Algebra 1 Factoring and Quadratics Workbook

6 lessons · printable · answer key at the back

Name _____ Date _____

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Factoring Out the GCF

WORKED EXAMPLE

48x + 12

GCF = 12

12(4x + 1)

1. Taking out a common factor

Take out the greatest common factor. Fill in every number.

1 **99x + 81**

2 **63x + 9**

3 **144x + 32**

4 **12x + 8**

5 **36x + 60**

6 **161x + 230**

7 **3x + 27**

8 **14x + 8**

9 **16x + 6**

10 **115x + 207**

11 **99x + 22**

12 **32x + 20**

13 **65x + 78**

14 **63x + 56**

15 **11x + 33**

16 **220x + 240**

17 **72x + 8**

18 **108x + 24**

19 **128x + 80**

20 **18x + 12**

2. Write the equation of a line

Write the equation of each line through the two points.

21 **through (2, 8) and (4, 14)**

22 **through (1, −2) and (3, −8)**

23 **through (2, 3) and (4, 9)**

24 **through (1, 3) and (3, 1)**

25 **through (2, 0) and (4, 2)**

26 **through (−1, 7) and (2, −5)**

2. Write the equation of a line (continued)

Write the equation of each line through the two points.

27 **through (−1, 4) and (2, 7)**

28 **through (−3, 7) and (−1, 3)**

29 **through (2, −2) and (4, 4)**

30 **through (−1, 3) and (2, 12)**

Difference of Two Squares

3. Difference of two squares

Each one is a difference of two squares. Factor it.

1 $x^2 - 16$

2 $x^2 - 484$

3 $x^2 - 100$

4 $x^2 - 4$

5 $x^2 - 1089$

6 $x^2 - 9$

7 $x^2 - 400$

8 $x^2 - 196$

9 $x^2 - 121$

10 $x^2 - 784$

11 $x^2 - 81$

12 $x^2 - 1444$

13 $x^2 - 225$

14 $x^2 - 576$

15 $x^2 - 64$

16 $x^2 - 529$

17 $x^2 - 25$

18 $x^2 - 841$

19 $x^2 - 169$

20 $x^2 - 625$

4. Function notation and composition

Work out the value each function asks for.

21 $f(x) = -x + 7$ $f(3) =$ _____

22 $f(x) = 3x - 4$ $f(-3) =$ _____

23 $f(x) = -4x - 8$ $f(-2) =$ _____

24 $f(x) = -x + 5$ $f(11) =$ _____

25 $f(x) = 5x - 9$ $f(-6) =$ _____

26 $f(x) = -4x + 5$ $f(-1) =$ _____

27 $f(x) = 8x - 10$ $f(4) =$ _____

28 $f(x) = -x - 2$ $f(-3) =$ _____

29 $f(x) = x - 20$ $f(4) =$ _____

30 $f(x) = -6x - 17$ $f(-5) =$ _____

Factoring Quadratics

WORKED EXAMPLE

$x^2 + 18x + 32$

multiply to 32 · add to 18
16 and 2
= (x + 16)(x + 2)

5. Factoring $x^2 + bx + c$

Factor each one.

1 $x^2 + 18x + 65$

2 $x^2 + 23x + 112$

3 $x^2 + 38x + 361$

4 $x^2 + 24x + 95$

5 $x^2 + 26x + 120$

6 $x^2 + 13x + 22$

7 $x^2 + 29x + 154$

8 $x^2 + 28x + 147$

9 $x^2 + 21x + 20$

10 $x^2 + 21x + 108$

11 $x^2 + 7x + 6$

12 $x^2 + 35x + 304$

13 $x^2 + 22x + 121$

14 $x^2 + 32x + 231$

15 $x^2 + 8x + 7$

16 $x^2 + 20x + 19$

17 $x^2 + 11x + 24$

18 $x^2 + 34x + 289$

19 $x^2 + 24x + 108$

20 $x^2 + 16x + 63$

6. Evaluate expressions

Put the number in place of the letter, then work each one out.

21 $x = 10$ $4x - 20 =$ ___

22 $x = 7$ $5x - 9 =$ ___

23 $x = 11$ $2x - 16 =$ ___

24 $a = 7, b = 5$ $6a + b =$ ___

25 $x = 2$ $8x + 2 =$ ___

26 $x = 7$ $3x + 7 =$ ___

27 $a = 2, b = 4$ $3a + b =$ ___

28 $x = 2$ $7x + 9 =$ ___

29 $x = 10$ $5x - 17 =$ ___

30 $a = 10, b = 6$ $4a + b =$ ___

Solving Quadratics by Factoring

WORKED EXAMPLE

$(x - 8)(x - 9) = 0$

$x - 8 = 0$ or $x - 9 = 0$

$x - 8 = 0 \rightarrow x = 8$

$x - 9 = 0 \rightarrow x = 9$

$x = 8$ or $x = 9$

7. Solving $x^2 + bx + c = 0$

Each product is zero. Write the two values of x.

1 $(x - 20)(x - 2) = 0$

2 $(x + 13)(x + 19) = 0$

3 $(x - 3)(x + 3) = 0$

4 $(x - 20)(x + 1) = 0$

5 $(x + 12)(x - 20) = 0$

6 $(x + 13)(x + 6) = 0$

7 $(x - 20)(x - 16) = 0$

8 $(x + 13)(x + 1) = 0$

9 $(x - 14)(x + 18) = 0$

10 $(x - 3)(x - 8) = 0$

11 $(x + 13)(x - 18) = 0$

12 $(x - 10)(x + 14) = 0$

13 $(x + 10)(x - 19) = 0$

14 $(x + 7)(x + 20) = 0$

15 $(x - 11)(x - 15) = 0$

16 $(x + 14)(x - 5) = 0$

8. Taking out a common factor

Take out the greatest common factor. Fill in every number.

17 **70x + 10**

18 **105x + 15**

19 **48x + 12**

20 **14x + 154**

21 **7x + 42**

22 **42x + 105**

23 **14x + 2**

24 **3x + 21**

25 **126x + 72**

26 **4x + 4**

Solving with Square Roots

WORKED EXAMPLE

$(x - 3)^2 = 25$

$x - 3 = \pm 5$

$x = 3 \pm 5$

$x = 8$ or $x = -2$

9. Solve quadratics by square roots

Solve each equation by taking square roots.

1 $x^2 = 4$

2 $(x + 5)^2 = 9$

3 $(x - 9)^2 = 36$

4 $(x - 3)^2 = 4$

5 $(x - 4)^2 = 9$

6 $(x - 3)^2 = 100$

7 $(x + 3)^2 = 25$

8 $(x - 1)^2 = 4$

9 $(x - 7)^2 = 64$

10 $(x - 9)^2 = 64$

11 $7x^2 = 252$

12 $5x^2 = 45$

13 $(x - 9)^2 = 49$

14 $(x - 8)^2 = 4$

15 $(x - 6)^2 = 64$

16 $(x - 4)^2 = 1$

10. Difference of two squares

Each one is a difference of two squares. Factor it.

17 $x^2 - 81$

18 $x^2 - 784$

19 $x^2 - 1296$

20 $x^2 - 64$

21 $x^2 - 529$

22 $x^2 - 1024$

23 $x^2 - 400$

24 $x^2 - 25$

25 $x^2 - 900$

26 $x^2 - 1444$

How Many Solutions?

WORKED EXAMPLE

$10x - 31 = 10x - 27$

x is gone
$-31 = -27$ is false
no solution

11. How many solutions?

How many solutions does each equation have? Show how you know.

1 $-14x + 35 = -14x + 34$

2 $11x + 49 = 11x + 55$

3 $-15x - 23 = -23 - 15x$

4 $x - 44 = -44 + x$

5 $15x - 37 = -12x + 4$

6 $-12x + 49 = -12x + 51$

7 $20x + 25 = 20x + 28$

8 $2x - 24 = -7x + 1$

9 $17x + 25 = 25 + 17x$

10 $-2x + 12 = -2x + 14$

11 $5x - 15 = 11x$

12 $18x - 27 = -2x + 45$

13 $7x + 40 = 40 + 7x$

14 $16x + 23 = -6x + 55$

15 $5x - 50 = 12x + 24$

16 $12x + 58 = 58 + 12x$

17 $-16x + 26 = 8x + 54$

18 $7x - 49 = -10x - 44$

19 $-9x = -9x + 5$

20 $17x + 7 = 14x + 10$

12. Factoring $x^2 + bx + c$

Factor each one.

21 $x^2 + 25x + 150$

22 $x^2 + 28x + 171$

23 $x^2 + 36x + 315$

24 $x^2 + 24x + 63$

25 $x^2 + 41x + 414$

26 $x^2 + 39x + 360$

27 $x^2 + 24x + 23$

28 $x^2 + 26x + 105$

29 $x^2 + 4x + 4$

30 $x^2 + 27x + 162$

Answer Key (1 of 3)

Factoring Out the GCF

Taking out a common factor

1. $99x + 81 = 9(11x + 9)$ ($99 \div 9 = 11$, $81 \div 9 = 9$)
2. $63x + 9 = 9(7x + 1)$ ($63 \div 9 = 7$, $9 \div 9 = 1$)
3. $144x + 32 = 16(9x + 2)$ ($144 \div 16 = 9$, $32 \div 16 = 2$)
4. $12x + 8 = 4(3x + 2)$ ($12 \div 4 = 3$, $8 \div 4 = 2$)
5. $36x + 60 = 12(3x + 5)$ ($36 \div 12 = 3$, $60 \div 12 = 5$)
6. $161x + 230 = 23(7x + 10)$ ($161 \div 23 = 7$, $230 \div 23 = 10$)
7. $3x + 27 = 3(x + 9)$ ($3 \div 3 = 1$, $27 \div 3 = 9$)
8. $14x + 8 = 2(7x + 4)$ ($14 \div 2 = 7$, $8 \div 2 = 4$)
9. $16x + 6 = 2(8x + 3)$ ($16 \div 2 = 8$, $6 \div 2 = 3$)
10. $115x + 207 = 23(5x + 9)$ ($115 \div 23 = 5$, $207 \div 23 = 9$)
11. $99x + 22 = 11(9x + 2)$ ($99 \div 11 = 9$, $22 \div 11 = 2$)
12. $32x + 20 = 4(8x + 5)$ ($32 \div 4 = 8$, $20 \div 4 = 5$)
13. $65x + 78 = 13(5x + 6)$ ($65 \div 13 = 5$, $78 \div 13 = 6$)
14. $63x + 56 = 7(9x + 8)$ ($63 \div 7 = 9$, $56 \div 7 = 8$)
15. $11x + 33 = 11(x + 3)$ ($11 \div 11 = 1$, $33 \div 11 = 3$)
16. $220x + 240 = 20(11x + 12)$ ($220 \div 20 = 11$, $240 \div 20 = 12$)
17. $72x + 8 = 8(9x + 1)$ ($72 \div 8 = 9$, $8 \div 8 = 1$)
18. $108x + 24 = 12(9x + 2)$ ($108 \div 12 = 9$, $24 \div 12 = 2$)
19. $128x + 80 = 16(8x + 5)$ ($128 \div 16 = 8$, $80 \div 16 = 5$)
20. $18x + 12 = 6(3x + 2)$ ($18 \div 6 = 3$, $12 \div 6 = 2$)

Write the equation of a line

21. through (2, 8) and (4, 14) → $y = 3x + 2$
22. through (1, −2) and (3, −8) → $y = -3x + 1$
23. through (2, 3) and (4, 9) → $y = 3x - 3$
24. through (1, 3) and (3, 1) → $y = -x + 4$
25. through (2, 0) and (4, 2) → $y = x - 2$
26. through (−1, 7) and (2, −5) → $y = -4x + 3$
27. through (−1, 4) and (2, 7) → $y = x + 5$
28. through (−3, 7) and (−1, 3) → $y = -2x + 1$
29. through (2, −2) and (4, 4) → $y = 3x - 8$
30. through (−1, 3) and (2, 12) → $y = 3x + 6$

Difference of Two Squares

Difference of two squares

1. $x^2 - 16 = (x + 4)(x - 4)$ (16 is 4 squared, so the two brackets are (x + 4) and (x − 4))
2. $x^2 - 484 = (x + 22)(x - 22)$ (484 is 22 squared, so the two brackets are (x + 22) and (x − 22))
3. $x^2 - 100 = (x + 10)(x - 10)$ (100 is 10 squared, so the two brackets are (x + 10) and (x − 10))
4. $x^2 - 4 = (x + 2)(x - 2)$ (4 is 2 squared, so the two brackets are (x + 2) and (x − 2))
5. $x^2 - 1089 = (x + 33)(x - 33)$ (1089 is 33 squared, so the two brackets are (x + 33) and (x − 33))
6. $x^2 - 9 = (x + 3)(x - 3)$ (9 is 3 squared, so the two brackets are (x + 3) and (x − 3))
7. $x^2 - 400 = (x + 20)(x - 20)$ (400 is 20 squared, so the two brackets are (x + 20) and (x − 20))

Difference of Two Squares (continued)

Difference of two squares (continued)

8. $x^2 - 196 = (x + 14)(x - 14)$ (196 is 14 squared, so the two brackets are (x + 14) and (x − 14))
9. $x^2 - 121 = (x + 11)(x - 11)$ (121 is 11 squared, so the two brackets are (x + 11) and (x − 11))
10. $x^2 - 784 = (x + 28)(x - 28)$ (784 is 28 squared, so the two brackets are (x + 28) and (x − 28))
11. $x^2 - 81 = (x + 9)(x - 9)$ (81 is 9 squared, so the two brackets are (x + 9) and (x − 9))
12. $x^2 - 1444 = (x + 38)(x - 38)$ (1444 is 38 squared, so the two brackets are (x + 38) and (x − 38))
13. $x^2 - 225 = (x + 15)(x - 15)$ (225 is 15 squared, so the two brackets are (x + 15) and (x − 15))
14. $x^2 - 576 = (x + 24)(x - 24)$ (576 is 24 squared, so the two brackets are (x + 24) and (x − 24))
15. $x^2 - 64 = (x + 8)(x - 8)$ (64 is 8 squared, so the two brackets are (x + 8) and (x − 8))
16. $x^2 - 529 = (x + 23)(x - 23)$ (529 is 23 squared, so the two brackets are (x + 23) and (x − 23))
17. $x^2 - 25 = (x + 5)(x - 5)$ (25 is 5 squared, so the two brackets are (x + 5) and (x − 5))
18. $x^2 - 841 = (x + 29)(x - 29)$ (841 is 29 squared, so the two brackets are (x + 29) and (x − 29))
19. $x^2 - 169 = (x + 13)(x - 13)$ (169 is 13 squared, so the two brackets are (x + 13) and (x − 13))
20. $x^2 - 625 = (x + 25)(x - 25)$ (625 is 25 squared, so the two brackets are (x + 25) and (x − 25))

Function notation and composition

21. $f(x) = -x + 7$ $f(3) = 4$
22. $f(x) = 3x - 4$ $f(-3) = -13$
23. $f(x) = -4x - 8$ $f(-2) = 0$
24. $f(x) = -x + 5$ $f(11) = -6$
25. $f(x) = 5x - 9$ $f(-6) = -39$
26. $f(x) = -4x + 5$ $f(-1) = 9$
27. $f(x) = 8x - 10$ $f(4) = 22$
28. $f(x) = -x - 2$ $f(-3) = 1$
29. $f(x) = x - 20$ $f(4) = -16$
30. $f(x) = -6x - 17$ $f(-5) = 13$

Factoring Quadratics

Factoring $x^2 + bx + c$

1. $x^2 + 18x + 65 = (x + 13)(x + 5)$ (13 and 5)
2. $x^2 + 23x + 112 = (x + 7)(x + 16)$ (7 and 16)
3. $x^2 + 38x + 361 = (x + 19)(x + 19)$ (19 and 19)
4. $x^2 + 24x + 95 = (x + 5)(x + 19)$ (5 and 19)
5. $x^2 + 26x + 120 = (x + 20)(x + 6)$ (20 and 6)
6. $x^2 + 13x + 22 = (x + 11)(x + 2)$ (11 and 2)
7. $x^2 + 29x + 154 = (x + 7)(x + 22)$ (7 and 22)
8. $x^2 + 28x + 147 = (x + 21)(x + 7)$ (21 and 7)
9. $x^2 + 21x + 20 = (x + 20)(x + 1)$ (20 and 1)

Answer Key (2 of 3)

Factoring Quadratics (continued)

Factoring $x^2 + bx + c$ (continued)

10. $x^2 + 21x + 108 = (x + 9)(x + 12)$ (9 and 12)
11. $x^2 + 7x + 6 = (x + 1)(x + 6)$ (1 and 6)
12. $x^2 + 35x + 304 = (x + 16)(x + 19)$ (16 and 19)
13. $x^2 + 22x + 121 = (x + 11)(x + 11)$ (11 and 11)
14. $x^2 + 32x + 231 = (x + 21)(x + 11)$ (21 and 11)
15. $x^2 + 8x + 7 = (x + 7)(x + 1)$ (7 and 1)
16. $x^2 + 20x + 19 = (x + 19)(x + 1)$ (19 and 1)
17. $x^2 + 11x + 24 = (x + 3)(x + 8)$ (3 and 8)
18. $x^2 + 34x + 289 = (x + 17)(x + 17)$ (17 and 17)
19. $x^2 + 24x + 108 = (x + 18)(x + 6)$ (18 and 6)
20. $x^2 + 16x + 63 = (x + 7)(x + 9)$ (7 and 9)

Evaluate expressions

21. $4x - 20$ $(x = 10) \rightarrow 20$
22. $5x - 9$ $(x = 7) \rightarrow 26$
23. $2x - 16$ $(x = 11) \rightarrow 6$
24. $6a + b$ $(a = 7, b = 5) \rightarrow 47$
25. $8x + 2$ $(x = 2) \rightarrow 18$
26. $3x + 7$ $(x = 7) \rightarrow 28$
27. $3a + b$ $(a = 2, b = 4) \rightarrow 10$
28. $7x + 9$ $(x = 2) \rightarrow 23$
29. $5x - 17$ $(x = 10) \rightarrow 33$
30. $4a + b$ $(a = 10, b = 6) \rightarrow 46$

Solving Quadratics by Factoring

Solving $x^2 + bx + c = 0$

1. $(x - 20)(x - 2) = 0 \rightarrow (x - 20)(x - 2) = 0 \rightarrow x = 20$ or $x = 2$
2. $(x + 13)(x + 19) = 0 \rightarrow (x + 13)(x + 19) = 0 \rightarrow x = -13$ or $x = -19$
3. $(x - 3)(x + 3) = 0 \rightarrow (x - 3)(x + 3) = 0 \rightarrow x = 3$ or $x = -3$
4. $(x - 20)(x + 1) = 0 \rightarrow (x - 20)(x + 1) = 0 \rightarrow x = 20$ or $x = -1$
5. $(x + 12)(x - 20) = 0 \rightarrow (x + 12)(x - 20) = 0 \rightarrow x = -12$ or $x = 20$
6. $(x + 13)(x + 6) = 0 \rightarrow (x + 13)(x + 6) = 0 \rightarrow x = -13$ or $x = -6$
7. $(x - 20)(x - 16) = 0 \rightarrow (x - 20)(x - 16) = 0 \rightarrow x = 20$ or $x = 16$
8. $(x + 13)(x + 1) = 0 \rightarrow (x + 13)(x + 1) = 0 \rightarrow x = -13$ or $x = -1$
9. $(x - 14)(x + 18) = 0 \rightarrow (x - 14)(x + 18) = 0 \rightarrow x = 14$ or $x = -18$
10. $(x - 3)(x - 8) = 0 \rightarrow (x - 3)(x - 8) = 0 \rightarrow x = 3$ or $x = 8$
11. $(x + 13)(x - 18) = 0 \rightarrow (x + 13)(x - 18) = 0 \rightarrow x = -13$ or $x = 18$
12. $(x - 10)(x + 14) = 0 \rightarrow (x - 10)(x + 14) = 0 \rightarrow x = 10$ or $x = -14$
13. $(x + 10)(x - 19) = 0 \rightarrow (x + 10)(x - 19) = 0 \rightarrow x = -10$ or $x = 19$
14. $(x + 7)(x + 20) = 0 \rightarrow (x + 7)(x + 20) = 0 \rightarrow x = -7$ or $x = -20$

Solving Quadratics by Factoring (continued)

Solving $x^2 + bx + c = 0$ (continued)

15. $(x - 11)(x - 15) = 0 \rightarrow (x - 11)(x - 15) = 0 \rightarrow x = 11$ or $x = 15$
16. $(x + 14)(x - 5) = 0 \rightarrow (x + 14)(x - 5) = 0 \rightarrow x = -14$ or $x = 5$

Taking out a common factor

17. $70x + 10 = 10(7x + 1)$ $(70 \div 10 = 7, 10 \div 10 = 1)$
18. $105x + 15 = 15(7x + 1)$ $(105 \div 15 = 7, 15 \div 15 = 1)$
19. $48x + 12 = 12(4x + 1)$ $(48 \div 12 = 4, 12 \div 12 = 1)$
20. $14x + 154 = 14(x + 11)$ $(14 \div 14 = 1, 154 \div 14 = 11)$
21. $7x + 42 = 7(x + 6)$ $(7 \div 7 = 1, 42 \div 7 = 6)$
22. $42x + 105 = 21(2x + 5)$ $(42 \div 21 = 2, 105 \div 21 = 5)$
23. $14x + 2 = 2(7x + 1)$ $(14 \div 2 = 7, 2 \div 2 = 1)$
24. $3x + 21 = 3(x + 7)$ $(3 \div 3 = 1, 21 \div 3 = 7)$
25. $126x + 72 = 18(7x + 4)$ $(126 \div 18 = 7, 72 \div 18 = 4)$
26. $4x + 4 = 4(x + 1)$ $(4 \div 4 = 1, 4 \div 4 = 1)$

Solving with Square Roots

Solve quadratics by square roots

1. $x^2 = 4 \rightarrow x = -2$ or $x = 2$
2. $(x + 5)^2 = 9 \rightarrow x = -8$ or $x = -2$
3. $(x - 9)^2 = 36 \rightarrow x = 3$ or $x = 15$
4. $(x - 3)^2 = 4 \rightarrow x = 1$ or $x = 5$
5. $(x - 4)^2 = 9 \rightarrow x = 1$ or $x = 7$
6. $(x - 3)^2 = 100 \rightarrow x = -7$ or $x = 13$
7. $(x + 3)^2 = 25 \rightarrow x = -8$ or $x = 2$
8. $(x - 1)^2 = 4 \rightarrow x = -1$ or $x = 3$
9. $(x - 7)^2 = 64 \rightarrow x = -1$ or $x = 15$
10. $(x - 9)^2 = 64 \rightarrow x = 1$ or $x = 17$
11. $7x^2 = 252 \rightarrow x = -6$ or $x = 6$
12. $5x^2 = 45 \rightarrow x = -3$ or $x = 3$
13. $(x - 9)^2 = 49 \rightarrow x = 2$ or $x = 16$
14. $(x - 8)^2 = 4 \rightarrow x = 6$ or $x = 10$
15. $(x - 6)^2 = 64 \rightarrow x = -2$ or $x = 14$
16. $(x - 4)^2 = 1 \rightarrow x = 3$ or $x = 5$

Difference of two squares

17. $x^2 - 81 = (x + 9)(x - 9)$ (81 is 9 squared, so the two brackets are $(x + 9)$ and $(x - 9)$)
18. $x^2 - 784 = (x + 28)(x - 28)$ (784 is 28 squared, so the two brackets are $(x + 28)$ and $(x - 28)$)
19. $x^2 - 1296 = (x + 36)(x - 36)$ (1296 is 36 squared, so the two brackets are $(x + 36)$ and $(x - 36)$)
20. $x^2 - 64 = (x + 8)(x - 8)$ (64 is 8 squared, so the two brackets are $(x + 8)$ and $(x - 8)$)
21. $x^2 - 529 = (x + 23)(x - 23)$ (529 is 23 squared, so the two brackets are $(x + 23)$ and $(x - 23)$)
22. $x^2 - 1024 = (x + 32)(x - 32)$ (1024 is 32 squared, so the two brackets are $(x + 32)$ and $(x - 32)$)
23. $x^2 - 400 = (x + 20)(x - 20)$ (400 is 20 squared, so the two brackets are $(x + 20)$ and $(x - 20)$)

Answer Key (3 of 3)

Solving with Square Roots (continued)

Difference of two squares (continued)

24. $x^2 - 25 = (x + 5)(x - 5)$ (25 is 5 squared, so the two brackets are (x + 5) and (x − 5))
25. $x^2 - 900 = (x + 30)(x - 30)$ (900 is 30 squared, so the two brackets are (x + 30) and (x − 30))
26. $x^2 - 1444 = (x + 38)(x - 38)$ (1444 is 38 squared, so the two brackets are (x + 38) and (x − 38))

How Many Solutions?

How many solutions?

1. $-14x + 35 = -14x + 34$: no solution
2. $11x + 49 = 11x + 55$: no solution
3. $-15x - 23 = -23 - 15x$: infinitely many
4. $x - 44 = -44 + x$: infinitely many
5. $15x - 37 = -12x + 4$: one solution
6. $-12x + 49 = -12x + 51$: no solution
7. $20x + 25 = 20x + 28$: no solution
8. $2x - 24 = -7x + 1$: one solution
9. $17x + 25 = 25 + 17x$: infinitely many
10. $-2x + 12 = -2x + 14$: no solution
11. $5x - 15 = 11x$: one solution
12. $18x - 27 = -2x + 45$: one solution
13. $7x + 40 = 40 + 7x$: infinitely many
14. $16x + 23 = -6x + 55$: one solution
15. $5x - 50 = 12x + 24$: one solution
16. $12x + 58 = 58 + 12x$: infinitely many
17. $-16x + 26 = 8x + 54$: one solution
18. $7x - 49 = -10x - 44$: one solution
19. $-9x = -9x + 5$: no solution
20. $17x + 7 = 14x + 10$: one solution

Factoring $x^2 + bx + c$

21. $x^2 + 25x + 150 = (x + 15)(x + 10)$ (15 and 10)
22. $x^2 + 28x + 171 = (x + 19)(x + 9)$ (19 and 9)
23. $x^2 + 36x + 315 = (x + 21)(x + 15)$ (21 and 15)
24. $x^2 + 24x + 63 = (x + 21)(x + 3)$ (21 and 3)
25. $x^2 + 41x + 414 = (x + 18)(x + 23)$ (18 and 23)
26. $x^2 + 39x + 360 = (x + 24)(x + 15)$ (24 and 15)
27. $x^2 + 24x + 23 = (x + 1)(x + 23)$ (1 and 23)
28. $x^2 + 26x + 105 = (x + 5)(x + 21)$ (5 and 21)
29. $x^2 + 4x + 4 = (x + 2)(x + 2)$ (2 and 2)
30. $x^2 + 27x + 162 = (x + 9)(x + 18)$ (9 and 18)

Solutions (1 of 8)

Factoring Out the GCF

Taking out a common factor

1. **99x + 81**

$$99x + 81 = \mathbf{9(11x + 9)}$$

Before anything else, look at whether every term shares a factor. The numbers here are 99, 81 — so the question is what the biggest number that divides ALL of them is.

List what divides each one and look for the biggest they all share. 99: 1, 3, 9, 11, 33, 99 81: 1, 3, 9, 27, 81.

The biggest number in all of those lists is 9. Take it out in front and divide every term by it: 99 ÷ 9 = 11, 81 ÷ 9 = 9.

Check the bracket before you stop. 11x + 9 — those numbers share nothing but 1, so there is no more to take out and 9 really was the greatest. 99x + 81 = 9(11x + 9).

2. **63x + 9**

$$63x + 9 = \mathbf{9(7x + 1)}$$

Before anything else, look at whether every term shares a factor. The numbers here are 63, 9 — so the question is what the biggest number that divides ALL of them is.

List what divides each one and look for the biggest they all share. 63: 1, 3, 7, 9, 21, 63 9: 1, 3, 9.

The biggest number in all of those lists is 9. Take it out in front and divide every term by it: 63 ÷ 9 = 7, 9 ÷ 9 = 1.

Check the bracket before you stop. 7x + 1 — those numbers share nothing but 1, so there is no more to take out and 9 really was the greatest. 63x + 9 = 9(7x + 1).

Solutions (2 of 8)

Factoring Out the GCF (continued)

Write the equation of a line

21. **through (2, 8) and (4, 14)**

slope = (14 − 8) ÷ (4 − 2)

6 ÷ 2 = **3**

b = 8 − (3)(2) = **2**

y = 3x + 2

Find the slope first: rise over run, (14 − 8) ÷ (4 − 2) = 3.

Put a point back in to find b: b = y − mx = 8 − (3)(2) = 2.

Write it as y = mx + b: y = 3x + 2.

22. **through (1, −2) and (3, −8)**

slope = (−8 − (−2)) ÷ (3 − 1)

−6 ÷ 2 = **−3**

b = −2 − (−3)(1) = **1**

y = −3x + 1

Find the slope first: rise over run, (−8 − (−2)) ÷ (3 − 1) = −3.

Put a point back in to find b: b = y − mx = −2 − (−3)(1) = 1.

Write it as y = mx + b: y = −3x + 1.

Solutions (3 of 8)

Difference of Two Squares

Difference of two squares

1. **$x^2 - 16$**

$$x^2 - 16 = (x + 4)(x - 4)$$

Before factoring anything, check the shape. There are two parts with a − between them, and each part has to be a perfect square: x^2 is x squared, and 16 is 4 squared. Both are, and it is a minus — so this is a difference of two squares.

That shape always factors the same way, and there is nothing to search for: it is the SUM times the DIFFERENCE of the two things being squared — $(x + 4)(x - 4)$.

So the missing number is 4: 16 is 4 squared, so the two brackets are $(x + 4)$ and $(x - 4)$.

2. **$x^2 - 484$**

$$x^2 - 484 = (x + 22)(x - 22)$$

Before factoring anything, check the shape. There are two parts with a − between them, and each part has to be a perfect square: x^2 is x squared, and 484 is 22 squared. Both are, and it is a minus — so this is a difference of two squares.

That shape always factors the same way, and there is nothing to search for: it is the SUM times the DIFFERENCE of the two things being squared — $(x + 22)(x - 22)$.

So the missing number is 22: 484 is 22 squared, so the two brackets are $(x + 22)$ and $(x - 22)$.

Function notation and composition

21. $f(x) = -x + 7$ **$f(3) =$** ___

$$f(x) = -x + 7 \quad \text{at} \quad x = 3$$
$$-1 \times 3 = -3$$
$$-3 + 7 = 4$$
$$f(3) = 4$$

f(3) is a name for the value of the rule when x = 3. Put 3 where the x is — it is not f times 3.

Multiply first, sign and all: $-1 \times 3 = -3$.

Then add 7: $-3 + 7 = 4$.

So $f(3) = 4$.

Solutions (4 of 8)

Difference of Two Squares (continued)

Function notation and composition (continued)

22. $f(x) = 3x - 4$ **$f(-3) =$** _____

$f(x) = 3x - 4$ at **$x = -3$**

$3 \times (-3) =$ **-9**

$-9 - 4 =$ **-13**

$f(-3) =$ **-13**

$f(-3)$ is a name for the value of the rule when $x = -3$. Put -3 where the x is — it is not f times -3.

Multiply first, sign and all: $3 \times (-3) = -9$.

Then subtract 4: $-9 - 4 = -13$.

So $f(-3) = -13$.

Factoring Quadratics

Factoring $x^2 + bx + c$

1. **$x^2 + 18x + 65$**

$x^2 + 18x + 65 =$

$(x + 13)(x + 5)$

Factoring this means finding two numbers to put in $(x + \square)(x + \square)$. Two conditions decide them, and both have to hold: they MULTIPLY to 65, the last term, and they ADD to 18, the number in front of x.

Now list the pairs that multiply to 65. There are only 2, so this is a short search rather than a guess: 1 and 65, 5 and 13.

Which of them adds to 18? $13 \times 5 = 65$ and $13 + 5 = 18$. So the two numbers are 13 and 5.

Put them in the brackets: $x^2 + 18x + 65 = (x + 13)(x + 5)$. Multiply it back out if you want to check — the x terms should add up to 18x again.

Solutions (5 of 8)

Factoring Quadratics (continued)

Factoring $x^2 + bx + c$ (continued)

2. **$x^2 + 23x + 112$**

$$x^2 + 23x + 112 = (x + 7)(x + 16)$$

Factoring this means finding two numbers to put in (x + □)(x + □). Two conditions decide them, and both have to hold: they MULTIPLY to 112, the last term, and they ADD to 23, the number in front of x.

Now list the pairs that multiply to 112. There are only 5, so this is a short search rather than a guess: 1 and 112, 2 and 56, 4 and 28, 7 and 16, 8 and 14.

Which of them adds to 23? 7 × 16 = 112 and 7 + 16 = 23. So the two numbers are 7 and 16.

Put them in the brackets: $x^2 + 23x + 112 = (x + 7)(x + 16)$. Multiply it back out if you want to check — the x terms should add up to 23x again.

Evaluate expressions

21. **x = 10** **4x − 20 = ___**

$$4x - 20 \text{ with } x = 10$$
$$4 \times 10 = 40$$
$$40 - 20 = 20$$
$$4x - 20 = 20$$

A letter is a number that has not been written down yet, and this question writes it down: x = 10. So put the value in where the letter is — in BRACKETS, which keeps whatever was being done to the letter being done to the number.

4x means 4 TIMES x, and multiplying happens before adding or taking away however the line is written. So this multiplication comes first: 4 × 10 = 40.

Now the last bit: 40 − 20 = 20. Doing it the other way round — taking away first and multiplying after — gives -40, which is why the order is the whole of this question.

So 4x − 20 is 20 when x = 10.

Solutions (6 of 8)

Factoring Quadratics (continued)

Evaluate expressions (continued)

22. $x = 7$ **$5x - 9 =$ ___**

$5x - 9$ with **$x = 7$**

$5 \times 7 =$ **35**

$35 - 9 =$ **26**

$5x - 9 =$ **26**

A letter is a number that has not been written down yet, and this question writes it down: x = 7. So put the value in where the letter is — in BRACKETS, which keeps whatever was being done to the letter being done to the number.

5x means 5 TIMES x, and multiplying happens before adding or taking away however the line is written. So this multiplication comes first: 5 × 7 = 35.

Now the last bit: 35 – 9 = 26. Doing it the other way round — taking away first and multiplying after — gives -10, which is why the order is the whole of this question.

So 5x – 9 is 26 when x = 7.

Solving Quadratics by Factoring

Solving $x^2 + bx + c = 0$

1. **$(x - 20)(x - 2) = 0$**

$x = 20$ or **$x = 2$**

The left side is already a product of two brackets, and the right side is zero. That is the useful shape, so no factoring is needed here.

Now the step that gives the answers: a product is zero ONLY when one of the things being multiplied is zero. Two numbers that multiply to zero — one of them has to be zero. So either (x – 20) is zero, or (x – 2) is zero.

Solve each one. x – 20 = 0 gives x = 20, and x – 2 = 0 gives x = 2. Notice the sign turns over: a – in the bracket gives a POSITIVE answer.

So x = 20 or x = 2. Either one makes the equation true, and it is "or" rather than "and" — x cannot be both at once.

Solutions (7 of 8)

Solving Quadratics by Factoring (continued)

Solving $x^2 + bx + c = 0$ (continued)

2. $(x + 13)(x + 19) = 0$

$x = -13$ or $x = -19$

The left side is already a product of two brackets, and the right side is zero. That is the useful shape, so no factoring is needed here.

Now the step that gives the answers: a product is zero ONLY when one of the things being multiplied is zero. Two numbers that multiply to zero — one of them has to be zero. So either $(x + 13)$ is zero, or $(x + 19)$ is zero.

Solve each one. $x + 13 = 0$ gives $x = -13$, and $x + 19 = 0$ gives $x = -19$. Notice the sign turns over: a + in the bracket gives a NEGATIVE answer.

So $x = -13$ or $x = -19$. Either one makes the equation true, and it is "or" rather than "and" — x cannot be both at once.

Solving with Square Roots

Solve quadratics by square roots

1. $x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

$x = 2$ or $x = -2$

Take the square root of both sides — and keep both signs: $x = \pm\sqrt{4} = \pm 2$.

So $x = 2$ or $x = -2$ — both roots.

2. $(x + 5)^2 = 9$

$x + 5 = \pm 3$

$x = -5 \pm 3$

$x = -2$ or $x = -8$

Take the square root of both sides — both signs: $x + 5 = \pm\sqrt{9} = \pm 3$.

Then subtract 5 to both sides: $x = -5 \pm 3$.

So $x = -2$ or $x = -8$ — both roots.

Solutions (8 of 8)

How Many Solutions?

How many solutions?

1. $-14x + 35 = -14x + 34$

35 = 34 is false

no solution

Do not start by hunting for x. Ask a smaller question first: what is left when the x terms are collected on one side?

Both sides have the SAME number of x, so taking it off both sides removes x completely. That is not a mistake — it is the answer arriving early.

What is left is 35 = 34, which is never true, whatever x is. So there is no solution.

2. $11x + 49 = 11x + 55$

49 = 55 is false

no solution

Do not start by hunting for x. Ask a smaller question first: what is left when the x terms are collected on one side?

Both sides have the SAME number of x, so taking it off both sides removes x completely. That is not a mistake — it is the answer arriving early.

What is left is 49 = 55, which is never true, whatever x is. So there is no solution.