Algebra 2 Advanced Workbook

18 lessons · printable · answer key at the back

Name _____ Date _____

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1-1

Simplifying Radicals

WORKED EXAMPLE

$\sqrt{75}$

$75 = 25 \times 3$

$= \sqrt{25} \times \sqrt{3}$

$= 5\sqrt{3}$

1. Simplifying radicals

Simplify each square root.

1 $\sqrt{360}$

2 $\sqrt{486}$

3 $\sqrt{125}$

4 $\sqrt{300}$

5 $\sqrt{72}$

6 $\sqrt{605}$

7 $\sqrt{162}$

8 $\sqrt{175}$

9 $\sqrt{847}$

10 $\sqrt{704}$

11 $\sqrt{504}$

12 $\sqrt{384}$

13 $\sqrt{726}$

14 $\sqrt{98}$

15 $\sqrt{600}$

16 $\sqrt{243}$

17 $\sqrt{128}$

18 $\sqrt{896}$

19 $\sqrt{810}$

20 $\sqrt{539}$

21 $\sqrt{500}$

22 $\sqrt{320}$

23 $\sqrt{288}$

24 $\sqrt{396}$

25 $\sqrt{960}$

26 $\sqrt{242}$

27 $\sqrt{245}$

28 $\sqrt{405}$

Cube Roots

WORKED EXAMPLE

$\sqrt[3]{56}$

$56 = 8 \times 7$

$= \sqrt[3]{8} \times \sqrt[3]{7}$

$= 2\sqrt[3]{7}$

2. Simplifying radicals

Simplify each cube root.

1 $\sqrt[3]{297}$

2 $\sqrt[3]{250}$

3 $\sqrt[3]{320}$

4 $\sqrt[3]{625}$

5 $\sqrt[3]{243}$

6 $\sqrt[3]{640}$

7 $\sqrt[3]{375}$

8 $\sqrt[3]{324}$

9 $\sqrt[3]{875}$

10 $\sqrt[3]{750}$

11 $\sqrt[3]{128}$

12 $\sqrt[3]{768}$

13 $\sqrt[3]{704}$

14 $\sqrt[3]{256}$

15 $\sqrt[3]{576}$

16 $\sqrt[3]{448}$

17 $\sqrt[3]{192}$

18 $\sqrt[3]{500}$

19 $\sqrt[3]{384}$

20 $\sqrt[3]{40}$

21 $\sqrt[3]{270}$

22 $\sqrt[3]{48}$

23 $\sqrt[3]{54}$

24 $\sqrt[3]{96}$

Rational Exponents

WORKED EXAMPLE

$216^{1/3} = 6$

$\sqrt[3]{216} = 6$

$= \sqrt[3]{216}$

$= 6$

3. Rational exponents

Work out each one. Take the root first, then raise it.

1 $216^{2/3} =$ _____

2 $1000^{4/3} =$ _____

3 $256^{1/4} =$ _____

4 $729^{4/3} =$ _____

5 $100^{1/2} =$ _____

6 $6561^{1/4} =$ _____

7 $10000^{1/4} =$ _____

8 $16^{3/4} =$ _____

9 $729^{1/3} =$ _____

10 $512^{1/3} =$ _____

11 $256^{5/4} =$ _____

12 $1000^{1/3} =$ _____

13 $2401^{1/4} =$ _____

14 $81^{5/4} =$ _____

15 $512^{4/3} =$ _____

16 $1296^{5/4} =$ _____

17 $100^{3/2} =$ _____

18 $625^{3/4} =$ _____

19 $1296^{1/4} =$ _____

20 $125^{1/3} =$ _____

21 $4096^{1/4} =$ _____

22 $625^{1/4} =$ _____

23 $64^{3/2} =$ _____

24 $4^{3/2} =$ _____

25 $64^{1/2} =$ _____

26 $343^{4/3} =$ _____

27 $64^{5/3} =$ _____

28 $216^{5/3} =$ _____

Logarithms

WORKED EXAMPLE

$\log_2 4 = 2$

$2^2 = 4$

$= 2$

4. Logarithms

Work out each logarithm, or find the missing number.

1 $\log_3 \frac{1}{729} =$ _____

2 $\log_3 2187 =$ _____

3 $\log_7 \frac{1}{2401} =$ _____

4 $\log_3$ _____ $= 9$

5 $\log_9 59049 =$ _____

6 $\log_2$ _____ $= 16$

7 $\log_2$ _____ $= 11$

8 $\log_4$ _____ $= 4$

9 $\log_3$ _____ $= 7$

10 $\log_2$ _____ $= 13$

11 $\log_3 \frac{1}{81} =$ _____

12 $\log_4$ _____ $= 7$

13 $\log_2$ _____ $= 8$

14 $\log_6 \frac{1}{7776} =$ _____

15 $\log_3$ _____ $= 8$

16 $\log_2 \frac{1}{2048} =$ _____

17 $\log_4 \frac{1}{1024} =$ _____

18 $\log_5$ _____ $= 7$

19 $\log_4 65536 =$ _____

20 $\log_4$ _____ $= 6$

21 $\log_2 16384 =$ _____

22 $\log_2 4096 =$ _____

23 $\log_3 19683 =$ _____

24 $\log_6$ _____ $= 6$

4. Logarithms (continued)

Work out each logarithm, or find the missing number.

25 $\log_4 16384 =$ _____

26 $\log_2 1024 =$ _____

27 $\log_5 \frac{1}{625} =$ _____

28 $\log_2 32768 =$ _____

5. Simplifying radicals

Simplify each square root.

29 $\sqrt{252}$

30 $\sqrt{864}$

31 $\sqrt{539}$

32 $\sqrt{640}$

33 $\sqrt{396}$

34 $\sqrt{720}$

35 $\sqrt{704}$

36 $\sqrt{108}$

37 $\sqrt{810}$

38 $\sqrt{162}$

The Laws of Logarithms

WORKED EXAMPLE

$\log_5 2 + \log_5 12 = \log_5 \underline{24}$

$\log_5 m + \log_5 n = \log_5 (m \times n)$

$2 \times 12 = 24$

$= \log_5 24$

6. Laws of logarithms

Write each of these as one logarithm.

1 $\log_5 6 + \log_5 7 = \log_5 \_\_\_\_$

2 $2 \log_3 11 = \log_3 \_\_\_\_$

3 $\log_2 6 + \log_2 8 = \log_2 \_\_\_\_$

4 $2 \log_{10} 9 = \log_{10} \_\_\_\_$

5 $2 \log_2 6 = \log_2 \_\_\_\_$

6 $\log_{10} 108 - \log_{10} 9 = \log_{10} \_\_\_\_$

7 $\log_5 132 - \log_5 12 = \log_5 \_\_\_\_$

8 $\log_2 3 + \log_2 11 = \log_2 \_\_\_\_$

9 $\log_2 72 - \log_2 6 = \log_2 \_\_\_\_$

10 $\log_{10} 88 - \log_{10} 8 = \log_{10} \_\_\_\_$

11 $\log_2 42 - \log_2 7 = \log_2 \_\_\_\_$

12 $\log_5 84 - \log_5 12 = \log_5 \_\_\_\_$

13 $\log_{10} 84 - \log_{10} 12 = \log_{10} \_\_\_\_$

14 $\log_2 44 - \log_2 11 = \log_2 \_\_\_\_$

6. Laws of logarithms (continued)

Write each of these as one logarithm.

15 $2\log_5 12 = \log_5 \_\_\_\_$

16 $\log_{10} 81 - \log_{10} 9 = \log_{10} \_\_\_\_$

17 $\log_2 30 - \log_2 5 = \log_2 \_\_\_\_$

18 $\log_5 9 + \log_5 10 = \log_5 \_\_\_\_$

19 $\log_2 66 - \log_2 6 = \log_2 \_\_\_\_$

20 $\log_5 48 - \log_5 12 = \log_5 \_\_\_\_$

21 $\log_3 90 - \log_3 9 = \log_3 \_\_\_\_$

22 $\log_2 120 - \log_2 12 = \log_2 \_\_\_\_$

23 $\log_3 121 - \log_3 11 = \log_3 \_\_\_\_$

24 $\log_5 4 + \log_5 10 = \log_5 \_\_\_\_$

7. Simplifying radicals

Simplify each cube root.

25 $\sqrt[3]{128}$

26 $\sqrt[3]{704}$

27 $\sqrt[3]{192}$

28 $\sqrt[3]{576}$

29 $\sqrt[3]{270}$

30 $\sqrt[3]{375}$

31 $\sqrt[3]{500}$

32 $\sqrt[3]{384}$

33 $\sqrt[3]{768}$

34 $\sqrt[3]{875}$

Exponential Functions

WORKED EXAMPLE

$f(x) = 12 \cdot 3^x$ **f(2) =** 108

$f(2) = 12 \cdot 3^2$

$3^2 = 9$

$12 \cdot 9 = 108$

8. Evaluate exponential functions

Evaluate each exponential function at the value given.

1 $f(x) = 7 \cdot 6^x$ **f(1) =** _____

2 $f(x) = 7 \cdot 2^x$ **f(6) =** _____

3 $f(x) = 5 \cdot 9^x$ **f(0) =** _____

4 $f(x) = 6 \cdot 4^x$ **f(4) =** _____

5 $f(x) = 8 \cdot 7^x$ **f(1) =** _____

6 $f(x) = 4 \cdot 2^x$ **f(5) =** _____

7 $f(x) = 6 \cdot 3^x$ **f(5) =** _____

8 $f(x) = 10 \cdot 8^x$ **f(1) =** _____

9 $f(x) = 4 \cdot 9^x$ **f(3) =** _____

10 $f(x) = 10 \cdot 9^x$ **f(2) =** _____

11 $f(x) = 3 \cdot 9^x$ **f(1) =** _____

12 $f(x) = 8 \cdot 2^x$ **f(6) =** _____

13 $f(x) = 12 \cdot 6^x$ **f(3) =** _____

14 $f(x) = 6 \cdot 4^x$ **f(2) =** _____

15 $f(x) = 2 \cdot 2^x$ **f(5) =** _____

16 $f(x) = 10 \cdot 6^x$ **f(3) =** _____

8. Evaluate exponential functions (continued)

Evaluate each exponential function at the value given.

17 $f(x) = 6 \cdot 3^x$ **f(2) =** _____

18 $f(x) = 10 \cdot 3^x$ **f(4) =** _____

19 $f(x) = 7 \cdot 10^x$ **f(2) =** _____

20 $f(x) = 4 \cdot 4^x$ **f(4) =** _____

21 $f(x) = 4 \cdot 3^x$ **f(5) =** _____

22 $f(x) = 6 \cdot 9^x$ **f(2) =** _____

23 $f(x) = 10 \cdot 5^x$ **f(3) =** _____

24 $f(x) = 7 \cdot 3^x$ **f(2) =** _____

9. Rational exponents

Work out each one. Take the root first, then raise it.

25 $81^{5/4}$ = _____

26 $1296^{1/4}$ = _____

27 $64^{3/2}$ = _____

28 $1000^{4/3}$ = _____

29 $81^{1/2}$ = _____

30 $729^{1/3}$ = _____

31 $1296^{5/4}$ = _____

32 $81^{1/4}$ = _____

33 $1000^{1/3}$ = _____

34 $4096^{1/4}$ = _____

Completing the Square

WORKED EXAMPLE

$x^2 - 2x - 8$

$-2 \div 2 = -1$

$(x - 1)^2 = x^2 - 2x + 1$

$= (x - 1)^2 - 9$

10. Completing the square

Write each expression as $(x + p)^2 + q$.

1 $x^2 + 16x + 65$

2 $x^2 - 22x + 145$

3 $x^2 + 16x + 67$

4 $x^2 + 24x + 147$

5 $x^2 - 16x + 66$

6 $x^2 - 24x + 119$

7 $x^2 - 16x + 24$

8 $x^2 - 8x - 24$

9 $x^2 - 20x + 128$

10 $x^2 + 24x + 124$

11 $x^2 + 12x + 27$

12 $x^2 + 24x + 150$

13 $x^2 + 22x + 129$

14 $x^2 - 20x + 102$

15 $x^2 + 24x + 174$

16 $x^2 + 24x + 139$

17 $x^2 - 12x + 20$

18 $x^2 + 20x + 130$

19 $x^2 + 22x + 133$

20 $x^2 - 12x + 56$

21 $x^2 + 22x + 96$

22 $x^2 - 24x + 128$

11. Logarithms

Work out each logarithm, or find the missing number.

23 $\log_8 32768 =$ _____

24 $\log_2 32768 =$ _____

25 $\log_9$ _____ $= 4$

26 $\log_2 4096 =$ _____

27 $\log_{10} 100000 =$ _____

28 $\log_2 1024 =$ _____

29 $\log_4$ _____ $= 8$

30 $\log_2 \frac{1}{128} =$ _____

31 $\log_3 2187 =$ _____

32 $\log_5$ _____ $= 7$

How Many Real Solutions

WORKED EXAMPLE

$x^2 + 24x + 144 = 0$

$a = 1, b = 24, c = 144$

$b^2 - 4ac = 24^2 - 4(1)(144)$

$= 0 \rightarrow$ one

12. How many real solutions?

How many real solutions does each equation have? Circle one.

1 $x^2 + 22x + 149 = 0$

2 $x^2 + 12x - 160 = 0$

3 $x^2 - 3x + 98 = 0$

4 $x^2 + 18x + 107 = 0$

5 $x^2 + 2x + 1 = 0$

6 $x^2 + 16x + 65 = 0$

12. How many real solutions? (continued)

How many real solutions does each equation have? Circle one.

7 $x^2 - 34x - 34 = 0$

8 $x^2 - 28x - 147 = 0$

9 $x^2 - 9x - 178 = 0$

10 $x^2 - 20x + 86 = 0$

11 $x^2 - 18x + 81 = 0$

12 $x^2 + 34x + 56 = 0$

13 $x^2 + 15x + 151 = 0$

14 $x^2 + 102 = 0$

15 $x^2 - 9x + 42 = 0$

12. How many real solutions? (continued)

How many real solutions does each equation have? Circle one.

16 $x^2 + 23x - 48 = 0$

17 $x^2 - 6x + 9 = 0$

18 $x^2 - 8x + 72 = 0$

19 $x^2 - 21x - 137 = 0$

20 $x^2 + 24x - 18 = 0$

21 $x^2 - 4x + 4 = 0$

22 $x^2 - 7x + 152 = 0$

23 $x^2 + 17x + 172 = 0$

24 $x^2 + 26x + 90 = 0$

13. Laws of logarithms

Write each of these as one logarithm.

25 $\log_3 48 - \log_3 6 = \log_3 \_\_\_\_$

26 $\log_5 8 + \log_5 9 = \log_5 \_\_\_\_$

27 $2 \log_3 6 = \log_3 \_\_\_\_$

28 $4 \log_2 3 = \log_2 \_\_\_\_$

29 $\log_3 50 - \log_3 5 = \log_3 \_\_\_\_$

30 $\log_2 11 + \log_2 11 = \log_2 \_\_\_\_$

31 $\log_2 77 - \log_2 11 = \log_2 \_\_\_\_$

32 $\log_5 3 + \log_5 12 = \log_5 \_\_\_\_$

33 $\log_5 110 - \log_5 10 = \log_5 \_\_\_\_$

34 $\log_2 96 - \log_2 12 = \log_2 \_\_\_\_$

Adding and Subtracting Complex Numbers

WORKED EXAMPLE

(2 − 5i) − (−1 − 3i)

2 − (−1) = 3
−5 − (−3) = −2
= 3 − 2i

14. Complex numbers

Add or subtract each pair of complex numbers.

1 **(5 + 2i) − (3 − 5i)**

2 **(−2 + 5i) − (−6 + 6i)**

3 **(3 + 4i) − (−5 − i)**

4 **(3 − 6i) + (6 + i)**

5 **(4 + 2i) + (3 − 5i)**

6 **(6 − 6i) + (−6 − 6i)**

7 **(4 + 6i) − (−6 − i)**

8 **(−4 − i) + (4 − 5i)**

9 **(4 + 5i) − (−6 + 6i)**

10 **(5 − 3i) − (5 − 6i)**

11 **(4 − 5i) − (−1 − 3i)**

12 **(1 − 6i) − (−6 − 6i)**

13 **(−6 + 6i) − (−6 − i)**

14 **(2 + 6i) − (−6 − i)**

15 **(−3 − 6i) − (−6 − 6i)**

16 **(−5 + 6i) + (−6 − i)**

17 **(−4 + 2i) + (3 − 5i)**

18 **(5 − 5i) + (6 − 6i)**

19 **(−6 − 2i) + (−3 + 5i)**

20 **(−4 − 5i) − (−1 − 3i)**

21 **(−3 + 3i) − (−5 + 6i)**

22 **(6 + 6i) − (−6 − i)**

15. Evaluate exponential functions

Evaluate each exponential function at the value given.

23 $f(x) = 4 \cdot 2^x$ **f(4) =** _____

24 $f(x) = 12 \cdot 8^x$ **f(2) =** _____

25 $f(x) = 7 \cdot 4^x$ **f(3) =** _____

26 $f(x) = 10 \cdot 2^x$ **f(4) =** _____

27 $f(x) = 10 \cdot 4^x$ **f(3) =** _____

28 $f(x) = 5 \cdot 4^x$ **f(4) =** _____

29 $f(x) = 6 \cdot 3^x$ **f(4) =** _____

30 $f(x) = 6 \cdot 9^x$ **f(1) =** _____

31 $f(x) = 10 \cdot 2^x$ **f(5) =** _____

32 $f(x) = 2 \cdot 10^x$ **f(3) =** _____

Multiplying Complex Numbers

WORKED EXAMPLE

$(-2 - 4i)(-2 + 4i)$

$= 4 - 8i + 8i - 16i^2$

$= 4 + 16 - 8i + 8i$

$= 20$

16. Complex numbers

Multiply each pair of complex numbers.

1 $(3 + 4i)(-5 - i)$

2 $(-6 + 6i)(-6 - i)$

3 $(6 - 3i)(-2 - 3i)$

4 $(6 - 2i)(-3 + 5i)$

5 $(-5 + 2i)(3 - 5i)$

6 $(-2 - 6i)(-6 - 6i)$

7 $(-6 - 5i)(6 - 6i)$

8 $(-3 - 4i)(5 + i)$

9 $(-5 - 5i)(6 - 6i)$

10 $(-5 - 6i)(6 + i)$

11 $(4 - 4i)(5 + i)$

12 $(3 + 5i)(-6 + 6i)$

13 $(-4 - 6i)(6 + i)$

14 $(4 - 4i)(-2 + 4i)$

15 $(4 - 6i)(-6 - 6i)$

16 $(3 - 3i)(5 - 6i)$

17 $(-5 + 5i)(1 + 3i)$

18 $(-6 - 3i)(5 - 6i)$

19 $(-4 + 3i)(-5 + 6i)$

20 $(5 - 5i)(-1 - 3i)$

21 $(6 - 3i)(5 - 6i)$

22 $(-1 - 5i)(6 - 6i)$

23 $(6 + 3i)(-5 + 6i)$

17. Completing the square

Write each expression as $(x + p)^2 + q$.

24 $x^2 + 10x - 15$

25 $x^2 - 18x + 61$

26 $x^2 + 14x + 57$

27 $x^2 + 24x + 150$

28 $x^2 - 20x + 97$

29 $x^2 - 22x + 119$

30 $x^2 + 20x + 68$

31 $x^2 + 18x + 81$

32 $x^2 + 16x + 94$

33 $x^2 - 24x + 180$

Adding, Subtracting and Multiplying Polynomials

WORKED EXAMPLE

$(4x^2 + 8x + 2) + (3x^2 + 4x + 6)$

$4x^2 + 3x^2 = 7x^2$
$8x + 4x = 12x$
$2 + 6 = 8$
$7x^2 + 12x + 8$

18. Add, subtract and multiply polynomials

Work each one out.

1 $(7x^2 + 6x + 12) - (5x^2 - 3x + 7)$

2 $(3x + 5)(x + 1)$

3 $(8x^2 + 5x + 10) - (x^2 - 9x + 6)$

4 $(4x + 3)(x + 2)$

5 $(6x^2 + 7x + 2) + (6x^2 + 8x + 7)$

6 $(x + 2)(x + 5)$

7 $(2x + 1)(x + 5)$

8 $(6x^2 + 7x + 12) - (4x^2 - 4x + 2)$

9 $(4x + 2)(x + 3)$

10 $(3x + 5)(x + 3)$

11 $(7x^2 + 4x + 11) - (x^2 - 8x + 3)$

12 $(2x + 3)(x + 5)$

18. Add, subtract and multiply polynomials (continued)

Work each one out.

13 $(3x + 6)(x + 3)$

14 $(9x^2 + 7x + 6) - (8x^2 - 8x + 1)$

15 $(3x + 8)(x + 2)$

16 $(3x + 4)(x + 2)$

17 $(9x^2 + 7x + 11) - (7x^2 - 9x + 9)$

18 $(2x + 1)(x + 4)$

19 $(4x + 1)(x + 1)$

20 $(5x^2 + x + 10) - (3x^2 - 7x + 5)$

21 $(x + 4)(x + 2)$

22 $(2x + 3)(x + 3)$

23 $(x + 9)(x + 1)$

24 $(8x^2 + 5x + 12) - (2x^2 - 7x + 10)$

19. How many real solutions?

How many real solutions does each equation have? Circle one.

25 $x^2 - 5x + 130 = 0$

26 $x^2 - 34x + 63 = 0$

19. How many real solutions? (continued)

How many real solutions does each equation have? Circle one.

27 $x^2 + 32x - 83 = 0$

28 $x^2 + 2x + 1 = 0$

29 $x^2 + 16x + 65 = 0$

30 $x^2 + 17x + 32 = 0$

31 $x^2 - 20x + 22 = 0$

32 $x^2 + 32x + 70 = 0$

33 $x^2 + 35x + 102 = 0$

34 $x^2 - 18x + 81 = 0$

Dividing Polynomials

WORKED EXAMPLE

$$
\begin{array}{r|l}
x+1 & x^3 + 2x - 1 \\
\hline
 & x^2 - x + 3 \\
 & -x^3 - x^2 \\
 & \quad -x^2 + 2x - 1 \\
 & \quad x^2 + x \\
 & \quad\quad 3x - 1 \\
 & \quad\quad -3x - 3 \\
 & \quad\quad\quad -4
\end{array}
$$

20. Dividing polynomials

Divide each polynomial. Write 0 for the remainder when it divides exactly.

1

$$
\begin{array}{r|l}
x+2 & x^3 + 8x^2 + 12x \\
\hline
 & x^2 + 6x + 0 \\
 & -x^3 - 2x^2 \\
 & \quad 6x^2 + 12x \\
 & \quad -6x^2 - 12x \\
 & \quad\quad 0
\end{array}
$$

2

$$
\begin{array}{r|l}
x+5 & x^3 + 8x^2 + 20x + 21 \\
\hline
 & x^2 + 3x + 5 \\
 & -x^3 - 5x^2 \\
 & \quad 3x^2 + 20x + 21 \\
 & \quad -3x^2 - 15x \\
 & \quad\quad 5x + 21 \\
 & \quad\quad -5x - 25 \\
 & \quad\quad\quad -4
\end{array}
$$

3

$$
\begin{array}{r|l}
x+3 & x^3 - 14x - 15 \\
\hline
 & x^2 - 3x - 5 \\
 & -x^3 - 3x^2 \\
 & \quad -3x^2 - 14x - 15 \\
 & \quad 3x^2 + 9x \\
 & \quad\quad -5x - 15 \\
 & \quad\quad 5x + 15 \\
 & \quad\quad\quad 0
\end{array}
$$

4

$$
\begin{array}{r|l}
x+5 & x^3 - 31x - 30 \\
\hline
 & x^2 - 5x - 6 \\
 & -x^3 - 5x^2 \\
 & \quad -5x^2 - 31x - 30 \\
 & \quad 5x^2 + 25x \\
 & \quad\quad -6x - 30 \\
 & \quad\quad 6x + 30 \\
 & \quad\quad\quad 0
\end{array}
$$

5

$$
\begin{array}{r|l}
x-5 & x^3 - 9x^2 + 21x - 5 \\
\hline
 & x^2 - 4x + 1 \\
 & -x^3 + 5x^2 \\
 & \quad -4x^2 + 21x - 5 \\
 & \quad 4x^2 - 20x \\
 & \quad\quad x - 5 \\
 & \quad\quad -x + 5 \\
 & \quad\quad\quad 0
\end{array}
$$

6

$$
\begin{array}{r|l}
x-6 & x^3 - 12x^2 + 36x \\
\hline
 & x^2 - 6x + 0 \\
 & -x^3 + 6x^2 \\
 & \quad -6x^2 + 36x \\
 & \quad 6x^2 - 36x \\
 & \quad\quad 0
\end{array}
$$

7

$$
\begin{array}{r|l}
x-6 & x^3 - 12x^2 + 36x - 1 \\
\hline
 & x^2 - 6x + 0 \\
 & -x^3 + 6x^2 \\
 & \quad -6x^2 + 36x - 1 \\
 & \quad 6x^2 - 36x \\
 & \quad\quad -1
\end{array}
$$

8

$$
\begin{array}{r|l}
x-2 & x^3 - 8x^2 + 12x \\
\hline
 & x^2 - 6x + 0 \\
 & -x^3 + 2x^2 \\
 & \quad -6x^2 + 12x \\
 & \quad 6x^2 - 12x \\
 & \quad\quad 0
\end{array}
$$

9

$$
\begin{array}{r|l}
x-4 & x^3 + x^2 - 14x - 24 \\
\hline
 & x^2 + 5x + 6 \\
 & -x^3 + 4x^2 \\
 & \quad 5x^2 - 14x - 24 \\
 & \quad -5x^2 + 20x \\
 & \quad\quad 6x - 24 \\
 & \quad\quad -6x + 24 \\
 & \quad\quad\quad 0
\end{array}
$$

10

$$
\begin{array}{r|l}
x-5 & x^3 - 8x^2 + 10x + 25 \\
\hline
 & x^2 - 3x - 5 \\
 & -x^3 + 5x^2 \\
 & \quad -3x^2 + 10x + 25 \\
 & \quad 3x^2 - 15x \\
 & \quad\quad -5x + 25 \\
 & \quad\quad 5x - 25 \\
 & \quad\quad\quad 0
\end{array}
$$

21. Complex numbers

Add or subtract each pair of complex numbers.

11 $(5 + 5i) + (1 + 3i)$

12 $(2 - 3i) + (5 - 6i)$

13 $(4 + 4i) + (2 - 4i)$

14 $(3 - 3i) - (5 - 6i)$

15 $(-5 - 2i) - (-3 + 5i)$

16 $(3 - 6i) - (-6 - 6i)$

17 $(4 - 3i) + (5 - 6i)$

18 $(5 + i) - (-4 + 5i)$

19 $(6 - 5i) - (6 - 6i)$

20 $(6 + 6i) - (1 - 4i)$

Simplifying Rational Expressions

WORKED EXAMPLE

$$\frac{x^2 - 2x - 120}{x^2 - 5x - 150}$$

① $x^2 - 2x - 120 = (x + 10)(x - 12)$

② $x^2 - 5x - 150 = (x + 10)(x - 15)$

①/② $\dfrac{(x + 10)(x - 12)}{(x + 10)(x - 15)}$

$= \dfrac{(x - 12)}{(x - 15)}$

22. Simplifying rational expressions

Factor the top and the bottom, then cancel.

1 $\dfrac{x^2 - 22x + 112}{x^2 - 14x}$

2 $\dfrac{x^2 - 7x - 228}{x^2 - 36x + 323}$

3 $\dfrac{x^2 - 19x + 88}{x^2 - x - 110}$

4 $\dfrac{x^2 + 10x - 75}{x^2 + 29x + 210}$

5 $\dfrac{x^2 + x - 30}{x^2 - 12x - 108}$

6 $\dfrac{x^2 - 2x - 323}{x^2 + 16x - 17}$

7 $\dfrac{x^2 + 14x - 51}{x^2 + 31x + 238}$

8 $\dfrac{x^2 - 6x - 112}{x^2 - 7x - 120}$

22. Simplifying rational expressions (continued)

Factor the top and the bottom, then cancel.

9 $\dfrac{x^2 + 21x + 80}{x^2 - 2x - 288}$

10 $\dfrac{x^2 - 4x - 192}{x^2 - 23x + 112}$

11 $\dfrac{x^2 - 13x - 114}{x^2 - 15x - 76}$

12 $\dfrac{x^2 - 25x + 126}{x^2 - 324}$

13 $\dfrac{x^2 + 37x + 340}{x^2 + 25x + 136}$

14 $\dfrac{x^2 - 8x - 180}{x^2 + 16x + 60}$

15 $\dfrac{x^2 - 35x + 304}{x^2 - 9x - 190}$

16 $\dfrac{x^2 - 21x + 80}{x^2 + 2x - 288}$

17 $\dfrac{x^2 + 9x - 136}{x^2 + 13x - 68}$

18 $\dfrac{x^2 + 25x + 126}{x^2 - 324}$

19 $\dfrac{x^2 + 27x + 182}{x^2 + 34x + 280}$

20 $\dfrac{x^2 + 2x - 99}{x^2 - 24x + 135}$

21 $\dfrac{x^2 - 6x - 112}{x^2 + x - 210}$

23. Simplifying radicals

Simplify each cube root.

22 $\sqrt[3]{324}$

23 $\sqrt[3]{500}$

24 $\sqrt[3]{320}$

25 $\sqrt[3]{625}$

26 $\sqrt[3]{243}$

27 $\sqrt[3]{875}$

28 $\sqrt[3]{704}$

29 $\sqrt[3]{384}$

30 $\sqrt[3]{768}$

31 $\sqrt[3]{750}$

Multiplying and Dividing Rational Expressions

WORKED EXAMPLE

$$\frac{(x+8)}{(x+6)} \div \frac{(x-7)}{(x+6)}$$

$(x + 8)/(x + 6) \times (x + 6)/(x - 7)$
$(x + 6)$ cancels
$(x + 8) / (x - 7)$
$x \neq -6, 7$

24. Multiplying and dividing rational expressions

Multiply or divide, then cancel. Write each answer as one fraction.

1 $\frac{(x+8)}{(x-3)} \div \frac{(x+4)}{(x-3)}$

2 $\frac{(x-9)}{(x-8)} \div \frac{(x-6)}{(x-8)}$

3 $\frac{(x-6)}{(x+8)} \times \frac{(x+8)}{(x+2)}$

4 $\frac{(x-6)}{(x+9)} \times \frac{(x+9)}{(x+6)}$

5 $\frac{(x-4)}{(x+3)} \times \frac{(x+3)}{(x-8)}$

6 $\frac{(x-8)}{(x-9)} \times \frac{(x-9)}{(x-5)}$

7 $\frac{(x-5)}{(x+4)} \div \frac{(x-9)}{(x+4)}$

8 $\frac{(x+8)}{(x+7)} \div \frac{(x-3)}{(x+7)}$

9 $\frac{(x+7)}{(x-9)} \div \frac{(x-1)}{(x-9)}$

10 $\frac{(x+2)}{(x-9)} \times \frac{(x-9)}{(x-2)}$

11 $\frac{(x+4)}{(x-7)} \div \frac{(x-8)}{(x-7)}$

12 $\frac{(x-9)}{(x-6)} \div \frac{(x+2)}{(x-6)}$

13 $\frac{(x+9)}{(x+6)} \div \frac{(x-2)}{(x+6)}$

14 $\frac{(x+1)}{(x-8)} \div \frac{(x-3)}{(x-8)}$

15 $\frac{(x+2)}{(x+9)} \div \frac{(x-1)}{(x+9)}$

16 $\frac{(x+6)}{(x+8)} \div \frac{(x-9)}{(x+8)}$

17 $\frac{x}{(x+2)} \times \frac{(x+2)}{(x+8)}$

18 $\frac{(x+1)}{(x+7)} \div \frac{(x+9)}{(x+7)}$

25. Add, subtract and multiply polynomials

Work each one out.

19 $(9x^2 + 7x + 11) - (7x^2 - 9x + 9)$

20 $(3x + 2)(x + 4)$

21 $(9x^2 + 8x + 11) - (4x^2 - 5x + 7)$

22 $(4x + 1)(x + 3)$

23 $(8x^2 + 7x + 12) - (5x^2 - 4x + 2)$

24 $(x + 6)(x + 3)$

25 $(4x + 6)(x + 1)$

26 $(6x^2 + 7x + 9) - (3x^2 - 6x + 6)$

27 $(4x + 3)(x + 4)$

28 $(2x + 1)(x + 3)$

Solving Rational Equations

WORKED EXAMPLE

$\frac{5}{(x-7)} = \frac{-5}{(x-3)}$, x = 5

$x \neq 3, 7$

$5(x-3) = -5(x-7)$

$x = 5$

$x = 5$ and $x \neq 3, 7$

26. Solving rational equations

Solve each equation for x.

1 $\frac{-21}{(x-6)} = 3$, x = ____

2 $\frac{56}{(x-8)} = \frac{-24}{(x+2)}$, x = ____

3 $\frac{-20}{(x-5)} = 2$, x = ____

4 $\frac{-65}{(x-5)} = 5$, x = ____

5 $\frac{21}{(x+5)} = \frac{-28}{(x-2)}$, x = ____

6 $\frac{-70}{(x-1)} = 7$, x = ____

7 $\frac{-42}{(x-4)} = 7$, x = ____

8 $\frac{24}{(x+4)} = 4$, x = ____

9 $\frac{18}{(x-5)} = \frac{42}{(x-1)}$, x = ____

10 $\frac{52}{(x+7)} = \frac{8}{(x-4)}$, x = ____

26. Solving rational equations (continued)

Solve each equation for x.

11 $\frac{22}{(x-8)} = \frac{16}{(x-5)}$, x = ____

12 $\frac{40}{(x+1)} = 8$, x = ____

13 $\frac{72}{(x+1)} = 9$, x = ____

14 $\frac{20}{x} = \frac{36}{(x-4)}$, x = ____

15 $\frac{40}{(x-8)} = \frac{-45}{(x+9)}$, x = ____

16 $\frac{55}{(x-3)} = \frac{5}{(x+7)}$, x = ____

17 $\frac{15}{(x-2)} = \frac{35}{(x+2)}$, x = ____

18 $\frac{-48}{(x-5)} = 6$, x = ____

19 $\frac{-60}{(x-6)} = 4$, x = ____

20 $\frac{-36}{(x-4)} = 9$, x = ____

21 $\frac{64}{(x+3)} = 8$, x = ____

22 $\frac{12}{(x+5)} = \frac{40}{(x-2)}$, x = ____

23 $\frac{80}{(x+9)} = \frac{24}{(x+2)}$, x = ____

24 $\frac{-16}{(x+1)} = 8$, x = ____

27. Dividing polynomials

Divide each polynomial. Write 0 for the remainder when it divides exactly.

25

$$x + 4 \overline{)\, x^3 - x^2 - 19x + 4}$$

$x^2 - 5x + 1$

$-x^3 - 4x^2$

$-5x^2 - 19x + 4$

$5x^2 + 20x$

$x + 4$

$-x - 4$

0

26

$$x - 5 \overline{)\, x^3 - 11x^2 + 30x + 9}$$

$x^2 - 6x + 0$

$-x^3 + 5x^2$

$-6x^2 + 30x + 9$

$6x^2 - 30x$

9

27

$$x + 2 \overline{)\, x^3 + 8x^2 + 12x - 9}$$

$x^2 + 6x + 0$

$-x^3 - 2x^2$

$6x^2 + 12x - 9$

$-6x^2 - 12x$

-9

28

$$x - 6 \overline{)\, x^3 - x^2 - 31x + 6}$$

$x^2 + 5x - 1$

$-x^3 + 6x^2$

$5x^2 - 31x + 6$

$-5x^2 + 30x$

$-x + 6$

$x - 6$

0

29

$$x - 4 \overline{)\, x^3 - 2x^2 - 4x - 17}$$

$x^2 + 2x + 4$

$-x^3 + 4x^2$

$2x^2 - 4x - 17$

$-2x^2 + 8x$

$4x - 17$

$-4x + 16$

-1

30

$$x + 6 \overline{)\, x^3 - 36x}$$

$x^2 - 6x + 0$

$-x^3 - 6x^2$

$-6x^2 - 36x$

$6x^2 + 36x$

0

31

$$x + 5 \overline{)\, x^3 + 11x^2 + 30x}$$

$x^2 + 6x + 0$

$-x^3 - 5x^2$

$6x^2 + 30x$

$-6x^2 - 30x$

0

32

$$x + 5 \overline{)\, x^3 + x^2 - 19x + 4}$$

$x^2 - 4x + 1$

$-x^3 - 5x^2$

$-4x^2 - 19x + 4$

$4x^2 + 20x$

$x + 4$

$-x - 5$

-1

Arithmetic and Geometric Sequences

WORKED EXAMPLE

$-4, -3, -2, -1, \ldots$ $a_{10} =$ 5

$d = 1$

$a_n = -4 + (n - 1) \times 1$

$a_{10} = -4 + 9 \times 1$

$-4 + 9 = 5$

$a_{10} = 5$

28. Arithmetic and geometric sequences

Find the term shown for each sequence.

1 $5, 9, 13, 17, \ldots$ $a_{17} =$ _____

2 $5, 10, 15, 20, \ldots$ $a_{24} =$ _____

3 $12, 9, 6, 3, \ldots$ $a_{20} =$ _____

4 $-7, -16, -25, -34, \ldots$ $a_{22} =$ _____

5 $8, 1, -6, -13, \ldots$ $a_{15} =$ _____

6 $10, 6, 2, -2, \ldots$ $a_{24} =$ _____

7 $11, 14, 17, 20, \ldots$ $a_{25} =$ _____

8 $-4, -6, -8, -10, \ldots$ $a_{19} =$ _____

9 $0, 9, 18, 27, \ldots$ $a_{19} =$ _____

10 $-2, 6, 14, 22, \ldots$ $a_{23} =$ _____

28. Arithmetic and geometric sequences (continued)

Find the term shown for each sequence.

11 $-1, -10, -19, -28, \ldots$ $a_{14} =$ _____

12 $4, -5, -14, -23, \ldots$ $a_{22} =$ _____

13 $4, 10, 16, 22, \ldots$ $a_{24} =$ _____

14 $-7, -2, 3, 8, \ldots$ $a_{18} =$ _____

15 $6, 10, 14, 18, \ldots$ $a_{23} =$ _____

16 $12, 14, 16, 18, \ldots$ $a_{24} =$ _____

17 $-3, -1, 1, 3, \ldots$ $a_{22} =$ _____

18 $1, -5, -11, -17, \ldots$ $a_{23} =$ _____

19 $8, 2, -4, -10, \ldots$ $a_{21} =$ _____

20 $-8, -9, -10, -11, \ldots$ $a_{23} =$ _____

21 $-1, 6, 13, 20, \ldots$ $a_{23} =$ _____

22 $11, 17, 23, 29, \ldots$ $a_{22} =$ _____

23 $4, 13, 22, 31, \ldots$ $a_{21} =$ _____

24 $0, -6, -12, -18, \ldots$ $a_{17} =$ _____

29. Simplifying rational expressions

Factor the top and the bottom, then cancel.

25 $\dfrac{x^2 - 12x - 160}{x^2 + 14x + 48}$

26 $\dfrac{x^2 - 5x - 234}{x^2 + 31x + 234}$

27 $\dfrac{x^2 + 24x + 95}{x^2 + 11x + 30}$

28 $\dfrac{x^2 - 34x + 280}{x^2 - 12x - 28}$

29 $\dfrac{x^2 - 19x + 88}{x^2 - x - 110}$

30 $\dfrac{x^2 + 34x + 280}{x^2 + 20x}$

31 $\dfrac{x^2 - 9x - 220}{x^2 + 29x + 198}$

32 $\dfrac{x^2 - 5x - 24}{x^2 + 10x - 144}$

33 $\dfrac{x^2 - 12x - 160}{x^2 - 27x + 140}$

34 $\dfrac{x^2 + 3x - 238}{x^2 + 35x + 306}$

Sums of Series

WORKED EXAMPLE

5, 10, 20, … S_5 = 155

$S_5 = 5(2^5 - 1) \div (2 - 1)$

$S_5 = 5 \times 31 \div 1 = 155$

30. Sums of series

S_n is the sum of the first n terms. Does each sequence add or multiply?

1 **6, 24, 96, … S_6 = _____**

2 **2, 11, 20, … S_8 = _____**

3 **9, 27, 81, … S_7 = _____**

4 **7, 11, 15, … S_8 = _____**

5 **8, 32, 128, … S_6 = _____**

6 **9, 12, 15, … S_9 = _____**

7 **1, 4, 16, … S_7 = _____**

8 **8, 16, 32, … S_7 = _____**

9 **11, 1, −9, … S_9 = _____**

10 **11, 17, 23, … S_9 = _____**

11 **5, −5, −15, … S_5 = _____**

12 **2, −1, −4, … S_{10} = _____**

13 **9, 5, 1, … S_7 = _____**

14 **2, 8, 32, … S_6 = _____**

15 **1, 11, 21, … S_6 = _____**

16 **6, 3, 0, … S_{10} = _____**

17 **2, 4, 8, … S_7 = _____**

18 **5, 8, 11, … S_9 = _____**

19 **10, 5, 0, … S_9 = _____**

20 **3, 9, 27, … S_7 = _____**

21 **1, 4, 7, … S_9 = _____**

31. Multiplying and dividing rational expressions

Multiply or divide, then cancel. Write each answer as one fraction.

22 $\dfrac{(x+8)}{(x+2)} \div \dfrac{(x+1)}{(x+2)}$

23 $\dfrac{(x-3)}{(x+6)} \div \dfrac{(x+9)}{(x+6)}$

24 $\dfrac{(x-7)}{(x+5)} \div \dfrac{(x+8)}{(x+5)}$

25 $\dfrac{(x-9)}{x} \times \dfrac{x}{(x+2)}$

26 $\dfrac{(x+3)}{(x-8)} \times \dfrac{(x-8)}{(x+6)}$

27 $\dfrac{(x-1)}{(x+9)} \times \dfrac{(x+9)}{(x+7)}$

28 $\dfrac{(x+9)}{(x+8)} \div \dfrac{(x+6)}{(x+8)}$

29 $\dfrac{(x-4)}{(x-3)} \times \dfrac{(x-3)}{(x-8)}$

30 $\dfrac{(x-6)}{(x+4)} \div \dfrac{(x+9)}{(x+4)}$

31 $\dfrac{(x+2)}{(x+1)} \div \dfrac{(x-9)}{(x+1)}$

Check the Log Laws

32. Check the log laws

Read each line of working and choose the answer that is correct.

1 **Check this line: $\log(x^2) + \log(x^5) = 7 \log x$. Which is correct?**

$\log(x^2 + x^5)$ | $10 \log x$ | $7 \log x$ | $10(\log x)^2$

Why? .....

2 **Ben wrote $\log(7x) - \log 7 = \log x \div \log 7$. Which is correct?**

$\log x$ | $7 \log x$ | $\log x \div \log 7$ | $\log 7 - \log x$

Why? .....

3 **$\log(x^3) + \log(x^5) = \log(x^3 + x^5)$ is wrong. Which is right?**

$\log(x^3 + x^5)$ | $8 \log x$ | $15 \log x$ | $15(\log x)^2$

Why? .....

4 **Spot the mistake: $\log(x^4) + \log(x^5) = 20(\log x)^2$. Which answer is right?**

$\log(x^4 + x^5)$ | $20 \log x$ | $20(\log x)^2$ | $9 \log x$

Why? .....

5 **Noor's answer to $\log(x^3) + \log(x^4)$ is $12(\log x)^2$. Which is correct?**

$7 \log x$ | $12(\log x)^2$ | $\log(x^3 + x^4)$ | $12 \log x$

Why? .....

6 **Check this line: $\log(2x) - \log 2 = \log x$. Which is correct?**

$\log x \div \log 2$ | $\log x$ | $\log 2 - \log x$ | $2 \log x$

Why? .....

7 **Ben says $\log(x^2) + \log(x^4)$ comes to $6 \log x$. Which is correct?**

$8(\log x)^2$ | $8 \log x$ | $\log(x^2 + x^4)$ | $6 \log x$

Why? .....

8 **Imani wrote $\log(x^2) + \log(x^3) = 6(\log x)^2$. What should it be?**

$6(\log x)^2$ | $5 \log x$ | $\log(x^2 + x^3)$ | $6 \log x$

Why? .....

Answer Key (1 of 7)

Simplifying Radicals

Simplifying radicals

1. $\sqrt{360} = 6\sqrt{10}$
2. $\sqrt{486} = 9\sqrt{6}$
3. $\sqrt{125} = 5\sqrt{5}$
4. $\sqrt{300} = 10\sqrt{3}$
5. $\sqrt{72} = 6\sqrt{2}$
6. $\sqrt{605} = 11\sqrt{5}$
7. $\sqrt{162} = 9\sqrt{2}$
8. $\sqrt{175} = 5\sqrt{7}$
9. $\sqrt{847} = 11\sqrt{7}$
10. $\sqrt{704} = 8\sqrt{11}$
11. $\sqrt{504} = 6\sqrt{14}$
12. $\sqrt{384} = 8\sqrt{6}$
13. $\sqrt{726} = 11\sqrt{6}$
14. $\sqrt{98} = 7\sqrt{2}$
15. $\sqrt{600} = 10\sqrt{6}$
16. $\sqrt{243} = 9\sqrt{3}$
17. $\sqrt{128} = 8\sqrt{2}$
18. $\sqrt{896} = 8\sqrt{14}$
19. $\sqrt{810} = 9\sqrt{10}$
20. $\sqrt{539} = 7\sqrt{11}$
21. $\sqrt{500} = 10\sqrt{5}$
22. $\sqrt{320} = 8\sqrt{5}$
23. $\sqrt{288} = 12\sqrt{2}$
24. $\sqrt{396} = 6\sqrt{11}$
25. $\sqrt{960} = 8\sqrt{15}$
26. $\sqrt{242} = 11\sqrt{2}$
27. $\sqrt{245} = 7\sqrt{5}$
28. $\sqrt{405} = 9\sqrt{5}$

Cube Roots

Simplifying radicals

1. $\sqrt[3]{297} = 3\sqrt[3]{11}$
2. $\sqrt[3]{250} = 5\sqrt[3]{2}$
3. $\sqrt[3]{320} = 4\sqrt[3]{5}$
4. $\sqrt[3]{625} = 5\sqrt[3]{5}$
5. $\sqrt[3]{243} = 3\sqrt[3]{9}$
6. $\sqrt[3]{640} = 4\sqrt[3]{10}$
7. $\sqrt[3]{375} = 5\sqrt[3]{3}$
8. $\sqrt[3]{324} = 3\sqrt[3]{12}$
9. $\sqrt[3]{875} = 5\sqrt[3]{7}$
10. $\sqrt[3]{750} = 5\sqrt[3]{6}$
11. $\sqrt[3]{128} = 4\sqrt[3]{2}$
12. $\sqrt[3]{768} = 4\sqrt[3]{12}$
13. $\sqrt[3]{704} = 4\sqrt[3]{11}$
14. $\sqrt[3]{256} = 4\sqrt[3]{4}$
15. $\sqrt[3]{576} = 4\sqrt[3]{9}$
16. $\sqrt[3]{448} = 4\sqrt[3]{7}$
17. $\sqrt[3]{192} = 4\sqrt[3]{3}$

Cube Roots (continued)

Simplifying radicals (continued)

18. $\sqrt[3]{500} = 5\sqrt[3]{4}$
19. $\sqrt[3]{384} = 4\sqrt[3]{6}$
20. $\sqrt[3]{40} = 2\sqrt[3]{5}$
21. $\sqrt[3]{270} = 3\sqrt[3]{10}$
22. $\sqrt[3]{48} = 2\sqrt[3]{6}$
23. $\sqrt[3]{54} = 3\sqrt[3]{2}$
24. $\sqrt[3]{96} = 2\sqrt[3]{12}$

Rational Exponents

Rational exponents

1. 216^(2/3) = 36
2. 1000^(4/3) = 10000
3. 256^(1/4) = 4
4. 729^(4/3) = 6561
5. 100^(1/2) = 10
6. 6561^(1/4) = 9
7. 10000^(1/4) = 10
8. 16^(3/4) = 8
9. 729^(1/3) = 9
10. 512^(1/3) = 8
11. 256^(5/4) = 1024
12. 1000^(1/3) = 10
13. 2401^(1/4) = 7
14. 81^(5/4) = 243
15. 512^(4/3) = 4096
16. 1296^(5/4) = 7776
17. 100^(3/2) = 1000
18. 625^(3/4) = 125
19. 1296^(1/4) = 6
20. 125^(1/3) = 5
21. 4096^(1/4) = 8
22. 625^(1/4) = 5
23. 64^(3/2) = 512
24. 4^(3/2) = 8
25. 64^(1/2) = 8
26. 343^(4/3) = 2401
27. 64^(5/3) = 1024
28. 216^(5/3) = 7776

Logarithms

Logarithms

1. $\log_3 1/729 = -6$
2. $\log_3 2187 = 7$
3. $\log_7 1/2401 = -4$
4. $\log_3 19683 = 9$
5. $\log_9 59049 = 5$
6. $\log_2 65536 = 16$
7. $\log_2 2048 = 11$

Answer Key (2 of 7)

Logarithms (continued)

Logarithms (continued)

8. $\log_4 256 = 4$
9. $\log_3 2187 = 7$
10. $\log_2 8192 = 13$
11. $\log_3 1/81 = -4$
12. $\log_4 16384 = 7$
13. $\log_2 256 = 8$
14. $\log_6 1/7776 = -5$
15. $\log_3 6561 = 8$
16. $\log_2 1/2048 = -11$
17. $\log_4 1/1024 = -5$
18. $\log_5 78125 = 7$
19. $\log_4 65536 = 8$
20. $\log_4 4096 = 6$
21. $\log_2 16384 = 14$
22. $\log_2 4096 = 12$
23. $\log_3 19683 = 9$
24. $\log_6 46656 = 6$
25. $\log_4 16384 = 7$
26. $\log_2 1024 = 10$
27. $\log_5 1/625 = -4$
28. $\log_2 32768 = 15$

Simplifying radicals

29. $\sqrt{252} = 6\sqrt{7}$
30. $\sqrt{864} = 12\sqrt{6}$
31. $\sqrt{539} = 7\sqrt{11}$
32. $\sqrt{640} = 8\sqrt{10}$
33. $\sqrt{396} = 6\sqrt{11}$
34. $\sqrt{720} = 12\sqrt{5}$
35. $\sqrt{704} = 8\sqrt{11}$
36. $\sqrt{108} = 6\sqrt{3}$
37. $\sqrt{810} = 9\sqrt{10}$
38. $\sqrt{162} = 9\sqrt{2}$

The Laws of Logarithms

Laws of logarithms

1. $\log_5 6 + \log_5 7 = \log_5 42$
2. $2 \log_3 11 = \log_3 121$
3. $\log_2 6 + \log_2 8 = \log_2 48$
4. $2 \log_{10} 9 = \log_{10} 81$
5. $2 \log_2 6 = \log_2 36$
6. $\log_{10} 108 - \log_{10} 9 = \log_{10} 12$
7. $\log_5 132 - \log_5 12 = \log_5 11$
8. $\log_2 3 + \log_2 11 = \log_2 33$
9. $\log_2 72 - \log_2 6 = \log_2 12$
10. $\log_{10} 88 - \log_{10} 8 = \log_{10} 11$
11. $\log_2 42 - \log_2 7 = \log_2 6$
12. $\log_5 84 - \log_5 12 = \log_5 7$
13. $\log_{10} 84 - \log_{10} 12 = \log_{10} 7$

The Laws of Logarithms (continued)

Laws of logarithms (continued)

14. $\log_2 44 - \log_2 11 = \log_2 4$, and that is 2
15. $2 \log_5 12 = \log_5 144$
16. $\log_{10} 81 - \log_{10} 9 = \log_{10} 9$
17. $\log_2 30 - \log_2 5 = \log_2 6$
18. $\log_5 9 + \log_5 10 = \log_5 90$
19. $\log_2 66 - \log_2 6 = \log_2 11$
20. $\log_5 48 - \log_5 12 = \log_5 4$
21. $\log_3 90 - \log_3 9 = \log_3 10$
22. $\log_2 120 - \log_2 12 = \log_2 10$
23. $\log_3 121 - \log_3 11 = \log_3 11$
24. $\log_5 4 + \log_5 10 = \log_5 40$

Simplifying radicals

25. $\sqrt[3]{128} = 4\sqrt[3]{2}$
26. $\sqrt[3]{704} = 4\sqrt[3]{11}$
27. $\sqrt[3]{192} = 4\sqrt[3]{3}$
28. $\sqrt[3]{576} = 4\sqrt[3]{9}$
29. $\sqrt[3]{270} = 3\sqrt[3]{10}$
30. $\sqrt[3]{375} = 5\sqrt[3]{3}$
31. $\sqrt[3]{500} = 5\sqrt[3]{4}$
32. $\sqrt[3]{384} = 4\sqrt[3]{6}$
33. $\sqrt[3]{768} = 4\sqrt[3]{12}$
34. $\sqrt[3]{875} = 5\sqrt[3]{7}$

Exponential Functions

Evaluate exponential functions

1. $f(x) = 7 \cdot 6^x \rightarrow f(1) = 42$
2. $f(x) = 7 \cdot 2^x \rightarrow f(6) = 448$
3. $f(x) = 5 \cdot 9^x \rightarrow f(0) = 5$
4. $f(x) = 6 \cdot 4^x \rightarrow f(4) = 1536$
5. $f(x) = 8 \cdot 7^x \rightarrow f(1) = 56$
6. $f(x) = 4 \cdot 2^x \rightarrow f(5) = 128$
7. $f(x) = 6 \cdot 3^x \rightarrow f(5) = 1458$
8. $f(x) = 10 \cdot 8^x \rightarrow f(1) = 80$
9. $f(x) = 4 \cdot 9^x \rightarrow f(3) = 2916$
10. $f(x) = 10 \cdot 9^x \rightarrow f(2) = 810$
11. $f(x) = 3 \cdot 9^x \rightarrow f(1) = 27$
12. $f(x) = 8 \cdot 2^x \rightarrow f(6) = 512$
13. $f(x) = 12 \cdot 6^x \rightarrow f(3) = 2592$
14. $f(x) = 6 \cdot 4^x \rightarrow f(2) = 96$
15. $f(x) = 2 \cdot 2^x \rightarrow f(5) = 64$
16. $f(x) = 10 \cdot 6^x \rightarrow f(3) = 2160$
17. $f(x) = 6 \cdot 3^x \rightarrow f(2) = 54$
18. $f(x) = 10 \cdot 3^x \rightarrow f(4) = 810$
19. $f(x) = 7 \cdot 10^x \rightarrow f(2) = 700$
20. $f(x) = 4 \cdot 4^x \rightarrow f(4) = 1024$
21. $f(x) = 4 \cdot 3^x \rightarrow f(5) = 972$
22. $f(x) = 6 \cdot 9^x \rightarrow f(2) = 486$
23. $f(x) = 10 \cdot 5^x \rightarrow f(3) = 1250$

Answer Key (3 of 7)

Exponential Functions (continued)

Evaluate exponential functions (continued)

24. $f(x) = 7 \cdot 3^x \rightarrow f(2) = 63$

Rational exponents

25. 81^(5/4) = 243
26. 1296^(1/4) = 6
27. 64^(3/2) = 512
28. 1000^(4/3) = 10000
29. 81^(1/2) = 9
30. 729^(1/3) = 9
31. 1296^(5/4) = 7776
32. 81^(1/4) = 3
33. 1000^(1/3) = 10
34. 4096^(1/4) = 8

Completing the Square

Completing the square

1. $x^2 + 16x + 65 = (x + 8)^2 + 1$
2. $x^2 - 22x + 145 = (x - 11)^2 + 24$
3. $x^2 + 16x + 67 = (x + 8)^2 + 3$
4. $x^2 + 24x + 147 = (x + 12)^2 + 3$
5. $x^2 - 16x + 66 = (x - 8)^2 + 2$
6. $x^2 - 24x + 119 = (x - 12)^2 - 25$
7. $x^2 - 16x + 24 = (x - 8)^2 - 40$
8. $x^2 - 8x - 24 = (x - 4)^2 - 40$
9. $x^2 - 20x + 128 = (x - 10)^2 + 28$
10. $x^2 + 24x + 124 = (x + 12)^2 - 20$
11. $x^2 + 12x + 27 = (x + 6)^2 - 9$
12. $x^2 + 24x + 150 = (x + 12)^2 + 6$
13. $x^2 + 22x + 129 = (x + 11)^2 + 8$
14. $x^2 - 20x + 102 = (x - 10)^2 + 2$
15. $x^2 + 24x + 174 = (x + 12)^2 + 30$
16. $x^2 + 24x + 139 = (x + 12)^2 - 5$
17. $x^2 - 12x + 20 = (x - 6)^2 - 16$
18. $x^2 + 20x + 130 = (x + 10)^2 + 30$
19. $x^2 + 22x + 133 = (x + 11)^2 + 12$
20. $x^2 - 12x + 56 = (x - 6)^2 + 20$
21. $x^2 + 22x + 96 = (x + 11)^2 - 25$
22. $x^2 - 24x + 128 = (x - 12)^2 - 16$

Logarithms

23. $\log_8 32768 = 5$
24. $\log_2 32768 = 15$
25. $\log_9 6561 = 4$
26. $\log_2 4096 = 12$
27. $\log_{10} 100000 = 5$
28. $\log_2 1024 = 10$
29. $\log_4 65536 = 8$
30. $\log_2 1/128 = -7$
31. $\log_3 2187 = 7$
32. $\log_5 78125 = 7$

How Many Real Solutions

How many real solutions?

1. $x^2 + 22x + 149 = 0$: $22^2 - 4 \times 1 \times 149 = 484 - 596 = -112$ → None
2. $x^2 + 12x - 160 = 0$: $12^2 - 4 \times 1 \times -160 = 144 - -640 = 784$ → Two
3. $x^2 - 3x + 98 = 0$: $-3^2 - 4 \times 1 \times 98 = 9 - 392 = -383$ → None
4. $x^2 + 18x + 107 = 0$: $18^2 - 4 \times 1 \times 107 = 324 - 428 = -104$ → None
5. $x^2 + 2x + 1 = 0$: $2^2 - 4 \times 1 \times 1 = 4 - 4 = 0$ → One
6. $x^2 + 16x + 65 = 0$: $16^2 - 4 \times 1 \times 65 = 256 - 260 = -4$ → None
7. $x^2 - 34x - 34 = 0$: $-34^2 - 4 \times 1 \times -34 = 1156 - -136 = 1292$ → Two
8. $x^2 - 28x - 147 = 0$: $-28^2 - 4 \times 1 \times -147 = 784 - -588 = 1372$ → Two
9. $x^2 - 9x - 178 = 0$: $-9^2 - 4 \times 1 \times -178 = 81 - -712 = 793$ → Two
10. $x^2 - 20x + 86 = 0$: $-20^2 - 4 \times 1 \times 86 = 400 - 344 = 56$ → Two
11. $x^2 - 18x + 81 = 0$: $-18^2 - 4 \times 1 \times 81 = 324 - 324 = 0$ → One
12. $x^2 + 34x + 56 = 0$: $34^2 - 4 \times 1 \times 56 = 1156 - 224 = 932$ → Two
13. $x^2 + 15x + 151 = 0$: $15^2 - 4 \times 1 \times 151 = 225 - 604 = -379$ → None
14. $x^2 + 102 = 0$: $0^2 - 4 \times 1 \times 102 = 0 - 408 = -408$ → None
15. $x^2 - 9x + 42 = 0$: $-9^2 - 4 \times 1 \times 42 = 81 - 168 = -87$ → None
16. $x^2 + 23x - 48 = 0$: $23^2 - 4 \times 1 \times -48 = 529 - -192 = 721$ → Two
17. $x^2 - 6x + 9 = 0$: $-6^2 - 4 \times 1 \times 9 = 36 - 36 = 0$ → One
18. $x^2 - 8x + 72 = 0$: $-8^2 - 4 \times 1 \times 72 = 64 - 288 = -224$ → None
19. $x^2 - 21x - 137 = 0$: $-21^2 - 4 \times 1 \times -137 = 441 - -548 = 989$ → Two
20. $x^2 + 24x - 18 = 0$: $24^2 - 4 \times 1 \times -18 = 576 - -72 = 648$ → Two
21. $x^2 - 4x + 4 = 0$: $-4^2 - 4 \times 1 \times 4 = 16 - 16 = 0$ → One
22. $x^2 - 7x + 152 = 0$: $-7^2 - 4 \times 1 \times 152 = 49 - 608 = -559$ → None
23. $x^2 + 17x + 172 = 0$: $17^2 - 4 \times 1 \times 172 = 289 - 688 = -399$ → None
24. $x^2 + 26x + 90 = 0$: $26^2 - 4 \times 1 \times 90 = 676 - 360 = 316$ → Two

Laws of logarithms

25. $\log_3 48 - \log_3 6 = \log_3 8$
26. $\log_5 8 + \log_5 9 = \log_5 72$
27. $2 \log_3 6 = \log_3 36$
28. $4 \log_2 3 = \log_2 81$
29. $\log_3 50 - \log_3 5 = \log_3 10$
30. $\log_2 11 + \log_2 11 = \log_2 121$
31. $\log_2 77 - \log_2 11 = \log_2 7$
32. $\log_5 3 + \log_5 12 = \log_5 36$
33. $\log_5 110 - \log_5 10 = \log_5 11$

Answer Key (4 of 7)

How Many Real Solutions (continued)

Laws of logarithms (continued)

34. $\log_2 96 - \log_2 12 = \log_2 8$, and that is 3

Adding and Subtracting Complex Numbers

Complex numbers

1. $(5 + 2i) - (3 - 5i) = 2 + 7i$
2. $(-2 + 5i) - (-6 + 6i) = 4 - i$
3. $(3 + 4i) - (-5 - i) = 8 + 5i$
4. $(3 - 6i) + (6 + i) = 9 - 5i$
5. $(4 + 2i) + (3 - 5i) = 7 - 3i$
6. $(6 - 6i) + (-6 - 6i) = -12i$
7. $(4 + 6i) - (-6 - i) = 10 + 7i$
8. $(-4 - i) + (4 - 5i) = -6i$
9. $(4 + 5i) - (-6 + 6i) = 10 - i$
10. $(5 - 3i) - (5 - 6i) = 3i$
11. $(4 - 5i) - (-1 - 3i) = 5 - 2i$
12. $(1 - 6i) - (-6 - 6i) = 7$
13. $(-6 + 6i) - (-6 - i) = 7i$
14. $(2 + 6i) - (-6 - i) = 8 + 7i$
15. $(-3 - 6i) - (-6 - 6i) = 3$
16. $(-5 + 6i) + (-6 - i) = -11 + 5i$
17. $(-4 + 2i) + (3 - 5i) = -1 - 3i$
18. $(5 - 5i) + (6 - 6i) = 11 - 11i$
19. $(-6 - 2i) + (-3 + 5i) = -9 + 3i$
20. $(-4 - 5i) - (-1 - 3i) = -3 - 2i$
21. $(-3 + 3i) - (-5 + 6i) = 2 - 3i$
22. $(6 + 6i) - (-6 - i) = 12 + 7i$

Evaluate exponential functions

23. $f(x) = 4 \cdot 2^x \rightarrow f(4) = 64$
24. $f(x) = 12 \cdot 8^x \rightarrow f(2) = 768$
25. $f(x) = 7 \cdot 4^x \rightarrow f(3) = 448$
26. $f(x) = 10 \cdot 2^x \rightarrow f(4) = 160$
27. $f(x) = 10 \cdot 4^x \rightarrow f(3) = 640$
28. $f(x) = 5 \cdot 4^x \rightarrow f(4) = 1280$
29. $f(x) = 6 \cdot 3^x \rightarrow f(4) = 486$
30. $f(x) = 6 \cdot 9^x \rightarrow f(1) = 54$
31. $f(x) = 10 \cdot 2^x \rightarrow f(5) = 320$
32. $f(x) = 2 \cdot 10^x \rightarrow f(3) = 2000$

Multiplying Complex Numbers

Complex numbers

1. $(3 + 4i)(-5 - i) = -11 - 23i$
2. $(-6 + 6i)(-6 - i) = 42 - 30i$
3. $(6 - 3i)(-2 - 3i) = -21 - 12i$
4. $(6 - 2i)(-3 + 5i) = -8 + 36i$
5. $(-5 + 2i)(3 - 5i) = -5 + 31i$
6. $(-2 - 6i)(-6 - 6i) = -24 + 48i$
7. $(-6 - 5i)(6 - 6i) = -66 + 6i$
8. $(-3 - 4i)(5 + i) = -11 - 23i$

Multiplying Complex Numbers (continued)

Complex numbers (continued)

9. $(-5 - 5i)(6 - 6i) = -60$
10. $(-5 - 6i)(6 + i) = -24 - 41i$
11. $(4 - 4i)(5 + i) = 24 - 16i$
12. $(3 + 5i)(-6 + 6i) = -48 - 12i$
13. $(-4 - 6i)(6 + i) = -18 - 40i$
14. $(4 - 4i)(-2 + 4i) = 8 + 24i$
15. $(4 - 6i)(-6 - 6i) = -60 + 12i$
16. $(3 - 3i)(5 - 6i) = -3 - 33i$
17. $(-5 + 5i)(1 + 3i) = -20 - 10i$
18. $(-6 - 3i)(5 - 6i) = -48 + 21i$
19. $(-4 + 3i)(-5 + 6i) = 2 - 39i$
20. $(5 - 5i)(-1 - 3i) = -20 - 10i$
21. $(6 - 3i)(5 - 6i) = 12 - 51i$
22. $(-1 - 5i)(6 - 6i) = -36 - 24i$
23. $(6 + 3i)(-5 + 6i) = -48 + 21i$

Completing the square

24. $x^2 + 10x - 15 = (x + 5)^2 - 40$
25. $x^2 - 18x + 61 = (x - 9)^2 - 20$
26. $x^2 + 14x + 57 = (x + 7)^2 + 8$
27. $x^2 + 24x + 150 = (x + 12)^2 + 6$
28. $x^2 - 20x + 97 = (x - 10)^2 - 3$
29. $x^2 - 22x + 119 = (x - 11)^2 - 2$
30. $x^2 + 20x + 68 = (x + 10)^2 - 32$
31. $x^2 + 18x + 81 = (x + 9)^2$
32. $x^2 + 16x + 94 = (x + 8)^2 + 30$
33. $x^2 - 24x + 180 = (x - 12)^2 + 36$

Adding, Subtracting and Multiplying Polynomials

Add, subtract and multiply polynomials

1. $(7x^2 + 6x + 12) - (5x^2 - 3x + 7) \rightarrow 2x^2 + 9x + 5$
2. $(3x + 5)(x + 1) \rightarrow 3x^2 + 8x + 5$
3. $(8x^2 + 5x + 10) - (x^2 - 9x + 6) \rightarrow 7x^2 + 14x + 4$
4. $(4x + 3)(x + 2) \rightarrow 4x^2 + 11x + 6$
5. $(6x^2 + 7x + 2) + (6x^2 + 8x + 7) \rightarrow 12x^2 + 15x + 9$
6. $(x + 2)(x + 5) \rightarrow 1x^2 + 7x + 10$
7. $(2x + 1)(x + 5) \rightarrow 2x^2 + 11x + 5$
8. $(6x^2 + 7x + 12) - (4x^2 - 4x + 2) \rightarrow 2x^2 + 11x + 10$
9. $(4x + 2)(x + 3) \rightarrow 4x^2 + 14x + 6$
10. $(3x + 5)(x + 3) \rightarrow 3x^2 + 14x + 15$
11. $(7x^2 + 4x + 11) - (x^2 - 8x + 3) \rightarrow 6x^2 + 12x + 8$
12. $(2x + 3)(x + 5) \rightarrow 2x^2 + 13x + 15$
13. $(3x + 6)(x + 3) \rightarrow 3x^2 + 15x + 18$
14. $(9x^2 + 7x + 6) - (8x^2 - 8x + 1) \rightarrow 1x^2 + 15x + 5$
15. $(3x + 8)(x + 2) \rightarrow 3x^2 + 14x + 16$
16. $(3x + 4)(x + 2) \rightarrow 3x^2 + 10x + 8$
17. $(9x^2 + 7x + 11) - (7x^2 - 9x + 9) \rightarrow 2x^2 + 16x + 2$
18. $(2x + 1)(x + 4) \rightarrow 2x^2 + 9x + 4$
19. $(4x + 1)(x + 1) \rightarrow 4x^2 + 5x + 1$

Answer Key (5 of 7)

Adding, Subtracting and Multiplying Polynomials (continued)

Add, subtract and multiply polynomials (continued)

20. $(5x^2 + x + 10) - (3x^2 - 7x + 5) \rightarrow 2x^2 + 8x + 5$
21. $(x + 4)(x + 2) \rightarrow 1x^2 + 6x + 8$
22. $(2x + 3)(x + 3) \rightarrow 2x^2 + 9x + 9$
23. $(x + 9)(x + 1) \rightarrow 1x^2 + 10x + 9$
24. $(8x^2 + 5x + 12) - (2x^2 - 7x + 10) \rightarrow 6x^2 + 12x + 2$

How many real solutions?

25. $x^2 - 5x + 130 = 0$: $-5^2 - 4 \times 1 \times 130 = 25 - 520 = -495 \rightarrow$ None
26. $x^2 - 34x + 63 = 0$: $-34^2 - 4 \times 1 \times 63 = 1156 - 252 = 904 \rightarrow$ Two
27. $x^2 + 32x - 83 = 0$: $32^2 - 4 \times 1 \times -83 = 1024 - -332 = 1356 \rightarrow$ Two
28. $x^2 + 2x + 1 = 0$: $2^2 - 4 \times 1 \times 1 = 4 - 4 = 0 \rightarrow$ One
29. $x^2 + 16x + 65 = 0$: $16^2 - 4 \times 1 \times 65 = 256 - 260 = -4 \rightarrow$ None
30. $x^2 + 17x + 32 = 0$: $17^2 - 4 \times 1 \times 32 = 289 - 128 = 161 \rightarrow$ Two
31. $x^2 - 20x + 22 = 0$: $-20^2 - 4 \times 1 \times 22 = 400 - 88 = 312 \rightarrow$ Two
32. $x^2 + 32x + 70 = 0$: $32^2 - 4 \times 1 \times 70 = 1024 - 280 = 744 \rightarrow$ Two
33. $x^2 + 35x + 102 = 0$: $35^2 - 4 \times 1 \times 102 = 1225 - 408 = 817 \rightarrow$ Two
34. $x^2 - 18x + 81 = 0$: $-18^2 - 4 \times 1 \times 81 = 324 - 324 = 0 \rightarrow$ One

Dividing Polynomials

Dividing polynomials

1. $(x^3 + 8x^2 + 12x) \div (x + 2) = x^2 + 6x$
2. $(x^3 + 8x^2 + 20x + 21) \div (x + 5) = x^2 + 3x + 5$ remainder -4
3. $(x^3 - 14x - 15) \div (x + 3) = x^2 - 3x - 5$
4. $(x^3 - 31x - 30) \div (x + 5) = x^2 - 5x - 6$
5. $(x^3 - 9x^2 + 21x - 5) \div (x - 5) = x^2 - 4x + 1$
6. $(x^3 - 12x^2 + 36x) \div (x - 6) = x^2 - 6x$
7. $(x^3 - 12x^2 + 36x - 1) \div (x - 6) = x^2 - 6x$ remainder -1
8. $(x^3 - 8x^2 + 12x) \div (x - 2) = x^2 - 6x$
9. $(x^3 + x^2 - 14x - 24) \div (x - 4) = x^2 + 5x + 6$
10. $(x^3 - 8x^2 + 10x + 25) \div (x - 5) = x^2 - 3x - 5$

Complex numbers

11. $(5 + 5i) + (1 + 3i) = 6 + 8i$
12. $(2 - 3i) + (5 - 6i) = 7 - 9i$
13. $(4 + 4i) + (2 - 4i) = 6$
14. $(3 - 3i) - (5 - 6i) = -2 + 3i$
15. $(-5 - 2i) - (-3 + 5i) = -2 - 7i$
16. $(3 - 6i) - (-6 - 6i) = 9$
17. $(4 - 3i) + (5 - 6i) = 9 - 9i$
18. $(5 + i) - (-4 + 5i) = 9 - 4i$
19. $(6 - 5i) - (6 - 6i) = i$
20. $(6 + 6i) - (1 - 4i) = 5 + 10i$

Simplifying Rational Expressions

Simplifying rational expressions

1. $(x^2 - 22x + 112) / (x^2 - 14x) = (x - 8) / x$
2. $(x^2 - 7x - 228) / (x^2 - 36x + 323) = (x + 12) / (x - 17)$
3. $(x^2 - 19x + 88) / (x^2 - x - 110) = (x - 8) / (x + 10)$
4. $(x^2 + 10x - 75) / (x^2 + 29x + 210) = (x - 5) / (x + 14)$
5. $(x^2 + x - 30) / (x^2 - 12x - 108) = (x - 5) / (x - 18)$
6. $(x^2 - 2x - 323) / (x^2 + 16x - 17) = (x - 19) / (x - 1)$
7. $(x^2 + 14x - 51) / (x^2 + 31x + 238) = (x - 3) / (x + 14)$
8. $(x^2 - 6x - 112) / (x^2 - 7x - 120) = (x - 14) / (x - 15)$
9. $(x^2 + 21x + 80) / (x^2 - 2x - 288) = (x + 5) / (x - 18)$
10. $(x^2 - 4x - 192) / (x^2 - 23x + 112) = (x + 12) / (x - 7)$
11. $(x^2 - 13x - 114) / (x^2 - 15x - 76) = (x + 6) / (x + 4)$
12. $(x^2 - 25x + 126) / (x^2 - 324) = (x - 7) / (x + 18)$
13. $(x^2 + 37x + 340) / (x^2 + 25x + 136) = (x + 20) / (x + 8)$
14. $(x^2 - 8x - 180) / (x^2 + 16x + 60) = (x - 18) / (x + 6)$
15. $(x^2 - 35x + 304) / (x^2 - 9x - 190) = (x - 16) / (x + 10)$
16. $(x^2 - 21x + 80) / (x^2 + 2x - 288) = (x - 5) / (x + 18)$
17. $(x^2 + 9x - 136) / (x^2 + 13x - 68) = (x - 8) / (x - 4)$
18. $(x^2 + 25x + 126) / (x^2 - 324) = (x + 7) / (x - 18)$
19. $(x^2 + 27x + 182) / (x^2 + 34x + 280) = (x + 13) / (x + 20)$
20. $(x^2 + 2x - 99) / (x^2 - 24x + 135) = (x + 11) / (x - 15)$
21. $(x^2 - 6x - 112) / (x^2 + x - 210) = (x + 8) / (x + 15)$

Simplifying radicals

22. $\sqrt[3]{324} = 3\sqrt[3]{12}$
23. $\sqrt[3]{500} = 5\sqrt[3]{4}$
24. $\sqrt[3]{320} = 4\sqrt[3]{5}$
25. $\sqrt[3]{625} = 5\sqrt[3]{5}$
26. $\sqrt[3]{243} = 3\sqrt[3]{9}$
27. $\sqrt[3]{875} = 5\sqrt[3]{7}$
28. $\sqrt[3]{704} = 4\sqrt[3]{11}$
29. $\sqrt[3]{384} = 4\sqrt[3]{6}$
30. $\sqrt[3]{768} = 4\sqrt[3]{12}$
31. $\sqrt[3]{750} = 5\sqrt[3]{6}$

Multiplying and Dividing Rational Expressions

Multiplying and dividing rational expressions

1. $(x + 8)/(x - 3) \div (x + 4)/(x - 3) = (x + 8) / (x + 4)$ $(x \neq -4, 3)$
2. $(x - 9)/(x - 8) \div (x - 6)/(x - 8) = (x - 9) / (x - 6)$ $(x \neq 6, 8)$
3. $(x - 6)/(x + 8) \times (x + 8)/(x + 2) = (x - 6) / (x + 2)$ $(x \neq -8, -2)$
4. $(x - 6)/(x + 9) \times (x + 9)/(x + 6) = (x - 6) / (x + 6)$ $(x \neq -9, -6)$
5. $(x - 4)/(x + 3) \times (x + 3)/(x - 8) = (x - 4) / (x - 8)$ $(x \neq -3, 8)$
6. $(x - 8)/(x - 9) \times (x - 9)/(x - 5) = (x - 8) / (x - 5)$ $(x \neq 5, 9)$
7. $(x - 5)/(x + 4) \div (x - 9)/(x + 4) = (x - 5) / (x - 9)$ $(x \neq -4, 9)$
8. $(x + 8)/(x + 7) \div (x - 3)/(x + 7) = (x + 8) / (x - 3)$ $(x \neq -7, 3)$
9. $(x + 7)/(x - 9) \div (x - 1)/(x - 9) = (x + 7) / (x - 1)$ $(x \neq 1, 9)$
10. $(x + 2)/(x - 9) \times (x - 9)/(x - 2) = (x + 2) / (x - 2)$ $(x \neq 2, 9)$
11. $(x + 4)/(x - 7) \div (x - 8)/(x - 7) = (x + 4) / (x - 8)$ $(x \neq 7, 8)$
12. $(x - 9)/(x - 6) \div (x + 2)/(x - 6) = (x - 9) / (x + 2)$ $(x \neq -2, 6)$
13. $(x + 9)/(x + 6) \div (x - 2)/(x + 6) = (x + 9) / (x - 2)$ $(x \neq -6, 2)$

Answer Key (6 of 7)

Multiplying and Dividing Rational Expressions (continued)

Multiplying and dividing rational expressions (continued)

14. $(x + 1)/(x - 8) \div (x - 3)/(x - 8) = (x + 1) / (x - 3)$ $(x \neq 3, 8)$
15. $(x + 2)/(x + 9) \div (x - 1)/(x + 9) = (x + 2) / (x - 1)$ $(x \neq -9, 1)$
16. $(x + 6)/(x + 8) \div (x - 9)/(x + 8) = (x + 6) / (x - 9)$ $(x \neq -8, 9)$
17. $x/(x + 2) \times (x + 2)/(x + 8) = x / (x + 8)$ $(x \neq -8, -2)$
18. $(x + 1)/(x + 7) \div (x + 9)/(x + 7) = (x + 1) / (x + 9)$ $(x \neq -9, -7)$

Add, subtract and multiply polynomials

19. $(9x^2 + 7x + 11) - (7x^2 - 9x + 9) \rightarrow 2x^2 + 16x + 2$
20. $(3x + 2)(x + 4) \rightarrow 3x^2 + 14x + 8$
21. $(9x^2 + 8x + 11) - (4x^2 - 5x + 7) \rightarrow 5x^2 + 13x + 4$
22. $(4x + 1)(x + 3) \rightarrow 4x^2 + 13x + 3$
23. $(8x^2 + 7x + 12) - (5x^2 - 4x + 2) \rightarrow 3x^2 + 11x + 10$
24. $(x + 6)(x + 3) \rightarrow 1x^2 + 9x + 18$
25. $(4x + 6)(x + 1) \rightarrow 4x^2 + 10x + 6$
26. $(6x^2 + 7x + 9) - (3x^2 - 6x + 6) \rightarrow 3x^2 + 13x + 3$
27. $(4x + 3)(x + 4) \rightarrow 4x^2 + 19x + 12$
28. $(2x + 1)(x + 3) \rightarrow 2x^2 + 7x + 3$

Solving Rational Equations

Solving rational equations

1. $-21/(x - 6) = 3 \rightarrow x = -1$ $(x \neq 6)$
2. $56/(x - 8) = -24/(x + 2) \rightarrow x = 1$ $(x \neq -2, 8)$
3. $-20/(x - 5) = 2 \rightarrow x = -5$ $(x \neq 5)$
4. $-65/(x - 5) = 5 \rightarrow x = -8$ $(x \neq 5)$
5. $21/(x + 5) = -28/(x - 2) \rightarrow x = -2$ $(x \neq -5, 2)$
6. $-70/(x - 1) = 7 \rightarrow x = -9$ $(x \neq 1)$
7. $-42/(x - 4) = 7 \rightarrow x = -2$ $(x \neq 4)$
8. $24/(x + 4) = 4 \rightarrow x = 2$ $(x \neq -4)$
9. $18/(x - 5) = 42/(x - 1) \rightarrow x = 8$ $(x \neq 1, 5)$
10. $52/(x + 7) = 8/(x - 4) \rightarrow x = 6$ $(x \neq -7, 4)$
11. $22/(x - 8) = 16/(x - 5) \rightarrow x = -3$ $(x \neq 5, 8)$
12. $40/(x + 1) = 8 \rightarrow x = 4$ $(x \neq -1)$
13. $72/(x + 1) = 9 \rightarrow x = 7$ $(x \neq -1)$
14. $20/x = 36/(x - 4) \rightarrow x = -5$ $(x \neq 0, 4)$
15. $40/(x - 8) = -45/(x + 9) \rightarrow x = 0$ $(x \neq -9, 8)$
16. $55/(x - 3) = 5/(x + 7) \rightarrow x = -8$ $(x \neq -7, 3)$
17. $15/(x - 2) = 35/(x + 2) \rightarrow x = 5$ $(x \neq -2, 2)$
18. $-48/(x - 5) = 6 \rightarrow x = -3$ $(x \neq 5)$
19. $-60/(x - 6) = 4 \rightarrow x = -9$ $(x \neq 6)$
20. $-36/(x - 4) = 9 \rightarrow x = 0$ $(x \neq 4)$
21. $64/(x + 3) = 8 \rightarrow x = 5$ $(x \neq -3)$
22. $12/(x + 5) = 40/(x - 2) \rightarrow x = -8$ $(x \neq -5, 2)$
23. $80/(x + 9) = 24/(x + 2) \rightarrow x = 1$ $(x \neq -9, -2)$
24. $-16/(x + 1) = 8 \rightarrow x = -3$ $(x \neq -1)$

Dividing polynomials

25. $(x^3 - x^2 - 19x + 4) \div (x + 4) = x^2 - 5x + 1$
26. $(x^3 - 11x^2 + 30x + 9) \div (x - 5) = x^2 - 6x$ remainder 9
27. $(x^3 + 8x^2 + 12x - 9) \div (x + 2) = x^2 + 6x$ remainder -9
28. $(x^3 - x^2 - 31x + 6) \div (x - 6) = x^2 + 5x - 1$

Solving Rational Equations (continued)

Dividing polynomials (continued)

29. $(x^3 - 2x^2 - 4x - 17) \div (x - 4) = x^2 + 2x + 4$ remainder -1
30. $(x^3 - 36x) \div (x + 6) = x^2 - 6x$
31. $(x^3 + 11x^2 + 30x) \div (x + 5) = x^2 + 6x$
32. $(x^3 + x^2 - 19x + 4) \div (x + 5) = x^2 - 4x + 1$ remainder -1

Arithmetic and Geometric Sequences

Arithmetic and geometric sequences

1. 5, 9, 13, 17, ... the 17th term = 69
2. 5, 10, 15, 20, ... the 24th term = 120
3. 12, 9, 6, 3, ... the 20th term = −45
4. −7, −16, −25, −34, ... the 22nd term = −196
5. 8, 1, −6, −13, ... the 15th term = −90
6. 10, 6, 2, −2, ... the 24th term = −82
7. 11, 14, 17, 20, ... the 25th term = 83
8. −4, −6, −8, −10, ... the 19th term = −40
9. 0, 9, 18, 27, ... the 19th term = 162
10. −2, 6, 14, 22, ... the 23rd term = 174
11. −1, −10, −19, −28, ... the 14th term = −118
12. 4, −5, −14, −23, ... the 22nd term = −185
13. 4, 10, 16, 22, ... the 24th term = 142
14. −7, −2, 3, 8, ... the 18th term = 78
15. 6, 10, 14, 18, ... the 23rd term = 94
16. 12, 14, 16, 18, ... the 24th term = 58
17. −3, −1, 1, 3, ... the 22nd term = 39
18. 1, −5, −11, −17, ... the 23rd term = −131
19. 8, 2, −4, −10, ... the 21st term = −112
20. −8, −9, −10, −11, ... the 23rd term = −30
21. −1, 6, 13, 20, ... the 23rd term = 153
22. 11, 17, 23, 29, ... the 22nd term = 137
23. 4, 13, 22, 31, ... the 21st term = 184
24. 0, −6, −12, −18, ... the 17th term = −96

Simplifying rational expressions

25. $(x^2 - 12x - 160) / (x^2 + 14x + 48) = (x - 20) / (x + 6)$
26. $(x^2 - 5x - 234) / (x^2 + 31x + 234) = (x - 18) / (x + 18)$
27. $(x^2 + 24x + 95) / (x^2 + 11x + 30) = (x + 19) / (x + 6)$
28. $(x^2 - 34x + 280) / (x^2 - 12x - 28) = (x - 20) / (x + 2)$
29. $(x^2 - 19x + 88) / (x^2 - x - 110) = (x - 8) / (x + 10)$
30. $(x^2 + 34x + 280) / (x^2 + 20x) = (x + 14) / x$
31. $(x^2 - 9x - 220) / (x^2 + 29x + 198) = (x - 20) / (x + 18)$
32. $(x^2 - 5x - 24) / (x^2 + 10x - 144) = (x + 3) / (x + 18)$
33. $(x^2 - 12x - 160) / (x^2 - 27x + 140) = (x + 8) / (x - 7)$
34. $(x^2 + 3x - 238) / (x^2 + 35x + 306) = (x - 14) / (x + 18)$

Sums of Series

Sums of series

1. S_6 for 6, 24, 96, ... = 8190
2. S_8 for 2, 11, 20, ... = 268
3. S_7 for 9, 27, 81, ... = 9837

Answer Key (7 of 7)

Sums of Series (continued)

Sums of series (continued)

4. S_8 for 7, 11, 15, ... = 168
5. S_6 for 8, 32, 128, ... = 10920
6. S_9 for 9, 12, 15, ... = 189
7. S_7 for 1, 4, 16, ... = 5461
8. S_7 for 8, 16, 32, ... = 1016
9. S_9 for 11, 1, −9, ... = −261
10. S_9 for 11, 17, 23, ... = 315
11. S_5 for 5, −5, −15, ... = −75
12. S_{10} for 2, −1, −4, ... = −115
13. S_7 for 9, 5, 1, ... = −21
14. S_6 for 2, 8, 32, ... = 2730
15. S_6 for 1, 11, 21, ... = 156
16. S_{10} for 6, 3, 0, ... = −75
17. S_7 for 2, 4, 8, ... = 254
18. S_9 for 5, 8, 11, ... = 153
19. S_9 for 10, 5, 0, ... = −90
20. S_7 for 3, 9, 27, ... = 3279
21. S_9 for 1, 4, 7, ... = 117

Multiplying and dividing rational expressions

22. $(x + 8)/(x + 2) \div (x + 1)/(x + 2) = (x + 8) / (x + 1)$ $(x \neq -2, -1)$
23. $(x - 3)/(x + 6) \div (x + 9)/(x + 6) = (x - 3) / (x + 9)$ $(x \neq -9, -6)$
24. $(x - 7)/(x + 5) \div (x + 8)/(x + 5) = (x - 7) / (x + 8)$ $(x \neq -8, -5)$
25. $(x - 9)/x \times x/(x + 2) = (x - 9) / (x + 2)$ $(x \neq -2, 0)$
26. $(x + 3)/(x - 8) \times (x - 8)/(x + 6) = (x + 3) / (x + 6)$ $(x \neq -6, 8)$
27. $(x - 1)/(x + 9) \times (x + 9)/(x + 7) = (x - 1) / (x + 7)$ $(x \neq -9, -7)$
28. $(x + 9)/(x + 8) \div (x + 6)/(x + 8) = (x + 9) / (x + 6)$ $(x \neq -8, -6)$
29. $(x - 4)/(x - 3) \times (x - 3)/(x - 8) = (x - 4) / (x - 8)$ $(x \neq 3, 8)$
30. $(x - 6)/(x + 4) \div (x + 9)/(x + 4) = (x - 6) / (x + 9)$ $(x \neq -9, -4)$
31. $(x + 2)/(x + 1) \div (x - 9)/(x + 1) = (x + 2) / (x - 9)$ $(x \neq -1, 9)$

Check the Log Laws

Check the log laws

1. $\log(x^2) + \log(x^5) = 7 \log x$, which is what was written — this line is correct
2. $\log(7x) - \log 7 = \log x$, not $\log x \div \log 7$ (the logs were divided because one was being taken from the other)
3. $\log(x^3) + \log(x^5) = 8 \log x$, not $\log(x^3 + x^5)$ (the two insides were added under one log)
4. $\log(x^4) + \log(x^5) = 9 \log x$, not $20(\log x)^2$ (the two logs were multiplied once their powers had come down)
5. $\log(x^3) + \log(x^4) = 7 \log x$, not $12(\log x)^2$ (the two logs were multiplied once their powers had come down)
6. $\log(2x) - \log 2 = \log x$, which is what was written — this line is correct
7. $\log(x^2) + \log(x^4) = 6 \log x$, which is what was written — this line is correct
8. $\log(x^2) + \log(x^3) = 5 \log x$, not $6(\log x)^2$ (the two logs were multiplied once their powers had come down)

Solutions (1 of 14)

Simplifying Radicals

Simplifying radicals

1. $\sqrt{360}$

$\sqrt{360} = 6\sqrt{10}$

Split 360 into a perfect square times whatever is left: 360 = 36 × 10. 36 is 6 squared, and 10 has no square factor of its own — that last part is what makes this the LARGEST perfect square inside, and it is where the answer stops.

A root splits across a product, so the one root becomes two.

√36 is 6, so it comes out in front. √10 stays under the sign, because 10 has nothing more to give.

2. $\sqrt{486}$

$\sqrt{486} = 9\sqrt{6}$

Split 486 into a perfect square times whatever is left: 486 = 81 × 6. 81 is 9 squared, and 6 has no square factor of its own — that last part is what makes this the LARGEST perfect square inside, and it is where the answer stops.

A root splits across a product, so the one root becomes two.

√81 is 9, so it comes out in front. √6 stays under the sign, because 6 has nothing more to give.

Rational Exponents

Rational exponents

1. $216^{2/3} =$ _____

216^(2/3) = 36

The BOTTOM of the exponent says which root to take, so the cube root of 216 comes first. Root before power, always — the other order works and makes the numbers enormous.

The TOP says what to raise it to. 6 to the power 2 is 36.

So 216^(2/3) is 36.

Solutions (2 of 14)

Rational Exponents (continued)

Rational exponents (continued)

2. $1000^{4/3}$ = _____

1000^(4/3) = 10000

The BOTTOM of the exponent says which root to take, so the cube root of 1000 comes first. Root before power, always — the other order works and makes the numbers enormous.

The TOP says what to raise it to. 10 to the power 4 is 10000.

So 1000^(4/3) is 10000.

Logarithms

Logarithms

1. $\log_3 \frac{1}{729}$ = _____

$\log_3 1/729 = -6$

$\log_3 1/729$ asks a question: which power of 3 lands on 1/729? It is not an instruction to divide by 3 — that is the mistake this notation collects.

1/729 is 1 over 729, and a negative exponent is what writes a reciprocal: 3^{-6} means $1 \div 3^6$. So the power that lands on 1/729 is the same one, turned over.

So $\log_3 1/729$ is −6. A logarithm is negative exactly when the number is between 0 and 1 — nothing has gone wrong.

2. $\log_3 2187$ = _____

$\log_3 2187 = 7$

$\log_3 2187$ asks a question: which power of 3 lands on 2187? It is not an instruction to divide by 3 — that is the mistake this notation collects.

So $\log_3 2187$ is 7.

Solutions (3 of 14)

The Laws of Logarithms

Laws of logarithms

1. $\log_5 6 + \log_5 7 = \log_5$ _____

$\log_5 6 + \log_5 7$

$\log_5 m + \log_5 n = \log_5 (m \times n)$

$6 \times 7 = 42$

$\log_5 6 + \log_5 7 = \log_5 42$

Both logarithms are to the same base, 5. That is what makes a law available at all — two logarithms to different bases cannot be combined, and checking the bases match is the first thing to do rather than the last.

A SUM of two logarithms with the same base is the logarithm of the two arguments MULTIPLIED. Adding the logs multiplies the numbers — that is the law, and it is the one worth saying out loud, because the operation changes.

So the two arguments become one: $6 \times 7 = 42$. The answer is $\log_5 42$.

2. $2 \log_3 11 = \log_3$ _____

$2 \log_3 11$

$k \log_3 m = \log_3 (m^k)$

$11^2 = 121$

$2 \log_3 11 = \log_3 121$

Both logarithms are to the same base, 3. That is what makes a law available at all — two logarithms to different bases cannot be combined, and checking the bases match is the first thing to do rather than the last.

A logarithm MULTIPLIED by a number is the logarithm of the argument RAISED to that number. The multiplier comes down off the front and goes up as a power — which is why the answer grows so much faster than the question looks like it should.

So the two arguments become one: $11^2 = 121$. The answer is $\log_3 121$.

Solutions (4 of 14)

Exponential Functions

Evaluate exponential functions

1. $f(x) = 7 \cdot 6^x$ **$f(1) =$** ___

$$f(1) = 7 \cdot 6^1$$
$$6^1 = 6$$
$$7 \cdot 6 = 42$$

Substitute $x = 1$: $f(1) = 7 \cdot 6^1$.

Do the power first: $6^1 = 6$.

Then multiply by the coefficient: $7 \cdot 6 = 42$.

2. $f(x) = 7 \cdot 2^x$ **$f(6) =$** ___

$$f(6) = 7 \cdot 2^6$$
$$2^6 = 64$$
$$7 \cdot 64 = 448$$

Substitute $x = 6$: $f(6) = 7 \cdot 2^6$.

Do the power first: $2^6 = 64$.

Then multiply by the coefficient: $7 \cdot 64 = 448$.

Completing the Square

Completing the square

1. **$x^2 + 16x + 65$**

$$x^2 + 16x + 65 = (x + 8)^2 + 1$$

Halve the middle coefficient. Half of 16 is 8, and that number goes straight into the bracket — sign and all.

But $(x + 8)^2$ is not the expression yet. Multiply it out and it brings a 64 along with it, and the original has 65 there instead.

So give the 64 back: $65 - 64 = 1$. That is the number that goes after the bracket.

Solutions (5 of 14)

Completing the Square (continued)

Completing the square (continued)

2. $x^2 - 22x + 145$

$x^2 - 22x + 145 = (x - 11)^2 + 24$

Halve the middle coefficient. Half of −22 is −11, and that number goes straight into the bracket — sign and all.

But $(x - 11)^2$ is not the expression yet. Multiply it out and it brings a 121 along with it, and the original has 145 there instead.

So give the 121 back: 145 − 121 = 24. That is the number that goes after the bracket.

How Many Real Solutions

How many real solutions?

1. $x^2 + 22x + 149 = 0$

$= -112 < 0 \quad \rightarrow$ **none**

You do not have to solve this to answer it. One number decides how many real solutions a quadratic has, and it is $b^2 - 4ac$ — the part that ends up under the square root sign when you do solve it.

Read off a, b and c from $x^2 + 22x + 149 = 0$: a = 1, b = 22, c = 149. Write all three in, even the a — on this sheet it is 1, and forgetting it is the mistake that only shows up when it is not 1.

Work it out: 22^2 is 484, and $4 \times 1 \times 149$ is 596. That leaves 484 − 596 = −112.

−112 is below zero, and no real number squares to a negative, so there is nothing to take the square root of. So there are NO real solutions.

Solutions (6 of 14)

How Many Real Solutions (continued)

How many real solutions? (continued)

2. $x^2 + 12x - 160 = 0$

$= 784 > 0 \quad \rightarrow$ **two**

You do not have to solve this to answer it. One number decides how many real solutions a quadratic has, and it is $b^2 - 4ac$ — the part that ends up under the square root sign when you do solve it.

Read off a, b and c from $x^2 + 12x - 160 = 0$: $a = 1$, $b = 12$, $c = -160$. Write all three in, even the a — on this sheet it is 1, and forgetting it is the mistake that only shows up when it is not 1.

Work it out: 12^2 is 144, and $4 \times 1 \times -160$ is -640. That leaves $144 - -640 = 784$.

784 is above zero, and a positive number has two square roots — one either way from the ±. So there are TWO real solutions.

Adding and Subtracting Complex Numbers

Complex numbers

1. $(5 + 2i) - (3 - 5i)$

$(5 + 2i) - (3 - 5i) = 2 + 7i$

Subtract the two REAL parts, and leave the i alone. The real parts are 5 and 3.

Now the two imaginary parts, the same way — this works because i is a number like any other here, so 2 of them minus 5 of them is what it sounds like.

Put the two parts back together: $2 + 7i$.

2. $(-2 + 5i) - (-6 + 6i)$

$(-2 + 5i) - (-6 + 6i) = 4 - i$

Subtract the two REAL parts, and leave the i alone. The real parts are -2 and -6.

Now the two imaginary parts, the same way — this works because i is a number like any other here, so 5 of them minus 6 of them is what it sounds like.

Put the two parts back together: $4 - i$.

Solutions (7 of 14)

Adding, Subtracting and Multiplying Polynomials

Add, subtract and multiply polynomials

1. **$(7x^2 + 6x + 12) - (5x^2 - 3x + 7)$**

$2x^2 + 9x + 5$

Only matching terms combine: x^2 with x^2, x with x, numbers with numbers. Line the two brackets up by power before doing anything, and nothing can drift into the wrong column.

Subtracting a bracket flips the sign of EVERY term inside it, not just the first: taking away $5x^2 - 3x + 7$ is the same as adding $-5x^2 + 3x - 7$. Rewrite it that way before adding — it is slower and it is the step that gets marks.

Now take the columns one at a time, adding the rewritten signs. x^2: $7 + (-5) = 2$. x: $6 + 3 = 9$. Numbers: $12 + (-7) = 5$.

So the answer is $2x^2 + 9x + 5$. Read it back and count the terms — three went in on each side, and three come out.

2. **$(3x + 5)(x + 1)$**

$3x + 5x = 8x$

$= 3x^2 + 8x + 5$

Two brackets means every term in the first has to meet every term in the second — two times two is FOUR multiplications, not two. Miss one and the x term comes out short.

$3x \times x = 3x^2$, and $3x \times 1 = 3x$.

Then the other one: $5 \times x = 5x$, and $5 \times 1 = 5$.

Two of those four are x terms, so they collect: $3x + 5x = 8x$. The answer is $3x^2 + 8x + 5$.

Solutions (8 of 14)

Dividing Polynomials

Dividing polynomials

1.

$$\begin{array}{r} \quad x^2 + 6x + 0 \\ x + 2 \overline{)\, x^3 + 8x^2 + 12x} \\ \underline{-x^3 - 2x^2} \\ \quad 6x^2 + 12x \\ \underline{-6x^2 - 12x} \\ \quad 0 \end{array}$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x + 2)$ is $x^3 + 2x^2$ — and take that away, leaving $6x^2 + 12x$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$6x^2 \div x$ is $6x$, so $6x$ goes in the x place. Multiply the WHOLE divisor by it — $6x(x + 2)$ is $6x^2 + 12x$ — and take that away, leaving 0. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

Nothing is left to divide, so the constant place of the quotient is 0. A trailing zero is not written out, which is why the answer stops at $x^2 + 6x$.

Nothing is left, so $x + 2$ divides it exactly: $(x^3 + 8x^2 + 12x) \div (x + 2) = x^2 + 6x$.

2.

$$\begin{array}{r} \quad x^2 + 3x + 5 \\ x + 5 \overline{)\, x^3 + 8x^2 + 20x + 21} \\ \underline{-x^3 - 5x^2} \\ \quad 3x^2 + 20x + 21 \\ \underline{-3x^2 - 15x} \\ \quad 5x + 21 \\ \underline{-5x - 25} \\ \quad -4 \end{array}$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x + 5)$ is $x^3 + 5x^2$ — and take that away, leaving $3x^2 + 20x + 21$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$3x^2 \div x$ is $3x$, so $3x$ goes in the x place. Multiply the WHOLE divisor by it — $3x(x + 5)$ is $3x^2 + 15x$ — and take that away, leaving $5x + 21$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$5x \div x$ is 5, so 5 goes in the constant place. Multiply the WHOLE divisor by it — $5(x + 5)$ is $5x + 25$ — and take that away, leaving -4. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

-4 is left and there is no x in it to divide, so that is the remainder. $(x^3 + 8x^2 + 20x + 21) \div (x + 5) = x^2 + 3x + 5$ remainder -4.

Solutions (9 of 14)

Simplifying Rational Expressions

Simplifying rational expressions

1. $\dfrac{x^2 - 22x + 112}{x^2 - 14x}$

①/② $\dfrac{(x - 14)(x - 8)}{x(x - 14)}$

$= \dfrac{(x - 8)}{x}$

Nothing can cancel while these are sums. Factor the top first — two numbers that multiply to 112 and add to −22. Call it ①.

Now the bottom, the same way — call it ②. (x − 14) turns up again, and that is not a coincidence: it is the whole reason this expression simplifies.

The question is ① over ②. (x − 14) is a FACTOR of both, so it divides out and leaves 1. Only a factor can go this way — striking the x^2 off the top and the bottom of the sums above is the mistake this page exists to catch. And the cancelling changed something quietly: x may not be 0 or 14, because the original has no value there.

2. $\dfrac{x^2 - 7x - 228}{x^2 - 36x + 323}$

①/② $\dfrac{(x - 19)(x + 12)}{(x - 19)(x - 17)}$

$= \dfrac{(x + 12)}{(x - 17)}$

Nothing can cancel while these are sums. Factor the top first — two numbers that multiply to −228 and add to −7. Call it ①.

Now the bottom, the same way — call it ②. (x − 19) turns up again, and that is not a coincidence: it is the whole reason this expression simplifies.

The question is ① over ②. (x − 19) is a FACTOR of both, so it divides out and leaves 1. Only a factor can go this way — striking the x^2 off the top and the bottom of the sums above is the mistake this page exists to catch. And the cancelling changed something quietly: x may not be 17 or 19, because the original has no value there.

Solutions (10 of 14)

Multiplying and Dividing Rational Expressions

Multiplying and dividing rational expressions

1. $\frac{(x + 8)}{(x - 3)} \div \frac{(x + 4)}{(x - 3)}$

$(x + 8)/(x - 3) \div (x + 4)/(x - 3)$

$(x + 8)/(x - 3) \times (x - 3)/(x + 4)$

$(x + 8)/(x - 3) \times (x - 3)/(x + 4)$

$(x - 3)$ cancels

$(x + 8) / (x + 4)$

$x \neq -4, 3$

Dividing by a fraction is multiplying by it upside down, and that has to happen FIRST. Turn $(x + 4)/(x - 3)$ over and change the $\div$ to a $\times$. Cancelling before the flip is cancelling across a division sign, which is not a move.

Now it is one product, so anything on top may cancel with anything underneath — the two fractions are no longer separate. $(x - 3)$ appears in both, so it goes.

What is left is $(x + 8) / (x + 4)$.

One more thing, and it is the half that gets dropped: x may not be -4 or 3. Those are the numbers that made a denominator zero in the ORIGINAL — including $(x - 3)$, which cancelled. A bracket does not stop mattering because it left the page.

2. $\frac{(x - 9)}{(x - 8)} \div \frac{(x - 6)}{(x - 8)}$

$(x - 9)/(x - 8) \div (x - 6)/(x - 8)$

$(x - 9)/(x - 8) \times (x - 8)/(x - 6)$

$(x - 9)/(x - 8) \times (x - 8)/(x - 6)$

$(x - 8)$ cancels

$(x - 9) / (x - 6)$

$x \neq 6, 8$

Dividing by a fraction is multiplying by it upside down, and that has to happen FIRST. Turn $(x - 6)/(x - 8)$ over and change the $\div$ to a $\times$. Cancelling before the flip is cancelling across a division sign, which is not a move.

Now it is one product, so anything on top may cancel with anything underneath — the two fractions are no longer separate. $(x - 8)$ appears in both, so it goes.

What is left is $(x - 9) / (x - 6)$.

One more thing, and it is the half that gets dropped: x may not be 6 or 8. Those are the numbers that made a denominator zero in the ORIGINAL — including $(x - 8)$, which cancelled. A bracket does not stop mattering because it left the page.

Solutions (11 of 14)

Solving Rational Equations

Solving rational equations

1. $\frac{-21}{(x-6)} = 3$, x = ____

$x \neq 6$

$-21/(x-6) = 3$

$-21 = 3(x-6)$

$x = -1$

$x = -1$ and $x \neq 6$

Before anything else, note what x cannot be. (x – 6) is a denominator, so x may not be 6. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Now clear the fraction: multiply both sides by (x – 6). The left side is left with –21 and the right becomes 3((x – 6)).

Multiply out, gather the x terms on one side and the numbers on the other, and x = –1.

Last, check it against the first line: x may not be 6, and –1 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

2. $\frac{56}{(x-8)} = \frac{-24}{(x+2)}$, x = ____

$x \neq -2, 8$

$56/(x-8) = -24/(x+2)$

$56(x+2) = -24(x-8)$

$x = 1$

$x = 1$ and $x \neq -2, 8$

Before anything else, note what x cannot be. (x – 8) is a denominator, so x may not be 8, and the same for the other one: not –2. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Two fractions, one equals sign — so cross-multiply. Each numerator meets the OTHER denominator, which is the same "multiply both sides by both denominators" done in one move.

Multiply out, gather the x terms on one side and the numbers on the other, and x = 1.

Last, check it against the first line: x may not be –2 or 8, and 1 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

Solutions (12 of 14)

Arithmetic and Geometric Sequences

Arithmetic and geometric sequences

1. 5, 9, 13, 17, ... a_{17} = ____

$$d = 4$$

$$a_n = 5 + (n - 1) \times 4$$

$$a_{17} = 5 + 16 \times 4$$

$$5 + 64 = 69$$

$$a_{17} = 69$$

This is an arithmetic sequence: it changes by 4 each step (9 – 5 = 4).

The first term is already there, so to reach the 17th term you take the step only 16 times — that is (n – 1), not n.

16 steps of 4 is 64, and 5 + 64 = 69.

So the 17th term is 69.

2. 5, 10, 15, 20, ... a_{24} = ____

$$d = 5$$

$$a_n = 5 + (n - 1) \times 5$$

$$a_{24} = 5 + 23 \times 5$$

$$5 + 115 = 120$$

$$a_{24} = 120$$

This is an arithmetic sequence: it changes by 5 each step (10 – 5 = 5).

The first term is already there, so to reach the 24th term you take the step only 23 times — that is (n – 1), not n.

23 steps of 5 is 115, and 5 + 115 = 120.

So the 24th term is 120.

Solutions (13 of 14)

Sums of Series

Sums of series

1. **6, 24, 96, … S_6 = _____**

$S_6 = 6 \times 4095 \div 3 = 8190$

Each term is the one before times 4, so this is a geometric series and the sum is $a(r^n - 1) \div (r - 1)$. Here a is 6, r is 4 and n is 6 — substituting the wrong one of the three is the whole difficulty.

4 to the power 6 is 4096, so the top is 6 × 4095 and the bottom is 3.

2. **2, 11, 20, … S_8 = _____**

$S_8 = (2 + 65) \times 8 \div 2 = 268$

Find the LAST term first — nothing can be added up until you have it. Every term is 9 more than the one before, so the 8th is 7 steps past the first.

Now pair the ends. First plus last is 67; so is second plus second-last, and so on all the way in — that is why the sum is that pair times half the count, and it is where the formula comes from.

Check the Log Laws

Check the log laws

1. **Check this line: $\log(x^2) + \log(x^5) = 7 \log x$. Which is correct?**

$\log(x^2 + x^5)$ | $10 \log x$ | $7 \log x$ | $10(\log x)^2$

Why?

$\log(x^2) + \log(x^5) = 7 \log x$

they wrote: $7 \log x$

Do not check by reading their line. Somebody else's working is the most persuasive thing on the page, and rereading it is how you end up agreeing with it — do the step yourself first, then compare.

On your own, and writing the step nobody writes down: $\log(x^2) + \log(x^5)$ becomes $2 \log x + 5 \log x$.

Tidy that up and it is $7 \log x$.

Now compare — and this time the two agree. The line on the page was already right, which is a finding, not a missing mistake. Some of them are.

Solutions (14 of 14)

Check the Log Laws (continued)

Check the log laws (continued)

2. **Ben wrote log(7x) − log 7 = log x ÷ log 7. Which is correct?**

log x | 7 log x | log x ÷ log 7 | log 7 − log x

Why?

log(7x) − log 7 = **log x**

they wrote: **log x ÷ log 7**

Do not check by reading their line. Somebody else's working is the most persuasive thing on the page, and rereading it is how you end up agreeing with it — do the step yourself first, then compare.

On your own, and writing the step nobody writes down: log(7x) − log 7 becomes log 7 + log x − log 7.

Tidy that up and it is log x.

Now compare. They wrote log x ÷ log 7, and the logs were divided because one was being taken from the other — so the correct answer is log x.