Algebra 1 Foundation Workbook

17 lessons · printable · answer key at the back

Name _____ Date _____

Contents

Evaluating Expressions

WORKED EXAMPLE

$x = 5$ $4x - 13 =$ 7

$4x - 13$ with $x = 5$
$4 \times 5 = 20$
$20 - 13 = 7$
$4x - 13 = 7$

1. Evaluate expressions

Put the number in place of the letter, then work each one out.

1 $x = 4$ $8x - 16 =$ ___

2 $x = 12$ $2x - 6 =$ ___

3 $x = 9$ $2x + 15 =$ ___

4 $x = 3$ $4x + 9 =$ ___

5 $x = 9$ $2x - 1 =$ ___

6 $x = 8$ $2x + 4 =$ ___

7 $a = 3, b = 6$ $5a + b =$ ___

8 $x = 8$ $2x - 10 =$ ___

9 $x = 4$ $2x - 1 =$ ___

10 $x = 2$ $3x - 1 =$ ___

11 $x = 8$ $2x + 10 =$ ___

12 $x = 7$ $2x - 4 =$ ___

1. Evaluate expressions (continued)

Put the number in place of the letter, then work each one out.

13 $x = 9$ $4x - 19 =$ ___

14 $x = 8$ $3x + 10 =$ ___

15 $x = 6$ $3x - 11 =$ ___

16 $x = 3$ $4x - 9 =$ ___

17 $x = 4$ $2x - 4 =$ ___

18 $x = 6$ $4x + 8 =$ ___

19 $x = 9$ $3x - 20 =$ ___

20 $x = 4$ $6x - 17 =$ ___

21 $a = 4, b = 6$ $2a + b =$ ___

22 $x = 3$ $3x =$ ___

23 $x = 3$ $7x + 1 =$ ___

24 $x = 2$ $5x - 5 =$ ___

Combining Like Terms

WORKED EXAMPLE

2 + 5x + 7 + 7x

5 + 7 = 12

2 + 7 = 9

12x + 9

2. Collect like terms

Collect the like terms.

1 **1 + 5x + 4 + 3x**

2 **3 + 3x + 7 + 7x**

3 **9 + 5x + 8 + 5x**

4 **11 + 8x + 1 + 4x**

5 **12x + 1 − 4x**

6 **10 + 2x + 1 + 6x**

7 **6 + 6x + 10 + 4x**

8 **6 + 8x + 5 + 2x**

9 **8x + x + 10**

10 **9x + 5 − 5x**

11 **10x + 12 − 3x − 11**

12 **9 + 6x + 5 + 4x**

13 **5 + 4x + 5 + 8x**

14 **9x + 20 − 8x**

15 **5x + 18 − 4x − 17**

16 **10 + 3x + 1 + x**

17 **3x + 15 − 2x − 14**

18 **8x + 15 − 3x**

19 **9x + 15 − 8x − 14**

20 **1 + 5x + 7 + 7x**

21 **4x + 14 − 3x − 8**

22 **4 + 5x + 6 + 2x**

23 **7 + 4x + 4 + 5x**

24 **2x + 6x + 18**

The Distributive Property

WORKED EXAMPLE

2(5 + 2x)

2 × 5 = 10

2 × 2x = 4x

4x + 10

3. Multiply out brackets

Multiply out each bracket.

1 **3(x + 8)**

2 **4(2x + 5)**

3 **6(7x + 3)**

4 **2(4x + 4)**

5 **2(7x + 11)**

6 **2(9x + 10)**

7 **2(11 + 5x)**

8 **2(6x − 5)**

9 **6(x − 4)**

10 **3(3 + 3x)**

11 **4(3x − 6)**

12 **3(5x + 6)**

13 **2(7x + 10)**

14 **8(x + 6)**

15 **2(7 + x)**

16 **2(7x − 3)**

17 **2(7x + 2)**

18 **7(2x + 7)**

19 **4(4x + 7)**

20 **3(2 + 3x)**

21 **2(9 + 5x)**

22 **4(2x + 8)**

23 **7(x + 5)**

24 **4(5x + 5)**

Absolute Value

WORKED EXAMPLE

$|2 - 10| = 8$

4. Absolute value

Work out each one. The bars ask how far the number is from zero.

1 $|17| =$ ___

2 $|7 - 19| =$ ___

3 $|-4| + |2| =$ ___

4 $|7 - 15| =$ ___

5 $|3 - 9| =$ ___

6 $|3 - 11| =$ ___

7 $|-3| + |8| =$ ___

8 $|13 - 14| =$ ___

9 $|8 - 10| =$ ___

10 $|-8| =$ ___

11 $|-5| + |7| =$ ___

12 $|7 - 11| =$ ___

13 $|-5| =$ ___

14 $|11| + |-2| =$ ___

15 $|4 - 9| =$ ___

16 $|12 - 14| =$ ___

17 $|7 - 13| =$ ___

18 $|2| + |-3| =$ ___

19 $|8| =$ ___

20 $|1 - 8| =$ ___

21 $|3| + |-11| =$ ___

22 $|7 - 17| =$ ___

23 $|12 - 16| =$ ___

24 $|14| =$ ___

5. Evaluate expressions

Put the number in place of the letter, then work each one out.

25 $x = 8$ $2x - 14 =$ ___

26 $x = 6$ $6x - 18 =$ ___

27 $a = 2, b = 7$ $3a + b =$ ___

28 $x = 3$ $4x - 5 =$ ___

29 $x = 6$ $3x - 17 =$ ___

30 $x = 4$ $5x - 3 =$ ___

31 $x = 7$ $2x + 18 =$ ___

32 $x = 8$ $4x - 18 =$ ___

33 $x = 2$ $7x + 9 =$ ___

34 $x = 3$ $10x =$ ___

Absolute Value Equations

WORKED EXAMPLE

|x + 6| = 7

x + 6 = 7 or x + 6 = −7
x + 6 = 7 → x = 1
x + 6 = −7 → x = −13
x = −13 or x = 1

6. Absolute value equations

Solve each equation. Each has two answers.

1 **|x − 2| = 3**

2 **|x − 6| = 3**

3 **|x − 4| = 5**

4 **|x + 1| = 3**

5 **|x − 2| = 4**

6 **|x − 3| = 3**

7 **|x + 1| = 7**

8 **|x − 2| = 2**

9 **|x + 1| = 2**

10 **|x| = 2**

11 **|x − 1| = 6**

12 **|x| = 3**

13 **|x + 6| = 4**

14 **|x − 5| = 7**

15 **|x + 4| = 3**

6. Absolute value equations (continued)

Solve each equation. Each has two answers.

16 $|x + 6| = 2$

17 $|x - 3| = 4$

18 $|x + 5| = 5$

19 $|x + 3| = 4$

20 $|x - 5| = 4$

7. Collect like terms

Collect the like terms.

21 $7 + 9x + 2 + 2x$

22 $9 + 9x + 4 + 2x$

23 $9x + 13 - x - 12$

24 $9 + 5x + 8 + 3x$

25 $3 + 4x + 9 + 3x$

26 $3 + 5x + 5 + 5x$

27 $4x + 15 - 2x - 8$

28 $4 + 8x + 3 + 2x$

29 $7 + 3x + 5 + 8x$

30 $11 + 7x + 1 + x$

Solving Inequalities

WORKED EXAMPLE

$x + 11 \leq 25 \quad x \boxed{\leq} 14$

$25 - 11 = 14$

$x \leq 14$

8. Inequalities

Solve each one, then write >, <, ≥ or ≤ in the box.

1 $6x + 9 > -39 \quad x \boxed{} -8$

2 $x - 1 > 19 \quad x \boxed{} 20$

3 $3x + 2 < -79 \quad x \boxed{} -27$

4 $2x \geq -2 \quad x \boxed{} -1$

5 $2x + 10 < 36 \quad x \boxed{} 13$

6 $6x + 7 \geq -71 \quad x \boxed{} -13$

7 $2x > 64 \quad x \boxed{} 32$

8 $3x + 4 > 22 \quad x \boxed{} 6$

9 $2x + 10 \leq 18 \quad x \boxed{} 4$

10 $4x + 6 < 18 \quad x \boxed{} 3$

11 $-4x + 12 > 44 \quad x \boxed{} -8$

12 $9x + 7 < -2 \quad x \boxed{} -1$

13 $2x + 9 \leq -17 \quad x \boxed{} -13$

14 $-3x \leq -30 \quad x \boxed{} 10$

15 $2x + 4 \geq 48 \quad x \boxed{} 22$

16 $x + 9 \leq 17 \quad x \boxed{} 8$

17 $x + 5 \leq 19 \quad x \boxed{} 14$

18 $3x + 7 \geq -86 \quad x \boxed{} -31$

19 $3x \geq 12 \quad x \boxed{} 4$

20 $4x + 11 > -9 \quad x \boxed{} -5$

9. Multiply out brackets

Multiply out each bracket.

21 $7(4x + 2)$

22 $3(4x + 12)$

23 $4(2 + 5x)$

24 $5(2x - 2)$

25 $4(7x + 3)$

26 $4(x + 3)$

27 $4(9x + 4)$

28 $2(4x + 9)$

29 $3(2x - 9)$

30 $2(8x + 8)$

Intercepts

WORKED EXAMPLE

$2x - 5y = 10$ $x =$ 5

$2x - 5y = 10,\ y = 0$

$2x = 10$

$x = 5$

10. Intercepts of a line

Find the x-intercept of each line.

1 $2x + y = 6$ $x =$ ___

2 $8x + y = 8$ $x =$ ___

3 $x + y = -3$ $x =$ ___

4 $9x + 4y = 36$ $x =$ ___

5 $x - y = 5$ $x =$ ___

6 $3x - 5y = 15$ $x =$ ___

7 $4x + y = -8$ $x =$ ___

8 $x - 4y = 8$ $x =$ ___

9 $x + 5y = 5$ $x =$ ___

10 $3x - y = 6$ $x =$ ___

11 $5x - 3y = -15$ $x =$ ___

12 $x + 3y = 3$ $x =$ ___

13 $9x + 2y = 18$ $x =$ ___

14 $x - y = -5$ $x =$ ___

15 $7x - 2y = 14$ $x =$ ___

16 $4x + 3y = 24$ $x =$ ___

11. Absolute value

Work out each one. The bars ask how far the number is from zero.

17 $|3 - 9| =$ ___

18 $|15 - 17| =$ ___

19 $|3| + |-9| =$ ___

20 $|10| =$ ___

21 $|3 - 7| =$ ___

22 $|5 - 12| =$ ___

23 $|5| + |-8| =$ ___

24 $|2| + |-1| =$ ___

25 $|-1| + |2| =$ ___

26 $|7 - 16| =$ ___

Writing the Equation of a Line

WORKED EXAMPLE

through (2, 5) and (4, 11)

slope = (11 − 5) ÷ (4 − 2)
6 ÷ 2 = 3
b = 5 − (3)(2) = −1
y = 3x − 1

12. Write the equation of a line

Write the equation of each line through the two points.

1 **through (−2, 0) and (1, 9)**

2 **through (−2, 2) and (1, 5)**

3 **through (1, 7) and (3, 5)**

4 **through (−1, 6) and (2, 9)**

5 **through (−3, 8) and (−1, 4)**

6 **through (2, 8) and (4, 14)**

7 **through (1, −1) and (3, −7)**

8 **through (−1, 2) and (2, 5)**

9 **through (1, 10) and (3, 14)**

10 **through (−2, 4) and (1, 7)**

11 **through (−2, 10) and (1, 4)**

12 **through (1, 2) and (2, 4)**

12. Write the equation of a line (continued)

Write the equation of each line through the two points.

13 **through (2, −1) and (4, 1)**

14 **through (2, −5) and (4, −9)**

15 **through (1, −3) and (3, −9)**

16 **through (−3, −1) and (−1, 1)**

13. Absolute value equations

Solve each equation. Each has two answers.

17 $|x - 5| = 3$

18 $|x + 6| = 4$

19 $|x| = 5$

20 $|x - 4| = 4$

21 $|x - 5| = 2$

22 $|x + 5| = 4$

23 $|x - 2| = 6$

24 $|x - 6| = 3$

25 $|x - 1| = 3$

26 $|x| = 4$

Function Notation

WORKED EXAMPLE

$f(x) = x + 3$ $f(3) =$ 6

14. Function notation and composition

Work out the value each function asks for.

1 $f(x) = -x + 5$ $f(11) =$ ____

2 $f(x) = 9x - 14$ $f(2) =$ ____

3 $f(x) = -x - 16$ $f(7) =$ ____

4 $f(x) = x - 1$ $f(6) =$ ____

5 $f(x) = -5x - 4$ $f(3) =$ ____

6 $f(x) = x + 7$ $f(-1) =$ ____

7 $f(x) = -2x + 10$ $f(-3) =$ ____

8 $f(x) = -x + 7$ $f(3) =$ ____

9 $f(x) = 9x - 10$ $f(3) =$ ____

10 $f(x) = 3x + 3$ $f(-4) =$ ____

11 $f(x) = 7x + 7$ $f(-3) =$ ____

12 $f(x) = -x + 3$ $f(6) =$ ____

13 $f(x) = 6x + 7$ $f(2) =$ ____

14 $f(x) = 4x + 11$ $f(3) =$ ____

15 $f(x) = -2x - 8$ $f(5) =$ ____

16 $f(x) = x + 16$ $f(2) =$ ____

17 $f(x) = x + 1$ $f(10) =$ ____

18 $f(x) = -2x - 12$ $f(6) =$ ____

19 $f(x) = 6x + 9$ $f(-2) =$ ____

20 $f(x) = -2x + 13$ $f(8) =$ ____

15. Inequalities

Solve each one, then write >, <, ≥ or ≤ in the box.

21 $9x + 7 \geq -2$ x ☐ -1

22 $7x + 4 \geq -94$ x ☐ -14

23 $-7x < 28$ x ☐ -4

24 $6x + 4 \leq 64$ x ☐ 10

25 $6x < -60$ x ☐ -10

26 $2x \geq -4$ x ☐ -2

27 $2x \geq -96$ x ☐ -48

28 $x + 1 > 7$ x ☐ 6

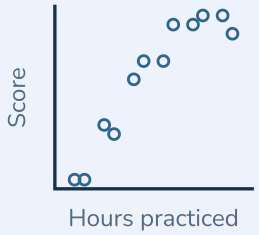
29 $x + 5 \leq 15$ x ☐ 10

30 $8x + 1 \geq -71$ x ☐ -9

Scatter Plots and Trend Lines

WORKED EXAMPLE

Practice and test scores



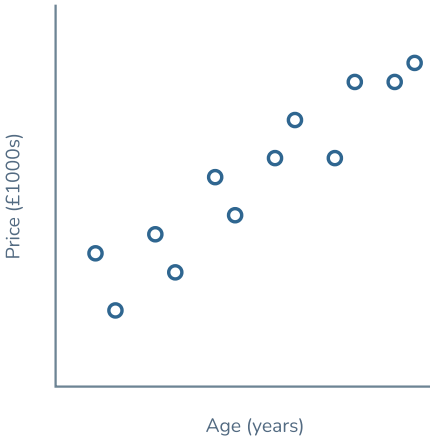
Do the points cluster along a line, or spread out?

(along a line) spread out

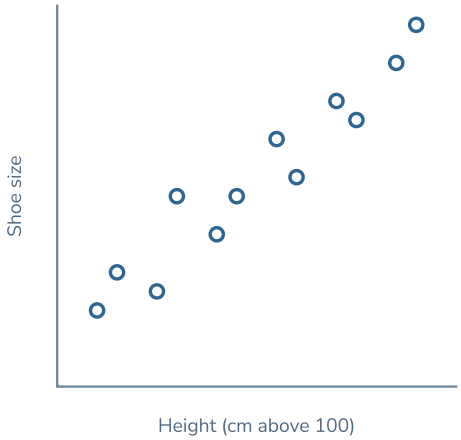
16. Scatter plots

Read each scatter plot and circle the answer.

Age and price of a used car



Height and shoe size



1 **Is there a point well away from the rest?**

yes no

2 **Do the points cluster along a line, or spread out?**

along a line spread out

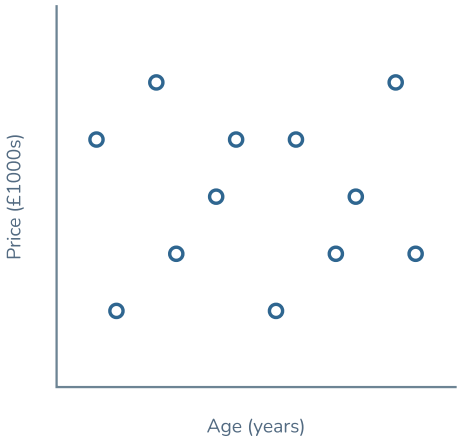
3 **Is the association positive, negative or none?**

positive negative none

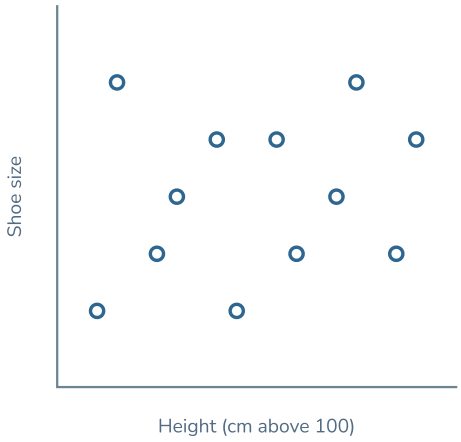
16. Scatter plots (continued)

Read each scatter plot and circle the answer.

Age and price of a used car



Height and shoe size



4 **Do the points cluster along a line, or spread out?**

along a line spread out

5 **Is the association positive, negative or none?**

positive negative none

6 **Is the pattern linear or nonlinear?**

linear nonlinear

17. Intercepts of a line

Find the x-intercept of each line.

7 $x - y = -1$ **x =** ____

8 $3x + 2y = 18$ **x =** ____

9 $5x - 6y = 30$ **x =** ____

10 $2x - y = 6$ **x =** ____

11 $x - y = 5$ **x =** ____

12 $x + y = 8$ **x =** ____

4-1

Factoring Out the GCF

WORKED EXAMPLE

48x + 12

GCF = 12

12(4x + 1)

18. Taking out a common factor

Take out the greatest common factor. Fill in every number.

1 **99x + 81**

2 **63x + 9**

3 **144x + 32**

4 **12x + 8**

5 **36x + 60**

6 **3x + 27**

7 **161x + 230**

8 **14x + 8**

9 **16x + 6**

10 **99x + 22**

11 **115x + 207**

12 **32x + 20**

13 **63x + 56**

14 **65x + 78**

15 **11x + 33**

16 **72x + 8**

17 **108x + 24**

18 **220x + 240**

19 **18x + 12**

20 **90x + 10**

19. Write the equation of a line

Write the equation of each line through the two points.

21 **through (2, 8) and (4, 14)**

22 **through (1, −2) and (3, −8)**

23 **through (2, 3) and (4, 9)**

24 **through (1, 3) and (3, 1)**

25 **through (2, 0) and (4, 2)**

26 **through (−1, 4) and (2, 7)**

19. Write the equation of a line (continued)

Write the equation of each line through the two points.

27 **through (−1, 7) and (2, −5)**

28 **through (−3, 7) and (−1, 3)**

29 **through (−1, 3) and (2, 12)**

30 **through (−2, −1) and (1, 8)**

Difference of Two Squares

20. Difference of two squares

Each one is a difference of two squares. Factor it.

1 $x^2 - 16$

2 $x^2 - 100$

3 $x^2 - 484$

4 $x^2 - 4$

5 $x^2 - 9$

6 $x^2 - 196$

7 $x^2 - 400$

8 $x^2 - 121$

9 $x^2 - 81$

10 $x^2 - 225$

11 $x^2 - 784$

12 $x^2 - 64$

13 $x^2 - 25$

14 $x^2 - 576$

15 $x^2 - 169$

16 $x^2 - 36$

17 $x^2 - 144$

18 $x^2 - 529$

19 $x^2 - 49$

20 $x^2 - 625$

21. Function notation and composition

Work out the value each function asks for.

21 $f(x) = -x + 7$ $f(3) =$ _____

22 $f(x) = -4x - 8$ $f(-2) =$ _____

23 $f(x) = 3x - 4$ $f(-3) =$ _____

24 $f(x) = -x + 5$ $f(11) =$ _____

25 $f(x) = -4x + 5$ $f(-1) =$ _____

26 $f(x) = -x - 2$ $f(-3) =$ _____

27 $f(x) = 8x - 10$ $f(4) =$ _____

28 $f(x) = x - 20$ $f(4) =$ _____

29 $f(x) = -3x + 9$ $f(3) =$ _____

30 $f(x) = 2x + 2$ $f(-2) =$ _____

Factoring Quadratics

WORKED EXAMPLE

$x^2 + 18x + 32$

multiply to 32 · add to 18
16 and 2
= (x + 16)(x + 2)

22. Factoring $x^2 + bx + c$

Factor each one.

1 $x^2 + 18x + 65$

2 $x^2 + 38x + 361$

3 $x^2 + 23x + 112$

4 $x^2 + 24x + 95$

5 $x^2 + 13x + 22$

6 $x^2 + 28x + 147$

7 $x^2 + 26x + 120$

8 $x^2 + 21x + 20$

9 $x^2 + 7x + 6$

10 $x^2 + 22x + 121$

11 $x^2 + 29x + 154$

12 $x^2 + 8x + 7$

13 $x^2 + 20x + 19$

14 $x^2 + 21x + 108$

15 $x^2 + 34x + 289$

16 $x^2 + 16x + 63$

17 $x^2 + 16x + 39$

18 $x^2 + 35x + 304$

19 $x^2 + 2x + 1$

20 $x^2 + 22x + 85$

23. Evaluate expressions

Put the number in place of the letter, then work each one out.

21 $x = 10$ $4x - 20 =$ ___

22 $x = 11$ $2x - 16 =$ ___

23 $x = 7$ $5x - 9 =$ ___

24 $x = 2$ $8x + 2 =$ ___

25 $a = 2, b = 4$ $3a + b =$ ___

26 $x = 2$ $7x + 9 =$ ___

27 $x = 7$ $3x + 7 =$ ___

28 $x = 2$ $4x - 2 =$ ___

29 $x = 6$ $3x + 6 =$ ___

30 $x = 4$ $6x - 14 =$ ___

Solving Quadratics by Factoring

WORKED EXAMPLE

$(x - 8)(x - 9) = 0$

$x - 8 = 0$ or $x - 9 = 0$
$x - 8 = 0 \rightarrow x = 8$
$x - 9 = 0 \rightarrow x = 9$
$x = 8$ or $x = 9$

24. Solving $x^2 + bx + c = 0$

Each product is zero. Write the two values of x.

1 $(x - 20)(x - 2) = 0$

2 $(x - 3)(x + 3) = 0$

3 $(x - 20)(x + 1) = 0$

4 $(x + 13)(x + 6) = 0$

5 $(x + 13)(x + 19) = 0$

6 $(x + 13)(x + 1) = 0$

7 $(x - 3)(x - 8) = 0$

8 $(x - 10)(x + 14) = 0$

9 $(x + 7)(x + 20) = 0$

10 $(x + 14)(x - 5) = 0$

11 $(x - 14)(x - 6) = 0$

12 $(x - 8)(x - 12) = 0$

13 $(x + 12)(x - 20) = 0$

14 $(x + 8)(x - 14) = 0$

15 $(x + 2)(x - 19) = 0$

16 $(x + 8)(x - 10) = 0$

25. Taking out a common factor

Take out the greatest common factor. Fill in every number.

17 $70x + 10$

18 $48x + 12$

19 $105x + 15$

20 $7x + 42$

21 $14x + 2$

22 $3x + 21$

23 $14x + 154$

24 $4x + 4$

25 $55x + 45$

26 $24x + 84$

Solving with Square Roots

WORKED EXAMPLE

$(x - 3)^2 = 25$

$x - 3 = \pm 5$

$x = 3 \pm 5$

$x = 8$ or $x = -2$

26. Solve quadratics by square roots

Solve each equation by taking square roots.

1 $(x + 1)^2 = 25$

2 $(x - 3)^2 = 9$

3 $(x - 2)^2 = 64$

4 $(x + 5)^2 = 1$

5 $6x^2 = 216$

6 $5x^2 = 80$

7 $(x + 2)^2 = 36$

8 $3x^2 = 243$

9 $(x + 3)^2 = 1$

10 $4x^2 = 36$

11 $(x + 8)^2 = 9$

12 $(x + 3)^2 = 4$

13 $7x^2 = 63$

14 $(x - 7)^2 = 4$

15 $4x^2 = 64$

16 $3x^2 = 27$

27. Difference of two squares

Each one is a difference of two squares. Factor it.

17 $x^2 - 196$

18 $x^2 - 9$

19 $x^2 - 441$

20 $x^2 - 144$

21 $x^2 - 121$

22 $x^2 - 100$

23 $x^2 - 676$

24 $x^2 - 25$

25 $x^2 - 36$

26 $x^2 - 81$

How Many Solutions?

WORKED EXAMPLE

10x − 31 = 10x − 27

x is gone
−31 = −27 is false
no solution

28. How many solutions?

How many solutions does each equation have? Show how you know.

1 **−14x + 35 = −14x + 34**

2 **11x + 49 = 11x + 55**

3 **−15x − 23 = −23 − 15x**

4 **x − 44 = −44 + x**

5 **15x − 37 = −12x + 4**

6 **−12x + 49 = −12x + 51**

7 **20x + 25 = 20x + 28**

8 **2x − 24 = −7x + 1**

9 **17x + 25 = 25 + 17x**

10 **−2x + 12 = −2x + 14**

11 **5x − 15 = 11x**

12 **18x − 27 = −2x + 45**

13 **7x + 40 = 40 + 7x**

14 **16x + 23 = −6x + 55**

15 **5x − 50 = 12x + 24**

16 **12x + 58 = 58 + 12x**

17 **−16x + 26 = 8x + 54**

18 **7x − 49 = −10x − 44**

19 **−9x = −9x + 5**

20 **17x + 7 = 14x + 10**

29. Factoring $x^2 + bx + c$

Factor each one.

21 $x^2 + 28x + 171$

22 $x^2 + 24x + 63$

23 $x^2 + 25x + 150$

24 $x^2 + 24x + 23$

25 $x^2 + 4x + 4$

26 $x^2 + 6x + 8$

27 $x^2 + 36x + 315$

28 $x^2 + 20x + 51$

29 $x^2 + 24x + 119$

30 $x^2 + 14x + 33$

Check the Expansion

30. Check the expansion

Read each line of working and choose the answer that is correct.

1 **On Sam's paper: $(x + 6)^2 - 36 = 2x - 24$. Which line should replace it?**

$x^2 + 12x + 36$ | $2x - 24$ | $x^2 + 12x$ | $x^2 + 6x$

Why? .....

2 **Theo wrote $(x + 12)^2 - 144 = 2x - 120$. Which is correct?**

$2x - 120$ | $x^2 + 24x + 144$ | $x^2 + 24x$ | $x^2 + 12x$

Why? .....

3 **Imani says $(x + 10)^2 - 100$ comes to $x^2 + 10x$. Which is correct?**

$x^2 + 20x$ | x^2 | $x^2 + 10x$ | $x^2 + 20x + 100$

Why? .....

4 **Sam says $(x + 3)^2 - 9$ comes to $x^2 + 6x$. Which is correct?**

$x^2 + 6x$ | $x^2 + 6x + 9$ | $x^2 + 3x$ | x^2

Why? .....

5 **Ana wrote $(x + 5)^2 - 25 = x^2 + 10x + 25$. Which is correct?**

$x^2 + 10x + 25$ | x^2 | $x^2 + 5x$ | $x^2 + 10x$

Why? .....

6 **Noor says $(x + 7)^2 - 49$ comes to $x^2 + 14x$. Which is correct?**

$x^2 + 14x$ | x^2 | $x^2 + 7x$ | $x^2 + 14x + 49$

Why? .....

7 **Check this line: $(x + 8)^2 - 64 = x^2 + 8x$. Which is correct?**

x^2 | $x^2 + 16x + 64$ | $x^2 + 8x$ | $x^2 + 16x$

Why? .....

8 **$(x + 11)^2 - 121 = x^2 + 22x + 121$ is wrong. Which is right?**

$x^2 + 22x + 121$ | x^2 | $x^2 + 11x$ | $x^2 + 22x$

Why? .....

Answer Key (1 of 6)

Evaluating Expressions

Evaluate expressions

1. $8x - 16$ ($x = 4$) → 16
2. $2x - 6$ ($x = 12$) → 18
3. $2x + 15$ ($x = 9$) → 33
4. $4x + 9$ ($x = 3$) → 21
5. $2x - 1$ ($x = 9$) → 17
6. $2x + 4$ ($x = 8$) → 20
7. $5a + b$ ($a = 3$, $b = 6$) → 21
8. $2x - 10$ ($x = 8$) → 6
9. $2x - 1$ ($x = 4$) → 7
10. $3x - 1$ ($x = 2$) → 5
11. $2x + 10$ ($x = 8$) → 26
12. $2x - 4$ ($x = 7$) → 10
13. $4x - 19$ ($x = 9$) → 17
14. $3x + 10$ ($x = 8$) → 34
15. $3x - 11$ ($x = 6$) → 7
16. $4x - 9$ ($x = 3$) → 3
17. $2x - 4$ ($x = 4$) → 4
18. $4x + 8$ ($x = 6$) → 32
19. $3x - 20$ ($x = 9$) → 7
20. $6x - 17$ ($x = 4$) → 7
21. $2a + b$ ($a = 4$, $b = 6$) → 14
22. $3x$ ($x = 3$) → 9
23. $7x + 1$ ($x = 3$) → 22
24. $5x - 5$ ($x = 2$) → 5

Combining Like Terms

Collect like terms

1. $1 + 5x + 4 + 3x \rightarrow 8x + 5$
2. $3 + 3x + 7 + 7x \rightarrow 10x + 10$
3. $9 + 5x + 8 + 5x \rightarrow 10x + 17$
4. $11 + 8x + 1 + 4x \rightarrow 12x + 12$
5. $12x + 1 - 4x \rightarrow 8x + 1$
6. $10 + 2x + 1 + 6x \rightarrow 8x + 11$
7. $6 + 6x + 10 + 4x \rightarrow 10x + 16$
8. $6 + 8x + 5 + 2x \rightarrow 10x + 11$
9. $8x + x + 10 \rightarrow 9x + 10$
10. $9x + 5 - 5x \rightarrow 4x + 5$
11. $10x + 12 - 3x - 11 \rightarrow 7x + 1$
12. $9 + 6x + 5 + 4x \rightarrow 10x + 14$
13. $5 + 4x + 5 + 8x \rightarrow 12x + 10$
14. $9x + 20 - 8x \rightarrow 1x + 20$
15. $5x + 18 - 4x - 17 \rightarrow 1x + 1$
16. $10 + 3x + 1 + x \rightarrow 4x + 11$
17. $3x + 15 - 2x - 14 \rightarrow 1x + 1$
18. $8x + 15 - 3x \rightarrow 5x + 15$
19. $9x + 15 - 8x - 14 \rightarrow 1x + 1$
20. $1 + 5x + 7 + 7x \rightarrow 12x + 8$

Combining Like Terms (continued)

Collect like terms (continued)

21. $4x + 14 - 3x - 8 \rightarrow 1x + 6$
22. $4 + 5x + 6 + 2x \rightarrow 7x + 10$
23. $7 + 4x + 4 + 5x \rightarrow 9x + 11$
24. $2x + 6x + 18 \rightarrow 8x + 18$

The Distributive Property

Multiply out brackets

1. $3(x + 8) \rightarrow 3x + 24$
2. $4(2x + 5) \rightarrow 8x + 20$
3. $6(7x + 3) \rightarrow 42x + 18$
4. $2(4x + 4) \rightarrow 8x + 8$
5. $2(7x + 11) \rightarrow 14x + 22$
6. $2(9x + 10) \rightarrow 18x + 20$
7. $2(11 + 5x) \rightarrow 10x + 22$
8. $2(6x - 5) \rightarrow 12x - 10$
9. $6(x - 4) \rightarrow 6x - 24$
10. $3(3 + 3x) \rightarrow 9x + 9$
11. $4(3x - 6) \rightarrow 12x - 24$
12. $3(5x + 6) \rightarrow 15x + 18$
13. $2(7x + 10) \rightarrow 14x + 20$
14. $8(x + 6) \rightarrow 8x + 48$
15. $2(7 + x) \rightarrow 2x + 14$
16. $2(7x - 3) \rightarrow 14x - 6$
17. $2(7x + 2) \rightarrow 14x + 4$
18. $7(2x + 7) \rightarrow 14x + 49$
19. $4(4x + 7) \rightarrow 16x + 28$
20. $3(2 + 3x) \rightarrow 9x + 6$
21. $2(9 + 5x) \rightarrow 10x + 18$
22. $4(2x + 8) \rightarrow 8x + 32$
23. $7(x + 5) \rightarrow 7x + 35$
24. $4(5x + 5) \rightarrow 20x + 20$

Absolute Value

Absolute value

1. $|17| \rightarrow 17$
2. $|7 - 19| \rightarrow 12$
3. $|-4| + |2| \rightarrow 6$
4. $|7 - 15| \rightarrow 8$
5. $|3 - 9| \rightarrow 6$
6. $|3 - 11| \rightarrow 8$
7. $|-3| + |8| \rightarrow 11$
8. $|13 - 14| \rightarrow 1$
9. $|8 - 10| \rightarrow 2$
10. $|-8| \rightarrow 8$
11. $|-5| + |7| \rightarrow 12$
12. $|7 - 11| \rightarrow 4$
13. $|-5| \rightarrow 5$

Answer Key (2 of 6)

Absolute Value (continued)

Absolute value (continued)

14. $|11| + |-2| \rightarrow 13$
15. $|4 - 9| \rightarrow 5$
16. $|12 - 14| \rightarrow 2$
17. $|7 - 13| \rightarrow 6$
18. $|2| + |-3| \rightarrow 5$
19. $|8| \rightarrow 8$
20. $|1 - 8| \rightarrow 7$
21. $|3| + |-11| \rightarrow 14$
22. $|7 - 17| \rightarrow 10$
23. $|12 - 16| \rightarrow 4$
24. $|14| \rightarrow 14$

Evaluate expressions

25. $2x - 14\ (x = 8) \rightarrow 2$
26. $6x - 18\ (x = 6) \rightarrow 18$
27. $3a + b\ (a = 2, b = 7) \rightarrow 13$
28. $4x - 5\ (x = 3) \rightarrow 7$
29. $3x - 17\ (x = 6) \rightarrow 1$
30. $5x - 3\ (x = 4) \rightarrow 17$
31. $2x + 18\ (x = 7) \rightarrow 32$
32. $4x - 18\ (x = 8) \rightarrow 14$
33. $7x + 9\ (x = 2) \rightarrow 23$
34. $10x\ (x = 3) \rightarrow 30$

Absolute Value Equations

Absolute value equations

1. $|x - 2| = 3 \rightarrow x = -1$ or $x = 5$
2. $|x - 6| = 3 \rightarrow x = 3$ or $x = 9$
3. $|x - 4| = 5 \rightarrow x = -1$ or $x = 9$
4. $|x + 1| = 3 \rightarrow x = -4$ or $x = 2$
5. $|x - 2| = 4 \rightarrow x = -2$ or $x = 6$
6. $|x - 3| = 3 \rightarrow x = 0$ or $x = 6$
7. $|x + 1| = 7 \rightarrow x = -8$ or $x = 6$
8. $|x - 2| = 2 \rightarrow x = 0$ or $x = 4$
9. $|x + 1| = 2 \rightarrow x = -3$ or $x = 1$
10. $|x| = 2 \rightarrow x = -2$ or $x = 2$
11. $|x - 1| = 6 \rightarrow x = -5$ or $x = 7$
12. $|x| = 3 \rightarrow x = -3$ or $x = 3$
13. $|x + 6| = 4 \rightarrow x = -10$ or $x = -2$
14. $|x - 5| = 7 \rightarrow x = -2$ or $x = 12$
15. $|x + 4| = 3 \rightarrow x = -7$ or $x = -1$
16. $|x + 6| = 2 \rightarrow x = -8$ or $x = -4$
17. $|x - 3| = 4 \rightarrow x = -1$ or $x = 7$
18. $|x + 5| = 5 \rightarrow x = -10$ or $x = 0$
19. $|x + 3| = 4 \rightarrow x = -7$ or $x = 1$
20. $|x - 5| = 4 \rightarrow x = 1$ or $x = 9$

Collect like terms

21. $7 + 9x + 2 + 2x \rightarrow 11x + 9$

Absolute Value Equations (continued)

Collect like terms (continued)

22. $9 + 9x + 4 + 2x \rightarrow 11x + 13$
23. $9x + 13 - x - 12 \rightarrow 8x + 1$
24. $9 + 5x + 8 + 3x \rightarrow 8x + 17$
25. $3 + 4x + 9 + 3x \rightarrow 7x + 12$
26. $3 + 5x + 5 + 5x \rightarrow 10x + 8$
27. $4x + 15 - 2x - 8 \rightarrow 2x + 7$
28. $4 + 8x + 3 + 2x \rightarrow 10x + 7$
29. $7 + 3x + 5 + 8x \rightarrow 11x + 12$
30. $11 + 7x + 1 + x \rightarrow 8x + 12$

Solving Inequalities

Inequalities

1. $6x + 9 > -39 \rightarrow x > -8$
2. $x - 1 > 19 \rightarrow x > 20$
3. $3x + 2 < -79 \rightarrow x < -27$
4. $2x \geq -2 \rightarrow x \geq -1$
5. $2x + 10 < 36 \rightarrow x < 13$
6. $6x + 7 \geq -71 \rightarrow x \geq -13$
7. $2x > 64 \rightarrow x > 32$
8. $3x + 4 > 22 \rightarrow x > 6$
9. $2x + 10 \leq 18 \rightarrow x \leq 4$
10. $4x + 6 < 18 \rightarrow x < 3$
11. $-4x + 12 > 44 \rightarrow x < -8$
12. $9x + 7 < -2 \rightarrow x < -1$
13. $2x + 9 \leq -17 \rightarrow x \leq -13$
14. $-3x \leq -30 \rightarrow x \geq 10$
15. $2x + 4 \geq 48 \rightarrow x \geq 22$
16. $x + 9 \leq 17 \rightarrow x \leq 8$
17. $x + 5 \leq 19 \rightarrow x \leq 14$
18. $3x + 7 \geq -86 \rightarrow x \geq -31$
19. $3x \geq 12 \rightarrow x \geq 4$
20. $4x + 11 > -9 \rightarrow x > -5$

Multiply out brackets

21. $7(4x + 2) \rightarrow 28x + 14$
22. $3(4x + 12) \rightarrow 12x + 36$
23. $4(2 + 5x) \rightarrow 20x + 8$
24. $5(2x - 2) \rightarrow 10x - 10$
25. $4(7x + 3) \rightarrow 28x + 12$
26. $4(x + 3) \rightarrow 4x + 12$
27. $4(9x + 4) \rightarrow 36x + 16$
28. $2(4x + 9) \rightarrow 8x + 18$
29. $3(2x - 9) \rightarrow 6x - 27$
30. $2(8x + 8) \rightarrow 16x + 16$

Intercepts

Intercepts of a line

1. $2x + y = 6$ the x-intercept = 3

Answer Key (3 of 6)

Intercepts (continued)

Intercepts of a line (continued)

2. $8x + y = 8$ the x-intercept = 1
3. $x + y = -3$ the x-intercept = −3
4. $9x + 4y = 36$ the x-intercept = 4
5. $x - y = 5$ the x-intercept = 5
6. $3x - 5y = 15$ the x-intercept = 5
7. $4x + y = -8$ the x-intercept = −2
8. $x - 4y = 8$ the x-intercept = 8
9. $x + 5y = 5$ the x-intercept = 5
10. $3x - y = 6$ the x-intercept = 2
11. $5x - 3y = -15$ the x-intercept = −3
12. $x + 3y = 3$ the x-intercept = 3
13. $9x + 2y = 18$ the x-intercept = 2
14. $x - y = -5$ the x-intercept = −5
15. $7x - 2y = 14$ the x-intercept = 2
16. $4x + 3y = 24$ the x-intercept = 6

Absolute value

17. $|3 - 9| \rightarrow 6$
18. $|15 - 17| \rightarrow 2$
19. $|3| + |-9| \rightarrow 12$
20. $|10| \rightarrow 10$
21. $|3 - 7| \rightarrow 4$
22. $|5 - 12| \rightarrow 7$
23. $|5| + |-8| \rightarrow 13$
24. $|2| + |-1| \rightarrow 3$
25. $|-1| + |2| \rightarrow 3$
26. $|7 - 16| \rightarrow 9$

Writing the Equation of a Line

Write the equation of a line

1. through (−2, 0) and (1, 9) → $y = 3x + 6$
2. through (−2, 2) and (1, 5) → $y = x + 4$
3. through (1, 7) and (3, 5) → $y = -x + 8$
4. through (−1, 6) and (2, 9) → $y = x + 7$
5. through (−3, 8) and (−1, 4) → $y = -2x + 2$
6. through (2, 8) and (4, 14) → $y = 3x + 2$
7. through (1, −1) and (3, −7) → $y = -3x + 2$
8. through (−1, 2) and (2, 5) → $y = x + 3$
9. through (1, 10) and (3, 14) → $y = 2x + 8$
10. through (−2, 4) and (1, 7) → $y = x + 6$
11. through (−2, 10) and (1, 4) → $y = -2x + 6$
12. through (1, 2) and (2, 4) → $y = 2x$
13. through (2, −1) and (4, 1) → $y = x - 3$
14. through (2, −5) and (4, −9) → $y = -2x - 1$
15. through (1, −3) and (3, −9) → $y = -3x$
16. through (−3, −1) and (−1, 1) → $y = x + 2$

Absolute value equations

17. $|x - 5| = 3 \rightarrow x = 2$ or $x = 8$

Writing the Equation of a Line (continued)

Absolute value equations (continued)

18. $|x + 6| = 4 \rightarrow x = -10$ or $x = -2$
19. $|x| = 5 \rightarrow x = -5$ or $x = 5$
20. $|x - 4| = 4 \rightarrow x = 0$ or $x = 8$
21. $|x - 5| = 2 \rightarrow x = 3$ or $x = 7$
22. $|x + 5| = 4 \rightarrow x = -9$ or $x = -1$
23. $|x - 2| = 6 \rightarrow x = -4$ or $x = 8$
24. $|x - 6| = 3 \rightarrow x = 3$ or $x = 9$
25. $|x - 1| = 3 \rightarrow x = -2$ or $x = 4$
26. $|x| = 4 \rightarrow x = -4$ or $x = 4$

Function Notation

Function notation and composition

1. $f(x) = -x + 5$ $f(11) = -6$
2. $f(x) = 9x - 14$ $f(2) = 4$
3. $f(x) = -x - 16$ $f(7) = -23$
4. $f(x) = x - 1$ $f(6) = 5$
5. $f(x) = -5x - 4$ $f(3) = -19$
6. $f(x) = x + 7$ $f(-1) = 6$
7. $f(x) = -2x + 10$ $f(-3) = 16$
8. $f(x) = -x + 7$ $f(3) = 4$
9. $f(x) = 9x - 10$ $f(3) = 17$
10. $f(x) = 3x + 3$ $f(-4) = -9$
11. $f(x) = 7x + 7$ $f(-3) = -14$
12. $f(x) = -x + 3$ $f(6) = -3$
13. $f(x) = 6x + 7$ $f(2) = 19$
14. $f(x) = 4x + 11$ $f(3) = 23$
15. $f(x) = -2x - 8$ $f(5) = -18$
16. $f(x) = x + 16$ $f(2) = 18$
17. $f(x) = x + 1$ $f(10) = 11$
18. $f(x) = -2x - 12$ $f(6) = -24$
19. $f(x) = 6x + 9$ $f(-2) = -3$
20. $f(x) = -2x + 13$ $f(8) = -3$

Inequalities

21. $9x + 7 \geq -2 \rightarrow x \geq -1$
22. $7x + 4 \geq -94 \rightarrow x \geq -14$
23. $-7x < 28 \rightarrow x > -4$
24. $6x + 4 \leq 64 \rightarrow x \leq 10$
25. $6x < -60 \rightarrow x < -10$
26. $2x \geq -4 \rightarrow x \geq -2$
27. $2x \geq -96 \rightarrow x \geq -48$
28. $x + 1 > 7 \rightarrow x > 6$
29. $x + 5 \leq 15 \rightarrow x \leq 10$
30. $8x + 1 \geq -71 \rightarrow x \geq -9$

Answer Key (4 of 6)

Scatter Plots and Trend Lines

Scatter plots

1. Price (£1000s) against Age (years) — Is there a point well away from the rest? no
2. Price (£1000s) against Age (years) — Do the points cluster along a line, or spread out? along a line
3. Shoe size against Height (cm above 100) — Is the association positive, negative or none? positive
4. Price (£1000s) against Age (years) — Do the points cluster along a line, or spread out? spread out
5. Shoe size against Height (cm above 100) — Is the association positive, negative or none? none
6. Shoe size against Height (cm above 100) — Is the pattern linear or nonlinear? nonlinear

Intercepts of a line

7. $x - y = -1$ the x-intercept = -1
8. $3x + 2y = 18$ the x-intercept = 6
9. $5x - 6y = 30$ the x-intercept = 6
10. $2x - y = 6$ the x-intercept = 3
11. $x - y = 5$ the x-intercept = 5
12. $x + y = 8$ the x-intercept = 8

Factoring Out the GCF

Taking out a common factor

1. $99x + 81 = 9(11x + 9)$ ($99 \div 9 = 11$, $81 \div 9 = 9$)
2. $63x + 9 = 9(7x + 1)$ ($63 \div 9 = 7$, $9 \div 9 = 1$)
3. $144x + 32 = 16(9x + 2)$ ($144 \div 16 = 9$, $32 \div 16 = 2$)
4. $12x + 8 = 4(3x + 2)$ ($12 \div 4 = 3$, $8 \div 4 = 2$)
5. $36x + 60 = 12(3x + 5)$ ($36 \div 12 = 3$, $60 \div 12 = 5$)
6. $3x + 27 = 3(x + 9)$ ($3 \div 3 = 1$, $27 \div 3 = 9$)
7. $161x + 230 = 23(7x + 10)$ ($161 \div 23 = 7$, $230 \div 23 = 10$)
8. $14x + 8 = 2(7x + 4)$ ($14 \div 2 = 7$, $8 \div 2 = 4$)
9. $16x + 6 = 2(8x + 3)$ ($16 \div 2 = 8$, $6 \div 2 = 3$)
10. $99x + 22 = 11(9x + 2)$ ($99 \div 11 = 9$, $22 \div 11 = 2$)
11. $115x + 207 = 23(5x + 9)$ ($115 \div 23 = 5$, $207 \div 23 = 9$)
12. $32x + 20 = 4(8x + 5)$ ($32 \div 4 = 8$, $20 \div 4 = 5$)
13. $63x + 56 = 7(9x + 8)$ ($63 \div 7 = 9$, $56 \div 7 = 8$)
14. $65x + 78 = 13(5x + 6)$ ($65 \div 13 = 5$, $78 \div 13 = 6$)
15. $11x + 33 = 11(x + 3)$ ($11 \div 11 = 1$, $33 \div 11 = 3$)
16. $72x + 8 = 8(9x + 1)$ ($72 \div 8 = 9$, $8 \div 8 = 1$)
17. $108x + 24 = 12(9x + 2)$ ($108 \div 12 = 9$, $24 \div 12 = 2$)
18. $220x + 240 = 20(11x + 12)$ ($220 \div 20 = 11$, $240 \div 20 = 12$)
19. $18x + 12 = 6(3x + 2)$ ($18 \div 6 = 3$, $12 \div 6 = 2$)
20. $90x + 10 = 10(9x + 1)$ ($90 \div 10 = 9$, $10 \div 10 = 1$)

Write the equation of a line

21. through (2, 8) and (4, 14) → $y = 3x + 2$
22. through (1, −2) and (3, −8) → $y = -3x + 1$
23. through (2, 3) and (4, 9) → $y = 3x - 3$
24. through (1, 3) and (3, 1) → $y = -x + 4$
25. through (2, 0) and (4, 2) → $y = x - 2$

Factoring Out the GCF (continued)

Write the equation of a line (continued)

26. through (−1, 4) and (2, 7) → $y = x + 5$
27. through (−1, 7) and (2, −5) → $y = -4x + 3$
28. through (−3, 7) and (−1, 3) → $y = -2x + 1$
29. through (−1, 3) and (2, 12) → $y = 3x + 6$
30. through (−2, −1) and (1, 8) → $y = 3x + 5$

Difference of Two Squares

Difference of two squares

1. $x^2 - 16 = (x + 4)(x - 4)$ (16 is 4 squared, so the two brackets are (x + 4) and (x − 4))
2. $x^2 - 100 = (x + 10)(x - 10)$ (100 is 10 squared, so the two brackets are (x + 10) and (x − 10))
3. $x^2 - 484 = (x + 22)(x - 22)$ (484 is 22 squared, so the two brackets are (x + 22) and (x − 22))
4. $x^2 - 4 = (x + 2)(x - 2)$ (4 is 2 squared, so the two brackets are (x + 2) and (x − 2))
5. $x^2 - 9 = (x + 3)(x - 3)$ (9 is 3 squared, so the two brackets are (x + 3) and (x − 3))
6. $x^2 - 196 = (x + 14)(x - 14)$ (196 is 14 squared, so the two brackets are (x + 14) and (x − 14))
7. $x^2 - 400 = (x + 20)(x - 20)$ (400 is 20 squared, so the two brackets are (x + 20) and (x − 20))
8. $x^2 - 121 = (x + 11)(x - 11)$ (121 is 11 squared, so the two brackets are (x + 11) and (x − 11))
9. $x^2 - 81 = (x + 9)(x - 9)$ (81 is 9 squared, so the two brackets are (x + 9) and (x − 9))
10. $x^2 - 225 = (x + 15)(x - 15)$ (225 is 15 squared, so the two brackets are (x + 15) and (x − 15))
11. $x^2 - 784 = (x + 28)(x - 28)$ (784 is 28 squared, so the two brackets are (x + 28) and (x − 28))
12. $x^2 - 64 = (x + 8)(x - 8)$ (64 is 8 squared, so the two brackets are (x + 8) and (x − 8))
13. $x^2 - 25 = (x + 5)(x - 5)$ (25 is 5 squared, so the two brackets are (x + 5) and (x − 5))
14. $x^2 - 576 = (x + 24)(x - 24)$ (576 is 24 squared, so the two brackets are (x + 24) and (x − 24))
15. $x^2 - 169 = (x + 13)(x - 13)$ (169 is 13 squared, so the two brackets are (x + 13) and (x − 13))
16. $x^2 - 36 = (x + 6)(x - 6)$ (36 is 6 squared, so the two brackets are (x + 6) and (x − 6))
17. $x^2 - 144 = (x + 12)(x - 12)$ (144 is 12 squared, so the two brackets are (x + 12) and (x − 12))
18. $x^2 - 529 = (x + 23)(x - 23)$ (529 is 23 squared, so the two brackets are (x + 23) and (x − 23))
19. $x^2 - 49 = (x + 7)(x - 7)$ (49 is 7 squared, so the two brackets are (x + 7) and (x − 7))
20. $x^2 - 625 = (x + 25)(x - 25)$ (625 is 25 squared, so the two brackets are (x + 25) and (x − 25))

Function notation and composition

21. $f(x) = -x + 7$ $f(3) = 4$

Answer Key (5 of 6)

Difference of Two Squares (continued)

Function notation and composition (continued)

22. f(x) = −4x − 8 f(−2) = 0
23. f(x) = 3x − 4 f(−3) = −13
24. f(x) = −x + 5 f(11) = −6
25. f(x) = −4x + 5 f(−1) = 9
26. f(x) = −x − 2 f(−3) = 1
27. f(x) = 8x − 10 f(4) = 22
28. f(x) = x − 20 f(4) = −16
29. f(x) = −3x + 9 f(3) = 0
30. f(x) = 2x + 2 f(−2) = −2

Factoring Quadratics

Factoring $x^2 + bx + c$

1. $x^2 + 18x + 65 = (x + 13)(x + 5)$ (13 and 5)
2. $x^2 + 38x + 361 = (x + 19)(x + 19)$ (19 and 19)
3. $x^2 + 23x + 112 = (x + 7)(x + 16)$ (7 and 16)
4. $x^2 + 24x + 95 = (x + 5)(x + 19)$ (5 and 19)
5. $x^2 + 13x + 22 = (x + 11)(x + 2)$ (11 and 2)
6. $x^2 + 28x + 147 = (x + 21)(x + 7)$ (21 and 7)
7. $x^2 + 26x + 120 = (x + 20)(x + 6)$ (20 and 6)
8. $x^2 + 21x + 20 = (x + 20)(x + 1)$ (20 and 1)
9. $x^2 + 7x + 6 = (x + 1)(x + 6)$ (1 and 6)
10. $x^2 + 22x + 121 = (x + 11)(x + 11)$ (11 and 11)
11. $x^2 + 29x + 154 = (x + 7)(x + 22)$ (7 and 22)
12. $x^2 + 8x + 7 = (x + 7)(x + 1)$ (7 and 1)
13. $x^2 + 20x + 19 = (x + 19)(x + 1)$ (19 and 1)
14. $x^2 + 21x + 108 = (x + 9)(x + 12)$ (9 and 12)
15. $x^2 + 34x + 289 = (x + 17)(x + 17)$ (17 and 17)
16. $x^2 + 16x + 63 = (x + 7)(x + 9)$ (7 and 9)
17. $x^2 + 16x + 39 = (x + 13)(x + 3)$ (13 and 3)
18. $x^2 + 35x + 304 = (x + 16)(x + 19)$ (16 and 19)
19. $x^2 + 2x + 1 = (x + 1)(x + 1)$ (1 and 1)
20. $x^2 + 22x + 85 = (x + 17)(x + 5)$ (17 and 5)

Evaluate expressions

21. 4x − 20 (x = 10) → 20
22. 2x − 16 (x = 11) → 6
23. 5x − 9 (x = 7) → 26
24. 8x + 2 (x = 2) → 18
25. 3a + b (a = 2, b = 4) → 10
26. 7x + 9 (x = 2) → 23
27. 3x + 7 (x = 7) → 28
28. 4x − 2 (x = 2) → 6
29. 3x + 6 (x = 6) → 24
30. 6x − 14 (x = 4) → 10

Solving Quadratics by Factoring

Solving $x^2 + bx + c = 0$

1. (x − 20)(x − 2) = 0 → (x − 20)(x − 2) = 0 → x = 20 or x = 2

Solving Quadratics by Factoring (continued)

Solving $x^2 + bx + c = 0$ (continued)

2. (x − 3)(x + 3) = 0 → (x − 3)(x + 3) = 0 → x = 3 or x = −3
3. (x − 20)(x + 1) = 0 → (x − 20)(x + 1) = 0 → x = 20 or x = −1
4. (x + 13)(x + 6) = 0 → (x + 13)(x + 6) = 0 → x = −13 or x = −6
5. (x + 13)(x + 19) = 0 → (x + 13)(x + 19) = 0 → x = −13 or x = −19
6. (x + 13)(x + 1) = 0 → (x + 13)(x + 1) = 0 → x = −13 or x = −1
7. (x − 3)(x − 8) = 0 → (x − 3)(x − 8) = 0 → x = 3 or x = 8
8. (x − 10)(x + 14) = 0 → (x − 10)(x + 14) = 0 → x = 10 or x = −14
9. (x + 7)(x + 20) = 0 → (x + 7)(x + 20) = 0 → x = −7 or x = −20
10. (x + 14)(x − 5) = 0 → (x + 14)(x − 5) = 0 → x = −14 or x = 5
11. (x − 14)(x − 6) = 0 → (x − 14)(x − 6) = 0 → x = 14 or x = 6
12. (x − 8)(x − 12) = 0 → (x − 8)(x − 12) = 0 → x = 8 or x = 12
13. (x + 12)(x − 20) = 0 → (x + 12)(x − 20) = 0 → x = −12 or x = 20
14. (x + 8)(x − 14) = 0 → (x + 8)(x − 14) = 0 → x = −8 or x = 14
15. (x + 2)(x − 19) = 0 → (x + 2)(x − 19) = 0 → x = −2 or x = 19
16. (x + 8)(x − 10) = 0 → (x + 8)(x − 10) = 0 → x = −8 or x = 10

Taking out a common factor

17. 70x + 10 = 10(7x + 1) (70 ÷ 10 = 7, 10 ÷ 10 = 1)
18. 48x + 12 = 12(4x + 1) (48 ÷ 12 = 4, 12 ÷ 12 = 1)
19. 105x + 15 = 15(7x + 1) (105 ÷ 15 = 7, 15 ÷ 15 = 1)
20. 7x + 42 = 7(x + 6) (7 ÷ 7 = 1, 42 ÷ 7 = 6)
21. 14x + 2 = 2(7x + 1) (14 ÷ 2 = 7, 2 ÷ 2 = 1)
22. 3x + 21 = 3(x + 7) (3 ÷ 3 = 1, 21 ÷ 3 = 7)
23. 14x + 154 = 14(x + 11) (14 ÷ 14 = 1, 154 ÷ 14 = 11)
24. 4x + 4 = 4(x + 1) (4 ÷ 4 = 1, 4 ÷ 4 = 1)
25. 55x + 45 = 5(11x + 9) (55 ÷ 5 = 11, 45 ÷ 5 = 9)
26. 24x + 84 = 12(2x + 7) (24 ÷ 12 = 2, 84 ÷ 12 = 7)

Solving with Square Roots

Solve quadratics by square roots

1. $(x + 1)^2 = 25$ → x = −6 or x = 4
2. $(x − 3)^2 = 9$ → x = 0 or x = 6
3. $(x − 2)^2 = 64$ → x = −6 or x = 10
4. $(x + 5)^2 = 1$ → x = −6 or x = −4
5. $6x^2 = 216$ → x = −6 or x = 6
6. $5x^2 = 80$ → x = −4 or x = 4
7. $(x + 2)^2 = 36$ → x = −8 or x = 4
8. $3x^2 = 243$ → x = −9 or x = 9
9. $(x + 3)^2 = 1$ → x = −4 or x = −2
10. $4x^2 = 36$ → x = −3 or x = 3
11. $(x + 8)^2 = 9$ → x = −11 or x = −5
12. $(x + 3)^2 = 4$ → x = −5 or x = −1
13. $7x^2 = 63$ → x = −3 or x = 3

Answer Key (6 of 6)

Solving with Square Roots (continued)

Solve quadratics by square roots (continued)

14. $(x - 7)^2 = 4 \rightarrow x = 5$ or $x = 9$
15. $4x^2 = 64 \rightarrow x = -4$ or $x = 4$
16. $3x^2 = 27 \rightarrow x = -3$ or $x = 3$

Difference of two squares

17. $x^2 - 196 = (x + 14)(x - 14)$ (196 is 14 squared, so the two brackets are $(x + 14)$ and $(x - 14)$)
18. $x^2 - 9 = (x + 3)(x - 3)$ (9 is 3 squared, so the two brackets are $(x + 3)$ and $(x - 3)$)
19. $x^2 - 441 = (x + 21)(x - 21)$ (441 is 21 squared, so the two brackets are $(x + 21)$ and $(x - 21)$)
20. $x^2 - 144 = (x + 12)(x - 12)$ (144 is 12 squared, so the two brackets are $(x + 12)$ and $(x - 12)$)
21. $x^2 - 121 = (x + 11)(x - 11)$ (121 is 11 squared, so the two brackets are $(x + 11)$ and $(x - 11)$)
22. $x^2 - 100 = (x + 10)(x - 10)$ (100 is 10 squared, so the two brackets are $(x + 10)$ and $(x - 10)$)
23. $x^2 - 676 = (x + 26)(x - 26)$ (676 is 26 squared, so the two brackets are $(x + 26)$ and $(x - 26)$)
24. $x^2 - 25 = (x + 5)(x - 5)$ (25 is 5 squared, so the two brackets are $(x + 5)$ and $(x - 5)$)
25. $x^2 - 36 = (x + 6)(x - 6)$ (36 is 6 squared, so the two brackets are $(x + 6)$ and $(x - 6)$)
26. $x^2 - 81 = (x + 9)(x - 9)$ (81 is 9 squared, so the two brackets are $(x + 9)$ and $(x - 9)$)

How Many Solutions?

How many solutions?

1. $-14x + 35 = -14x + 34$: no solution
2. $11x + 49 = 11x + 55$: no solution
3. $-15x - 23 = -23 - 15x$: infinitely many
4. $x - 44 = -44 + x$: infinitely many
5. $15x - 37 = -12x + 4$: one solution
6. $-12x + 49 = -12x + 51$: no solution
7. $20x + 25 = 20x + 28$: no solution
8. $2x - 24 = -7x + 1$: one solution
9. $17x + 25 = 25 + 17x$: infinitely many
10. $-2x + 12 = -2x + 14$: no solution
11. $5x - 15 = 11x$: one solution
12. $18x - 27 = -2x + 45$: one solution
13. $7x + 40 = 40 + 7x$: infinitely many
14. $16x + 23 = -6x + 55$: one solution
15. $5x - 50 = 12x + 24$: one solution
16. $12x + 58 = 58 + 12x$: infinitely many
17. $-16x + 26 = 8x + 54$: one solution
18. $7x - 49 = -10x - 44$: one solution
19. $-9x = -9x + 5$: no solution
20. $17x + 7 = 14x + 10$: one solution

How Many Solutions? (continued)

Factoring $x^2 + bx + c$

21. $x^2 + 28x + 171 = (x + 19)(x + 9)$ (19 and 9)
22. $x^2 + 24x + 63 = (x + 21)(x + 3)$ (21 and 3)
23. $x^2 + 25x + 150 = (x + 15)(x + 10)$ (15 and 10)
24. $x^2 + 24x + 23 = (x + 1)(x + 23)$ (1 and 23)
25. $x^2 + 4x + 4 = (x + 2)(x + 2)$ (2 and 2)
26. $x^2 + 6x + 8 = (x + 4)(x + 2)$ (4 and 2)
27. $x^2 + 36x + 315 = (x + 21)(x + 15)$ (21 and 15)
28. $x^2 + 20x + 51 = (x + 3)(x + 17)$ (3 and 17)
29. $x^2 + 24x + 119 = (x + 7)(x + 17)$ (7 and 17)
30. $x^2 + 14x + 33 = (x + 11)(x + 3)$ (11 and 3)

Check the Expansion

Check the expansion

1. $(x + 6)^2 - 36 = x^2 + 12x$, not $2x - 24$ (the bracket was doubled instead of squared)
2. $(x + 12)^2 - 144 = x^2 + 24x$, not $2x - 120$ (the bracket was doubled instead of squared)
3. $(x + 10)^2 - 100 = x^2 + 20x$, not $x^2 + 10x$ (the middle term was not doubled when the bracket was squared)
4. $(x + 3)^2 - 9 = x^2 + 6x$, which is what was written — this line is correct
5. $(x + 5)^2 - 25 = x^2 + 10x$, not $x^2 + 10x + 25$ (the squared constant was never taken away again)
6. $(x + 7)^2 - 49 = x^2 + 14x$, which is what was written — this line is correct
7. $(x + 8)^2 - 64 = x^2 + 16x$, not $x^2 + 8x$ (the middle term was not doubled when the bracket was squared)
8. $(x + 11)^2 - 121 = x^2 + 22x$, not $x^2 + 22x + 121$ (the squared constant was never taken away again)

Solutions (1 of 16)

Evaluating Expressions

Evaluate expressions

1. $x = 4$ **$8x - 16 =$ ___**

$8x - 16$ with **$x = 4$**

$8 \times 4 = 32$

$32 - 16 = 16$

$8x - 16 = 16$

A letter is a number that has not been written down yet, and this question writes it down: x = 4. So put the value in where the letter is — in BRACKETS, which keeps whatever was being done to the letter being done to the number.

8x means 8 TIMES x, and multiplying happens before adding or taking away however the line is written. So this multiplication comes first: 8 × 4 = 32.

Now the last bit: 32 – 16 = 16. Doing it the other way round — taking away first and multiplying after — gives -96, which is why the order is the whole of this question.

So 8x – 16 is 16 when x = 4.

2. $x = 12$ **$2x - 6 =$ ___**

$2x - 6$ with **$x = 12$**

$2 \times 12 = 24$

$24 - 6 = 18$

$2x - 6 = 18$

A letter is a number that has not been written down yet, and this question writes it down: x = 12. So put the value in where the letter is — in BRACKETS, which keeps whatever was being done to the letter being done to the number.

2x means 2 TIMES x, and multiplying happens before adding or taking away however the line is written. So this multiplication comes first: 2 × 12 = 24.

Now the last bit: 24 – 6 = 18. Doing it the other way round — taking away first and multiplying after — gives 12, which is why the order is the whole of this question.

So 2x – 6 is 18 when x = 12.

Solutions (2 of 16)

Combining Like Terms

Collect like terms

1. **1 + 5x + 4 + 3x**

1 + 5x + 4 + 3x		
5 + 3	=	**8**
1 + 4	=	**5**
8x	+	**5**

There are two kinds of thing on this line and they cannot mix: the x terms, and the plain numbers. Sort them into two piles before adding anything — the x terms are 5x and 3x, and the numbers are 1 and 4.

Add up the x pile, keeping the signs: 5 + 3 = 8, so 8x.

Then the number pile, on its own: 1 + 4 = 5.

So it is 8x + 5 — and that is finished, even though it is still two things. Squashing them into 13x would be answering "8 whats and 5 whats?", and there is no answer to that until somebody says what x is.

2. **3 + 3x + 7 + 7x**

3 + 3x + 7 + 7x		
3 + 7	=	**10**
3 + 7	=	**10**
10x	+	**10**

There are two kinds of thing on this line and they cannot mix: the x terms, and the plain numbers. Sort them into two piles before adding anything — the x terms are 3x and 7x, and the numbers are 3 and 7.

Add up the x pile, keeping the signs: 3 + 7 = 10, so 10x.

Then the number pile, on its own: 3 + 7 = 10.

So it is 10x + 10 — and that is finished, even though it is still two things. Squashing them into 20x would be answering "10 whats and 10 whats?", and there is no answer to that until somebody says what x is.

Solutions (3 of 16)

The Distributive Property

Multiply out brackets

1. **3(x + 8)**

$$3(x + 8)$$
$$3 \times x = 3x$$
$$3 \times 8 = 24$$
$$3x + 24$$

The 3 outside multiplies EVERYTHING inside the bracket, not just the thing next to it. Draw an arrow from the 3 to each of the two terms before working anything out — that is what stops one of them being left behind.

First arrow: $3 \times x = 3x$.

Second arrow: $3 \times 8 = 24$.

Put the two together: $3x + 24$. Check it by putting a small number in for x on both forms — they should agree, because they are the same amount written two ways.

2. **4(2x + 5)**

$$4(2x + 5)$$
$$4 \times 2x = 8x$$
$$4 \times 5 = 20$$
$$8x + 20$$

The 4 outside multiplies EVERYTHING inside the bracket, not just the thing next to it. Draw an arrow from the 4 to each of the two terms before working anything out — that is what stops one of them being left behind.

First arrow: $4 \times 2x = 8x$.

Second arrow: $4 \times 5 = 20$.

Put the two together: $8x + 20$. Check it by putting a small number in for x on both forms — they should agree, because they are the same amount written two ways.

Solutions (4 of 16)

Absolute Value

Absolute value

1. **$|17| = \_\_\_$**

$$|17| = \mathbf{17}$$

The two bars ask for a DISTANCE: how far the thing between them is from zero. A distance is never negative, and that is a fact about distance rather than a rule about minus signs.

17 sits 17 steps from zero already, so there is nothing for the bars to change: $|17| = 17$. Half of these have no sign to remove, which is why "drop the minus" is not the lesson.

2. **$|7 - 19| = \_\_\_$**

$$\mathbf{|7 - 19|}$$
$$7 - 19 = \mathbf{-12}$$
$$|7 - 19| = \mathbf{12}$$

The two bars ask for a DISTANCE: how far the thing between them is from zero. A distance is never negative, and that is a fact about distance rather than a rule about minus signs.

The bars work like brackets: everything inside gets worked out BEFORE the distance is taken. $7 - 19 = -12$.

Now the distance of that from zero: 12. Which is also how far apart 7 and 19 are — the same question read the other way round.

Absolute Value Equations

Absolute value equations

1. **$|x - 2| = 3$**

$$\mathbf{x - 2 = 3} \text{ or } \mathbf{x - 2 = -3}$$
$$x - 2 = 3 \rightarrow \mathbf{x = 5}$$
$$x - 2 = -3 \rightarrow \mathbf{x = -1}$$
$$\mathbf{x = -1} \text{ or } \mathbf{x = 5}$$

The bars mean the inside is 3 away from zero — so it is 3 OR it is −3. Split into two equations.

Solve the first, $x - 2 = 3$: add 2 to both sides: $x = 5$.

Solve the second, $x - 2 = -3$: add 2 to both sides: $x = -1$.

So $x = -1$ or $x = 5$ — both answers.

Solutions (5 of 16)

Absolute Value Equations (continued)

Absolute value equations (continued)

2. $|x - 6| = 3$

$x - 6 = 3$ or $x - 6 = -3$

$x - 6 = 3 \rightarrow x = 9$

$x - 6 = -3 \rightarrow x = 3$

$x = 3$ or $x = 9$

The bars mean the inside is 3 away from zero — so it is 3 OR it is −3. Split into two equations.

Solve the first, x − 6 = 3: add 6 to both sides: x = 9.

Solve the second, x − 6 = −3: add 6 to both sides: x = 3.

So x = 3 or x = 9 — both answers.

Solving Inequalities

Inequalities

1. $6x + 9 > -39$ x ☐ -8

$x > -8$

An inequality comes apart exactly like an equation — the same moves, done to both sides. There is one extra rule and it arrives at the end, so start by ignoring the sign and solving for x.

9 was added last, so it comes off first — off BOTH sides: -39 − 9 = -48.

Now divide both sides by 6: -48 ÷ 6 = −8.

Nothing was divided by a negative here, so the sign has not moved: it is still >. That is the one thing to check on every one of these before writing the answer down.

So x > −8. That is a whole RANGE of numbers rather than one, which is what makes this different from an equation — and why the sign is half the answer.

Solutions (6 of 16)

Solving Inequalities (continued)

Inequalities (continued)

2. $x - 1 > 19 \quad x \square 20$

$x > 20$

An inequality comes apart exactly like an equation — the same moves, done to both sides. There is one extra rule and it arrives at the end, so start by ignoring the sign and solving for x.

1 was taken away last, so it goes back on first — onto BOTH sides: 19 + 1 = 20.

Nothing was divided by a negative here, so the sign has not moved: it is still >. That is the one thing to check on every one of these before writing the answer down.

So x > 20. That is a whole RANGE of numbers rather than one, which is what makes this different from an equation — and why the sign is half the answer.

Intercepts

Intercepts of a line

1. $2x + y = 6 \quad x =$ ___

$2x + y = 6$ $y = 0$

$2x = 6$

$x = 3$

The x-intercept is where the line crosses the x-axis, and there y = 0. Put y = 0 into 2x + y = 6.

The y-term disappears, leaving 2x = 6.

Divide by 2: 6 ÷ 2 = 3 — so the x-intercept is 3.

2. $8x + y = 8 \quad x =$ ___

$8x + y = 8$ $y = 0$

$8x = 8$

$x = 1$

The x-intercept is where the line crosses the x-axis, and there y = 0. Put y = 0 into 8x + y = 8.

The y-term disappears, leaving 8x = 8.

Divide by 8: 8 ÷ 8 = 1 — so the x-intercept is 1.

Solutions (7 of 16)

Writing the Equation of a Line

Write the equation of a line

1. **through (−2, 0) and (1, 9)**

$$\text{slope} = (9 - 0) \div (1 - (-2))$$
$$9 \div 3 = 3$$
$$b = 0 - (3)(-2) = 6$$
$$y = 3x + 6$$

Find the slope first: rise over run, $(9 - 0) \div (1 - (-2)) = 3$.

Put a point back in to find b: $b = y - mx = 0 - (3)(-2) = 6$.

Write it as $y = mx + b$: $y = 3x + 6$.

2. **through (−2, 2) and (1, 5)**

$$\text{slope} = (5 - 2) \div (1 - (-2))$$
$$3 \div 3 = 1$$
$$b = 2 - (1)(-2) = 4$$
$$y = x + 4$$

Find the slope first: rise over run, $(5 - 2) \div (1 - (-2)) = 1$.

Put a point back in to find b: $b = y - mx = 2 - (1)(-2) = 4$.

Write it as $y = mx + b$: $y = x + 4$.

Function Notation

Function notation and composition

1. $f(x) = -x + 5$ **$f(11) =$ ___**

$$f(x) = -x + 5 \text{ at } x = 11$$
$$-1 \times 11 = -11$$
$$-11 + 5 = -6$$
$$f(11) = -6$$

f(11) is a name for the value of the rule when x = 11. Put 11 where the x is — it is not f times 11.

Multiply first, sign and all: $-1 \times 11 = -11$.

Then add 5: $-11 + 5 = -6$.

So $f(11) = -6$.

Solutions (8 of 16)

Function Notation (continued)

Function notation and composition (continued)

2. $f(x) = 9x - 14$ **$f(2) =$** ___

$$f(x) = 9x - 14 \quad \text{at} \quad x = 2$$
$$9 \times 2 = 18$$
$$18 - 14 = 4$$
$$f(2) = 4$$

$f(2)$ is a name for the value of the rule when $x = 2$. Put 2 where the x is — it is not f times 2.

Multiply first, sign and all: $9 \times 2 = 18$.

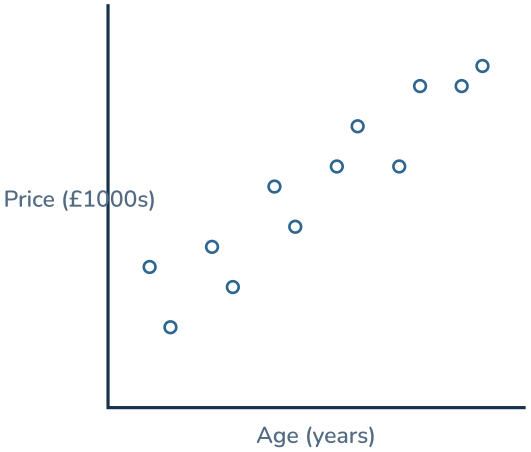
Then subtract 14: $18 - 14 = 4$.

So $f(2) = 4$.

Scatter Plots and Trend Lines

Scatter plots

1. **Age and price of a used car**



Is there a point well away from the rest?

yes no

no

A scatter plot puts two measurements against each other: Age (years) along the bottom, Price (£1000s) up the side, and one dot for each thing measured. There is no single dot to read off — every question here is about what the whole cloud is doing.

An outlier is one point standing well away from all the others — not merely the highest or the lowest, which every set has, but one that has left the crowd.

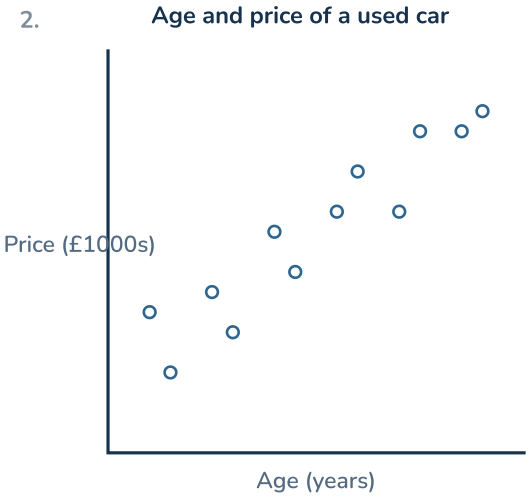
Every point is part of the same crowd. The end ones are further along, but none of them has left it — so no, there is no outlier here.

So the answer is "no". Say what that means for the two things rather than for the graph — and remember it is not a claim that one CAUSES the other. Two measurements can move together for a reason that is nothing to do with either of them.

Scatter Plots and Trend Lines (continued)

Scatter plots (continued)

2.



Do the points cluster along a line, or spread out?

along a line spread out

along a line

A scatter plot puts two measurements against each other: Age (years) along the bottom, Price (£1000s) up the side, and one dot for each thing measured. There is no single dot to read off — every question here is about what the whole cloud is doing.

This one is about how TIGHTLY the points hold to a line, which is a different question from which way they lean. A cloud can lean clearly and still be scattered wide.

They sit in a narrow band, close to one line, so they cluster ALONG A LINE.

So the answer is "along a line". Say what that means for the two things rather than for the graph — and remember it is not a claim that one CAUSES the other. Two measurements can move together for a reason that is nothing to do with either of them.

Solutions (10 of 16)

Factoring Out the GCF

Taking out a common factor

1. **99x + 81**

$$99x + 81 = 9(11x + 9)$$

Before anything else, look at whether every term shares a factor. The numbers here are 99, 81 — so the question is what the biggest number that divides ALL of them is.

List what divides each one and look for the biggest they all share. 99: 1, 3, 9, 11, 33, 99 81: 1, 3, 9, 27, 81.

The biggest number in all of those lists is 9. Take it out in front and divide every term by it: 99 ÷ 9 = 11, 81 ÷ 9 = 9.

Check the bracket before you stop. 11x + 9 — those numbers share nothing but 1, so there is no more to take out and 9 really was the greatest. 99x + 81 = 9(11x + 9).

2. **63x + 9**

$$63x + 9 = 9(7x + 1)$$

Before anything else, look at whether every term shares a factor. The numbers here are 63, 9 — so the question is what the biggest number that divides ALL of them is.

List what divides each one and look for the biggest they all share. 63: 1, 3, 7, 9, 21, 63 9: 1, 3, 9.

The biggest number in all of those lists is 9. Take it out in front and divide every term by it: 63 ÷ 9 = 7, 9 ÷ 9 = 1.

Check the bracket before you stop. 7x + 1 — those numbers share nothing but 1, so there is no more to take out and 9 really was the greatest. 63x + 9 = 9(7x + 1).

Solutions (11 of 16)

Difference of Two Squares

Difference of two squares

1. **$x^2 - 16$**

$$x^2 - 16 = (x + 4)(x - 4)$$

Before factoring anything, check the shape. There are two parts with a − between them, and each part has to be a perfect square: x^2 is x squared, and 16 is 4 squared. Both are, and it is a minus — so this is a difference of two squares.

That shape always factors the same way, and there is nothing to search for: it is the SUM times the DIFFERENCE of the two things being squared — $(x + 4)(x - 4)$.

So the missing number is 4: 16 is 4 squared, so the two brackets are $(x + 4)$ and $(x - 4)$.

2. **$x^2 - 100$**

$$x^2 - 100 = (x + 10)(x - 10)$$

Before factoring anything, check the shape. There are two parts with a − between them, and each part has to be a perfect square: x^2 is x squared, and 100 is 10 squared. Both are, and it is a minus — so this is a difference of two squares.

That shape always factors the same way, and there is nothing to search for: it is the SUM times the DIFFERENCE of the two things being squared — $(x + 10)(x - 10)$.

So the missing number is 10: 100 is 10 squared, so the two brackets are $(x + 10)$ and $(x - 10)$.

Solutions (12 of 16)

Factoring Quadratics

Factoring $x^2 + bx + c$

1. **$x^2 + 18x + 65$**

$$x^2 + 18x + 65 = (x + 13)(x + 5)$$

Factoring this means finding two numbers to put in $(x + \square)(x + \square)$. Two conditions decide them, and both have to hold: they MULTIPLY to 65, the last term, and they ADD to 18, the number in front of x.

Now list the pairs that multiply to 65. There are only 2, so this is a short search rather than a guess: 1 and 65, 5 and 13.

Which of them adds to 18? $13 \times 5 = 65$ and $13 + 5 = 18$. So the two numbers are 13 and 5.

Put them in the brackets: $x^2 + 18x + 65 = (x + 13)(x + 5)$. Multiply it back out if you want to check — the x terms should add up to 18x again.

2. **$x^2 + 38x + 361$**

$$x^2 + 38x + 361 = (x + 19)(x + 19)$$

Factoring this means finding two numbers to put in $(x + \square)(x + \square)$. Two conditions decide them, and both have to hold: they MULTIPLY to 361, the last term, and they ADD to 38, the number in front of x.

Now list the pairs that multiply to 361. There are only 2, so this is a short search rather than a guess: 1 and 361, 19 and 19.

Which of them adds to 38? $19 \times 19 = 361$ and $19 + 19 = 38$. So the two numbers are 19 and 19.

Put them in the brackets: $x^2 + 38x + 361 = (x + 19)(x + 19)$. Multiply it back out if you want to check — the x terms should add up to 38x again.

Solutions (13 of 16)

Solving Quadratics by Factoring

Solving $x^2 + bx + c = 0$

1. $(x - 20)(x - 2) = 0$

$x = 20$ or $x = 2$

The left side is already a product of two brackets, and the right side is zero. That is the useful shape, so no factoring is needed here.

Now the step that gives the answers: a product is zero ONLY when one of the things being multiplied is zero. Two numbers that multiply to zero — one of them has to be zero. So either $(x - 20)$ is zero, or $(x - 2)$ is zero.

Solve each one. $x - 20 = 0$ gives $x = 20$, and $x - 2 = 0$ gives $x = 2$. Notice the sign turns over: a $-$ in the bracket gives a POSITIVE answer.

So $x = 20$ or $x = 2$. Either one makes the equation true, and it is "or" rather than "and" — x cannot be both at once.

2. $(x - 3)(x + 3) = 0$

$x = 3$ or $x = -3$

The left side is already a product of two brackets, and the right side is zero. That is the useful shape, so no factoring is needed here.

Now the step that gives the answers: a product is zero ONLY when one of the things being multiplied is zero. Two numbers that multiply to zero — one of them has to be zero. So either $(x - 3)$ is zero, or $(x + 3)$ is zero.

Solve each one. $x - 3 = 0$ gives $x = 3$, and $x + 3 = 0$ gives $x = -3$. Notice the sign turns over: a $-$ in the bracket gives a positive answer and a $+$ gives a negative one.

So $x = 3$ or $x = -3$. Either one makes the equation true, and it is "or" rather than "and" — x cannot be both at once.

Solutions (14 of 16)

Solving with Square Roots

Solve quadratics by square roots

1. $(x + 1)^2 = 25$

$$x + 1 = \pm 5$$

$$x = -1 \pm 5$$

$$x = 4 \quad \text{or} \quad x = -6$$

Take the square root of both sides — both signs: $x + 1 = \pm\sqrt{25} = \pm 5$.

Then subtract 1 to both sides: $x = -1 \pm 5$.

So $x = 4$ or $x = -6$ — both roots.

2. $(x - 3)^2 = 9$

$$x - 3 = \pm 3$$

$$x = 3 \pm 3$$

$$x = 6 \quad \text{or} \quad x = 0$$

Take the square root of both sides — both signs: $x - 3 = \pm\sqrt{9} = \pm 3$.

Then add 3 to both sides: $x = 3 \pm 3$.

So $x = 6$ or $x = 0$ — both roots.

How Many Solutions?

How many solutions?

1. $-14x + 35 = -14x + 34$

$$35 = 34 \quad \text{is false}$$

no solution

Do not start by hunting for x. Ask a smaller question first: what is left when the x terms are collected on one side?

Both sides have the SAME number of x, so taking it off both sides removes x completely. That is not a mistake — it is the answer arriving early.

What is left is $35 = 34$, which is never true, whatever x is. So there is no solution.

Solutions (15 of 16)

How Many Solutions? (continued)

How many solutions? (continued)

2. **$11x + 49 = 11x + 55$**

49 = 55 **is false**

no solution

Do not start by hunting for x. Ask a smaller question first: what is left when the x terms are collected on one side?

Both sides have the SAME number of x, so taking it off both sides removes x completely. That is not a mistake — it is the answer arriving early.

What is left is 49 = 55, which is never true, whatever x is. So there is no solution.

Check the Expansion

Check the expansion

1. **On Sam's paper: $(x + 6)^2 - 36 = 2x - 24$. Which line should replace it?**

$x^2 + 12x + 36$ | $2x - 24$ | $x^2 + 12x$ | $x^2 + 6x$

Why?

$(x + 6)^2 - 36$ = **$x^2 + 12x$**

they wrote: **$2x - 24$**

Do not check by reading their line. Somebody else's working is the most persuasive thing on the page, and rereading it is how you end up agreeing with it — do the step yourself first, then compare.

On your own, and writing the step nobody writes down: $(x + 6)^2 - 36$ becomes $x^2 + 12x + 36 - 36$.

Tidy that up and it is $x^2 + 12x$.

Now compare. They wrote $2x - 24$, and the bracket was doubled instead of squared — so the correct answer is $x^2 + 12x$.

Solutions (16 of 16)

Check the Expansion (continued)

Check the expansion (continued)

2. **Theo wrote $(x + 12)^2 - 144 = 2x - 120$. Which is correct?**

$2x - 120$ | $x^2 + 24x + 144$ | $x^2 + 24x$ | $x^2 + 12x$

Why?

$(x + 12)^2 - 144$ = **$x^2 + 24x$**

they wrote: **$2x - 120$**

Do not check by reading their line. Somebody else's working is the most persuasive thing on the page, and rereading it is how you end up agreeing with it — do the step yourself first, then compare.

On your own, and writing the step nobody writes down: $(x + 12)^2 - 144$ becomes $x^2 + 24x + 144 - 144$.

Tidy that up and it is $x^2 + 24x$.

Now compare. They wrote $2x - 120$, and the bracket was doubled instead of squared — so the correct answer is $x^2 + 24x$.