that, times π: 4/3 × 3.14 × 27 = 113.04.

So the volume is 113.04 cubic cm. Cubic, not square — a volume fills a space, and the unit is the quickest way to notice you have answered a surface-area question instead.

Between which whole numbers

17. $\sqrt{227} >$ ___

$\sqrt{227}$ > **15**

Find the perfect squares on either side. 225 is 15^2 and 256 is 16^2, and 227 sits between them.

So the root is between those two whole numbers: $\sqrt{227}$ is between 15 and 16.

The smaller of the two is 15.

Solutions (16 of 16)

Volume of Spheres (continued)

Between which whole numbers (continued)

18. $\sqrt{128} > \_\_\_$

$\sqrt{128} > \mathbf{11}$

Find the perfect squares on either side. 121 is 11^2 and 144 is 12^2, and 128 sits between them.

So the root is between those two whole numbers: $\sqrt{128}$ is between 11 and 12.

The smaller of the two is 11.