Algebra 2 Math Review Workbook

6 lessons · printable · answer key at the back

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Contents

Rational Exponents

WORKED EXAMPLE

$216^{1/3}$ = 6 ______

$\sqrt[3]{216} = 6$
$= \sqrt[3]{216}$
$= 6$

1. Rational exponents

Work out each one. Take the root first, then raise it.

1 $4^{3/2}$ = ______

2 $64^{5/3}$ = ______

3 $216^{2/3}$ = ______

4 $25^{1/2}$ = ______

5 $25^{5/2}$ = ______

6 $256^{1/4}$ = ______

7 $36^{3/2}$ = ______

8 $9^{1/2}$ = ______

9 $1000^{4/3}$ = ______

10 $27^{1/3}$ = ______

11 $100^{1/2}$ = ______

12 $8^{1/3}$ = ______

13 $9^{3/2}$ = ______

14 $16^{3/4}$ = ______

15 $16^{1/2}$ = ______

16 $27^{5/3}$ = ______

17 $256^{5/4}$ = ______

18 $4^{1/2}$ = ______

19 $36^{1/2}$ = ______

20 $81^{5/4}$ = ______

21 $27^{2/3}$ = ______

22 $729^{4/3}$ = ______

23 $8^{4/3}$ = ______

24 $100^{3/2}$ = ______

25 $49^{1/2}$ = ______

26 $16^{5/2}$ = ______

27 $125^{1/3}$ = ______

28 $4^{5/2}$ = ______

Exponential Functions

WORKED EXAMPLE

$f(x) = 12 \cdot 3^x$ **$f(2) =$** 108

$f(2) = 12 \cdot 3^2$

$3^2 = 9$

$12 \cdot 9 = 108$

2. Evaluate exponential functions

Evaluate each exponential function at the value given.

1 $f(x) = 6 \cdot 3^x$ **f(0) =** ______

2 $f(x) = 5 \cdot 10^x$ **f(1) =** ______

3 $f(x) = 12 \cdot 2^x$ **f(1) =** ______

4 $f(x) = 4 \cdot 7^x$ **f(0) =** ______

5 $f(x) = 4 \cdot 8^x$ **f(3) =** ______

6 $f(x) = 2 \cdot 6^x$ **f(0) =** ______

7 $f(x) = 5 \cdot 7^x$ **f(2) =** ______

8 $f(x) = 8 \cdot 4^x$ **f(0) =** ______

9 $f(x) = 2 \cdot 10^x$ **f(0) =** ______

10 $f(x) = 4 \cdot 5^x$ **f(3) =** ______

11 $f(x) = 8 \cdot 8^x$ **f(0) =** ______

12 $f(x) = 10 \cdot 6^x$ **f(3) =** ______

13 $f(x) = 7 \cdot 3^x$ **f(1) =** ______

14 $f(x) = 7 \cdot 9^x$ **f(1) =** ______

15 $f(x) = 5 \cdot 4^x$ **f(1) =** ______

16 $f(x) = 3 \cdot 6^x$ **f(2) =** ______

17 $f(x) = 12 \cdot 8^x$ **f(0) =** ______

18 $f(x) = 12 \cdot 2^x$ **f(6) =** ______

19 $f(x) = 4 \cdot 4^x$ **f(0) =** ______

20 $f(x) = 6 \cdot 7^x$ **f(2) =** ______

Multiplying Complex Numbers

WORKED EXAMPLE

$(-2 - 4i)(-2 + 4i)$
$= 4 - 8i + 8i - 16i^2$
$= 4 + 16 - 8i + 8i$
$= 20$

3. Complex numbers

Multiply each pair of complex numbers.

1 $(2 - 4i)(-2 + 4i)$

2 $(3 + 4i)(-5 - i)$

3 $(4 - 3i)(-2 - 3i)$

4 $(-6 + 6i)(-6 - i)$

5 $(1 + 3i)(2 + 3i)$

6 $(6 - 3i)(-2 - 3i)$

7 $(-2 + 2i)(-4 - 2i)$

8 $(-4 + i)(3 + 2i)$

9 $(-5 + 2i)(3 - 5i)$

10 $(6 - 2i)(-3 + 5i)$

11 $(-4 + 2i)(-4 - 2i)$

12 $(-3 - 4i)(5 + i)$

13 $(-2 + 2i)(3 - 5i)$

14 $(-2 - 6i)(-6 - 6i)$

15 $(-1 - i)(-3 - 2i)$

16 $(4 - 4i)(5 + i)$

17 $(-6 - 5i)(6 - 6i)$

18 $(1 - 3i)(-2 - 3i)$

19 $(4 - 4i)(-2 + 4i)$

20 $(-2 - 2i)(4 + 2i)$

Dividing Polynomials

WORKED EXAMPLE

$$\begin{array}{rl} & x^2 \quad - x + 3 \\ \cline{2-2} x + 1 \,\big) & x^3 \qquad + 2x - 1 \\ & -x^3 - x^2 \\ \cline{2-2} & -x^2 + 2x - 1 \\ & x^2 \quad + x \\ \cline{2-2} & 3x - 1 \\ & -3x - 3 \\ \cline{2-2} & -4 \end{array}$$

4. Dividing polynomials

Divide each polynomial. Write 0 for the remainder when it divides exactly.

1

$$\begin{array}{rl} & x^2 + 3x - 2 \\ \cline{2-2} x + 1 \,\big) & x^3 + 4x^2 \quad + x - 2 \\ & -x^3 \quad - x^2 \\ & 3x^2 \quad + x - 2 \\ & -3x^2 - 3x \\ & -2x - 2 \\ & 2x + 2 \\ & 0 \end{array}$$

2

$$\begin{array}{rl} & x^2 \quad - 3x \quad + 2 \\ \cline{2-2} x - 6 \,\big) & x^3 - 9x^2 + 20x - 12 \\ & -x^3 + 6x^2 \\ & -3x^2 + 20x - 12 \\ & 3x^2 - 18x \\ & 2x - 12 \\ & -2x + 12 \\ & 0 \end{array}$$

3

$$\begin{array}{rl} & x^2 + 2x - 2 \\ \cline{2-2} x - 1 \,\big) & x^3 \quad + x^2 - 4x + 3 \\ & -x^3 \quad + x^2 \\ & 2x^2 - 4x + 3 \\ & -2x^2 + 2x \\ & -2x + 3 \\ & 2x - 2 \\ & 1 \end{array}$$

4

$$\begin{array}{rl} & x^2 \quad - 4x + 1 \\ \cline{2-2} x + 6 \,\big) & x^3 + 2x^2 - 23x + 6 \\ & -x^3 - 6x^2 \\ & -4x^2 - 23x + 6 \\ & 4x^2 + 24x \\ & x + 6 \\ & -x - 6 \\ & 0 \end{array}$$

5

$$\begin{array}{rl} & x^2 \quad + 0 \quad - 3 \\ \cline{2-2} x - 6 \,\big) & x^3 - 6x^2 - 3x + 19 \\ & -x^3 + 6x^2 \\ & -3x + 19 \\ & 3x - 18 \\ & 1 \end{array}$$

6

$$\begin{array}{rl} & x^2 \quad + 0 - 3 \\ \cline{2-2} x - 3 \,\big) & x^3 - 3x^2 - 3x \\ & -x^3 + 3x^2 \\ & -3x \\ & 3x - 9 \\ & -9 \end{array}$$

7

$$\begin{array}{rl} & x^2 \quad + 5x \quad + 6 \\ \cline{2-2} x + 4 \,\big) & x^3 + 9x^2 + 26x + 20 \\ & -x^3 - 4x^2 \\ & 5x^2 + 26x + 20 \\ & -5x^2 - 20x \\ & 6x + 20 \\ & -6x - 24 \\ & -4 \end{array}$$

8

$$\begin{array}{rl} & x^2 - 4x \quad - 5 \\ \cline{2-2} x + 1 \,\big) & x^3 - 3x^2 - 9x - 14 \\ & -x^3 \quad - x^2 \\ & -4x^2 - 9x - 14 \\ & 4x^2 + 4x \\ & -5x - 14 \\ & 5x \quad + 5 \\ & -9 \end{array}$$

9

$$\begin{array}{rl} & x^2 \quad + 0 \quad + 3 \\ \cline{2-2} x - 6 \,\big) & x^3 - 6x^2 + 3x \quad - 9 \\ & -x^3 + 6x^2 \\ & 3x \quad - 9 \\ & -3x + 18 \\ & 9 \end{array}$$

10

$$\begin{array}{rl} & x^2 \quad + 5x - 1 \\ \cline{2-2} x - 5 \,\big) & x^3 \qquad - 26x + 1 \\ & -x^3 + 5x^2 \\ & 5x^2 - 26x + 1 \\ & -5x^2 + 25x \\ & -x + 1 \\ & x - 5 \\ & -4 \end{array}$$

Solving Rational Equations

WORKED EXAMPLE

$$\frac{5}{(x-7)} = \frac{-5}{(x-3)}, \; x = \underline{5}$$

$x \neq 3, 7$

$5(x - 3) = -5(x - 7)$

$x = 5$

$x = 5$ and $x \neq 3, 7$

5. Solving rational equations

Solve each equation for x.

1 $\frac{11}{(x + 4)} = \frac{8}{(x + 1)}$, x = ____

2 $\frac{16}{(x - 4)} = \frac{-8}{(x - 7)}$, x = ____

3 $\frac{28}{(x + 2)} = \frac{64}{(x - 7)}$, x = ____

4 $\frac{4}{x} = 2$, x = ____

5 $\frac{3}{(x - 3)} = \frac{-18}{(x + 4)}$, x = ____

6 $\frac{-54}{(x + 3)} = 9$, x = ____

7 $\frac{32}{(x + 8)} = 8$, x = ____

8 $\frac{7}{(x + 6)} = \frac{-14}{(x - 3)}$, x = ____

9 $\frac{44}{(x + 6)} = 4$, x = ____

10 $\frac{80}{(x + 9)} = \frac{24}{(x + 2)}$, x = ____

11 $\frac{18}{(x + 2)} = 9$, x = ____

12 $\frac{4}{x} = \frac{12}{(x + 4)}$, x = ____

13 $\frac{48}{(x + 9)} = 6$, x = ____

14 $\frac{24}{(x - 2)} = \frac{33}{(x - 5)}$, x = ____

15 $\frac{72}{(x + 4)} = 8$, x = ____

16 $\frac{-6}{(x - 6)} = 6$, x = ____

Sums of Series

WORKED EXAMPLE

5, 10, 20, ... S_5 = 155

$S_5 = 5(2^5 - 1) \div (2 - 1)$

$S_5 = 5 \times 31 \div 1 = 155$

6. Sums of series

S_n is the sum of the first n terms. Does each sequence add or multiply?

1 6, 24, 96, ... S_6 = ________

2 5, 10, 20, ... S_6 = ________

3 2, 11, 20, ... S_8 = ________

4 9, 27, 81, ... S_7 = ________

5 7, 11, 15, ... S_8 = ________

6 8, 32, 128, ... S_6 = ________

7 9, 12, 15, ... S_9 = ________

8 8, 32, 128, ... S_5 = ________

9 1, 4, 16, ... S_7 = ________

10 8, 16, 32, ... S_7 = ________

11 11, 1, −9, ... S_9 = ________

12 11, 17, 23, ... S_9 = ________

13 5, −5, −15, ... S_5 = ________

14 2, −1, −4, ... S_{10} = ________

15 12, 24, 48, ... S_5 = ________

16 9, 5, 1, ... S_7 = ________

Answer Key (1 of 2)

Rational Exponents

Rational exponents

1. $4^{(3/2)} = 8$
2. $64^{(5/3)} = 1024$
3. $216^{(2/3)} = 36$
4. $25^{(1/2)} = 5$
5. $25^{(5/2)} = 3125$
6. $256^{(1/4)} = 4$
7. $36^{(3/2)} = 216$
8. $9^{(1/2)} = 3$
9. $1000^{(4/3)} = 10000$
10. $27^{(1/3)} = 3$
11. $100^{(1/2)} = 10$
12. $8^{(1/3)} = 2$
13. $9^{(3/2)} = 27$
14. $16^{(3/4)} = 8$
15. $16^{(1/2)} = 4$
16. $27^{(5/3)} = 243$
17. $256^{(5/4)} = 1024$
18. $4^{(1/2)} = 2$
19. $36^{(1/2)} = 6$
20. $81^{(5/4)} = 243$
21. $27^{(2/3)} = 9$
22. $729^{(4/3)} = 6561$
23. $8^{(4/3)} = 16$
24. $100^{(3/2)} = 1000$
25. $49^{(1/2)} = 7$
26. $16^{(5/2)} = 1024$
27. $125^{(1/3)} = 5$
28. $4^{(5/2)} = 32$

Exponential Functions

Evaluate exponential functions

1. $f(x) = 6 \cdot 3^x \rightarrow f(0) = 6$
2. $f(x) = 5 \cdot 10^x \rightarrow f(1) = 50$
3. $f(x) = 12 \cdot 2^x \rightarrow f(1) = 24$
4. $f(x) = 4 \cdot 7^x \rightarrow f(0) = 4$
5. $f(x) = 4 \cdot 8^x \rightarrow f(3) = 2048$
6. $f(x) = 2 \cdot 6^x \rightarrow f(0) = 2$
7. $f(x) = 5 \cdot 7^x \rightarrow f(2) = 245$
8. $f(x) = 8 \cdot 4^x \rightarrow f(0) = 8$
9. $f(x) = 2 \cdot 10^x \rightarrow f(0) = 2$
10. $f(x) = 4 \cdot 5^x \rightarrow f(3) = 500$
11. $f(x) = 8 \cdot 8^x \rightarrow f(0) = 8$
12. $f(x) = 10 \cdot 6^x \rightarrow f(3) = 2160$
13. $f(x) = 7 \cdot 3^x \rightarrow f(1) = 21$
14. $f(x) = 7 \cdot 9^x \rightarrow f(1) = 63$
15. $f(x) = 5 \cdot 4^x \rightarrow f(1) = 20$
16. $f(x) = 3 \cdot 6^x \rightarrow f(2) = 108$
17. $f(x) = 12 \cdot 8^x \rightarrow f(0) = 12$

Exponential Functions (continued)

Evaluate exponential functions (continued)

18. $f(x) = 12 \cdot 2^x \rightarrow f(6) = 768$
19. $f(x) = 4 \cdot 4^x \rightarrow f(0) = 4$
20. $f(x) = 6 \cdot 7^x \rightarrow f(2) = 294$

Multiplying Complex Numbers

Complex numbers

1. $(2 - 4i)(-2 + 4i) = 12 + 16i$
2. $(3 + 4i)(-5 - i) = -11 - 23i$
3. $(4 - 3i)(-2 - 3i) = -17 - 6i$
4. $(-6 + 6i)(-6 - i) = 42 - 30i$
5. $(1 + 3i)(2 + 3i) = -7 + 9i$
6. $(6 - 3i)(-2 - 3i) = -21 - 12i$
7. $(-2 + 2i)(-4 - 2i) = 12 - 4i$
8. $(-4 + i)(3 + 2i) = -14 - 5i$
9. $(-5 + 2i)(3 - 5i) = -5 + 31i$
10. $(6 - 2i)(-3 + 5i) = -8 + 36i$
11. $(-4 + 2i)(-4 - 2i) = 20$
12. $(-3 - 4i)(5 + i) = -11 - 23i$
13. $(-2 + 2i)(3 - 5i) = 4 + 16i$
14. $(-2 - 6i)(-6 - 6i) = -24 + 48i$
15. $(-1 - i)(-3 - 2i) = 1 + 5i$
16. $(4 - 4i)(5 + i) = 24 - 16i$
17. $(-6 - 5i)(6 - 6i) = -66 + 6i$
18. $(1 - 3i)(-2 - 3i) = -11 + 3i$
19. $(4 - 4i)(-2 + 4i) = 8 + 24i$
20. $(-2 - 2i)(4 + 2i) = -4 - 12i$

Dividing Polynomials

Dividing polynomials

1. $(x^3 + 4x^2 + x - 2) \div (x + 1) = x^2 + 3x - 2$
2. $(x^3 - 9x^2 + 20x - 12) \div (x - 6) = x^2 - 3x + 2$
3. $(x^3 + x^2 - 4x + 3) \div (x - 1) = x^2 + 2x - 2$ remainder 1
4. $(x^3 + 2x^2 - 23x + 6) \div (x + 6) = x^2 - 4x + 1$
5. $(x^3 - 6x^2 - 3x + 19) \div (x - 6) = x^2 - 3$ remainder 1
6. $(x^3 - 3x^2 - 3x) \div (x - 3) = x^2 - 3$ remainder -9
7. $(x^3 + 9x^2 + 26x + 20) \div (x + 4) = x^2 + 5x + 6$ remainder -4
8. $(x^3 - 3x^2 - 9x - 14) \div (x + 1) = x^2 - 4x - 5$ remainder -9
9. $(x^3 - 6x^2 + 3x - 9) \div (x - 6) = x^2 + 3$ remainder 9
10. $(x^3 - 26x + 1) \div (x - 5) = x^2 + 5x - 1$ remainder -4

Solving Rational Equations

Solving rational equations

1. $11/(x + 4) = 8/(x + 1) \rightarrow x = 7$ $(x \neq -4, -1)$
2. $16/(x - 4) = -8/(x - 7) \rightarrow x = 6$ $(x \neq 4, 7)$
3. $28/(x + 2) = 64/(x - 7) \rightarrow x = -9$ $(x \neq -2, 7)$
4. $4/x = 2 \rightarrow x = 2$ $(x \neq 0)$
5. $3/(x - 3) = -18/(x + 4) \rightarrow x = 2$ $(x \neq -4, 3)$
6. $-54/(x + 3) = 9 \rightarrow x = -9$ $(x \neq -3)$

Answer Key (2 of 2)

Solving Rational Equations (continued)

Solving rational equations (continued)

7. $32/(x + 8) = 8 \rightarrow x = -4$ ($x \neq -8$)
8. $7/(x + 6) = -14/(x - 3) \rightarrow x = -3$ ($x \neq -6, 3$)
9. $44/(x + 6) = 4 \rightarrow x = 5$ ($x \neq -6$)
10. $80/(x + 9) = 24/(x + 2) \rightarrow x = 1$ ($x \neq -9, -2$)
11. $18/(x + 2) = 9 \rightarrow x = 0$ ($x \neq -2$)
12. $4/x = 12/(x + 4) \rightarrow x = 2$ ($x \neq -4, 0$)
13. $48/(x + 9) = 6 \rightarrow x = -1$ ($x \neq -9$)
14. $24/(x - 2) = 33/(x - 5) \rightarrow x = -6$ ($x \neq 2, 5$)
15. $72/(x + 4) = 8 \rightarrow x = 5$ ($x \neq -4$)
16. $-6/(x - 6) = 6 \rightarrow x = 5$ ($x \neq 6$)

Sums of Series

Sums of series

1. S_6 for 6, 24, 96, … = 8190
2. S_6 for 5, 10, 20, … = 315
3. S_8 for 2, 11, 20, … = 268
4. S_7 for 9, 27, 81, … = 9837
5. S_8 for 7, 11, 15, … = 168
6. S_6 for 8, 32, 128, … = 10920
7. S_9 for 9, 12, 15, … = 189
8. S_5 for 8, 32, 128, … = 2728
9. S_7 for 1, 4, 16, … = 5461
10. S_7 for 8, 16, 32, … = 1016
11. S_9 for 11, 1, −9, … = −261
12. S_9 for 11, 17, 23, … = 315
13. S_5 for 5, −5, −15, … = −75
14. S_{10} for 2, −1, −4, … = −115
15. S_5 for 12, 24, 48, … = 372
16. S_7 for 9, 5, 1, … = −21

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Solutions (1 of 5)

Rational Exponents

Rational exponents

1. $4^{3/2}$ = ________

$4^{(3/2)} = 8$

The BOTTOM of the exponent says which root to take, so the square root of 4 comes first. Root before power, always — the other order works and makes the numbers enormous.

The TOP says what to raise it to. 2 to the power 3 is 8.

So 4^(3/2) is 8.

2. $64^{5/3}$ = ________

$64^{(5/3)} = 1024$

The BOTTOM of the exponent says which root to take, so the cube root of 64 comes first. Root before power, always — the other order works and makes the numbers enormous.

The TOP says what to raise it to. 4 to the power 5 is 1024.

So 64^(5/3) is 1024.

Exponential Functions

Evaluate exponential functions

1. $f(x) = 6 \cdot 3^x$ $f(0)$ = ___

$f(0) = 6 \cdot 3^0$

$6 \cdot 1 = 6$

Substitute x = 0: $f(0) = 6 \cdot 3^0$.

Anything to the power 0 is 1, so $3^0 = 1$ and $f(0) = 6 \cdot 1 = 6$.

Exponential Functions (continued)

Evaluate exponential functions (continued)

2. $f(x) = 5 \cdot 10^x$ **$f(1) =$ ___**

$$f(1) = 5 \cdot 10^1$$
$$10^1 = 10$$
$$5 \cdot 10 = 50$$

Substitute $x = 1$: $f(1) = 5 \cdot 10^1$.

Do the power first: $10^1 = 10$.

Then multiply by the coefficient: $5 \cdot 10 = 50$.

Multiplying Complex Numbers

Complex numbers

1. $(2 - 4i)(-2 + 4i)$

$$(2 - 4i)(-2 + 4i) = 12 + 16i$$

Expand the brackets exactly as you would with x in place of i — every term in the first bracket times every term in the second, four products.

Now the one step that is only true of i: i^2 is -1. So $-16i^2$ is 16, which is not imaginary at all — it joins the real part.

Collect the two real numbers and the two i terms: $12 + 16i$.

2. $(3 + 4i)(-5 - i)$

$$(3 + 4i)(-5 - i) = -11 - 23i$$

Expand the brackets exactly as you would with x in place of i — every term in the first bracket times every term in the second, four products.

Now the one step that is only true of i: i^2 is -1. So $-4i^2$ is 4, which is not imaginary at all — it joins the real part.

Collect the two real numbers and the two i terms: $-11 - 23i$.

Solutions (3 of 5)

Dividing Polynomials

Dividing polynomials

1.

$$
\begin{array}{r|l}
 & x^2 + 3x - 2 \\ \hline
x + 1 & x^3 + 4x^2 + x - 2 \\
 & -x^3 - x^2 \\ \hline
 & 3x^2 + x - 2 \\
 & -3x^2 - 3x \\ \hline
 & -2x - 2 \\
 & 2x + 2 \\ \hline
 & 0
\end{array}
$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x + 1)$ is $x^3 + x^2$ — and take that away, leaving $3x^2 + x - 2$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$3x^2 \div x$ is $3x$, so $3x$ goes in the x place. Multiply the WHOLE divisor by it — $3x(x + 1)$ is $3x^2 + 3x$ — and take that away, leaving $-2x - 2$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$-2x \div x$ is -2, so -2 goes in the constant place. Multiply the WHOLE divisor by it — $-2(x + 1)$ is $-2x - 2$ — and take that away, leaving 0. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

Nothing is left, so $x + 1$ divides it exactly: $(x^3 + 4x^2 + x - 2) \div (x + 1) = x^2 + 3x - 2$.

2.

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\ \hline
x - 6 & x^3 - 9x^2 + 20x - 12 \\
 & -x^3 + 6x^2 \\ \hline
 & -3x^2 + 20x - 12 \\
 & 3x^2 - 18x \\ \hline
 & 2x - 12 \\
 & -2x + 12 \\ \hline
 & 0
\end{array}
$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x - 6)$ is $x^3 - 6x^2$ — and take that away, leaving $-3x^2 + 20x - 12$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$-3x^2 \div x$ is $-3x$, so $-3x$ goes in the x place. Multiply the WHOLE divisor by it — $-3x(x - 6)$ is $-3x^2 + 18x$ — and take that away, leaving $2x - 12$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$2x \div x$ is 2, so 2 goes in the constant place. Multiply the WHOLE divisor by it — $2(x - 6)$ is $2x - 12$ — and take that away, leaving 0. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

Nothing is left, so $x - 6$ divides it exactly: $(x^3 - 9x^2 + 20x - 12) \div (x - 6) = x^2 - 3x + 2$.

Solutions (4 of 5)

Solving Rational Equations

Solving rational equations

1. $\dfrac{11}{(x + 4)} = \dfrac{8}{(x + 1)}$, x = ____

$x \neq -4, -1$

$11/(x + 4) = 8/(x + 1)$

$11(x + 1) = 8(x + 4)$

$x = 7$

$x = 7$ and $x \neq -4, -1$

Before anything else, note what x cannot be. (x + 4) is a denominator, so x may not be −4, and the same for the other one: not −1. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Two fractions, one equals sign — so cross-multiply. Each numerator meets the OTHER denominator, which is the same "multiply both sides by both denominators" done in one move.

Multiply out, gather the x terms on one side and the numbers on the other, and x = 7.

Last, check it against the first line: x may not be −4 or −1, and 7 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

2. $\dfrac{16}{(x - 4)} = \dfrac{-8}{(x - 7)}$, x = ____

$x \neq 4, 7$

$16/(x - 4) = -8/(x - 7)$

$16(x - 7) = -8(x - 4)$

$x = 6$

$x = 6$ and $x \neq 4, 7$

Before anything else, note what x cannot be. (x − 4) is a denominator, so x may not be 4, and the same for the other one: not 7. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Two fractions, one equals sign — so cross-multiply. Each numerator meets the OTHER denominator, which is the same "multiply both sides by both denominators" done in one move.

Multiply out, gather the x terms on one side and the numbers on the other, and x = 6.

Last, check it against the first line: x may not be 4 or 7, and 6 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

Solutions (5 of 5)

Sums of Series

Sums of series

1. 6, 24, 96, ... S_6 = ________

$$S_6 = 6 \times 4095 \div 3 = 8190$$

Each term is the one before times 4, so this is a geometric series and the sum is $a(r^n - 1) \div (r - 1)$. Here a is 6, r is 4 and n is 6 — substituting the wrong one of the three is the whole difficulty.

4 to the power 6 is 4096, so the top is 6 × 4095 and the bottom is 3.

2. 5, 10, 20, ... S_6 = ________

$$S_6 = 5 \times 63 \div 1 = 315$$

Each term is the one before times 2, so this is a geometric series and the sum is $a(r^n - 1) \div (r - 1)$. Here a is 5, r is 2 and n is 6 — substituting the wrong one of the three is the whole difficulty.

2 to the power 6 is 64, so the top is 5 × 63 and the bottom is 1.