Algebra 2 Foundation Workbook

18 lessons · printable · answer key at the back

Name _____ Date _____

Contents (1 of 2)

Contents (2 of 2)

1-1

Simplifying Radicals

WORKED EXAMPLE

$\sqrt{350}$

$350 = 25 \times 14$

$= \sqrt{25} \times \sqrt{14}$

$= 5\sqrt{14}$

1. Simplifying radicals

Simplify each square root.

1 $\sqrt{63}$

2 $\sqrt{126}$

3 $\sqrt{360}$

4 $\sqrt{224}$

5 $\sqrt{56}$

6 $\sqrt{8}$

7 $\sqrt{125}$

8 $\sqrt{32}$

9 $\sqrt{24}$

10 $\sqrt{96}$

11 $\sqrt{72}$

12 $\sqrt{12}$

13 $\sqrt{135}$

14 $\sqrt{175}$

15 $\sqrt{112}$

16 $\sqrt{75}$

17 $\sqrt{52}$

18 $\sqrt{504}$

19 $\sqrt{90}$

20 $\sqrt{44}$

21 $\sqrt{98}$

22 $\sqrt{48}$

23 $\sqrt{240}$

24 $\sqrt{99}$

25 $\sqrt{128}$

26 $\sqrt{45}$

27 $\sqrt{28}$

28 $\sqrt{539}$

Cube Roots

WORKED EXAMPLE

$\sqrt[3]{270}$

$270 = 27 \times 10$

$= \sqrt[3]{27} \times \sqrt[3]{10}$

$= 3\sqrt[3]{10}$

2. Simplifying radicals

Simplify each cube root.

1 $\sqrt[3]{40}$

2 $\sqrt[3]{48}$

3 $\sqrt[3]{297}$

4 $\sqrt[3]{54}$

5 $\sqrt[3]{96}$

6 $\sqrt[3]{108}$

7 $\sqrt[3]{320}$

8 $\sqrt[3]{88}$

9 $\sqrt[3]{80}$

10 $\sqrt[3]{81}$

11 $\sqrt[3]{243}$

12 $\sqrt[3]{24}$

13 $\sqrt[3]{16}$

14 $\sqrt[3]{324}$

15 $\sqrt[3]{135}$

16 $\sqrt[3]{189}$

17 $\sqrt[3]{56}$

18 $\sqrt[3]{128}$

19 $\sqrt[3]{72}$

20 $\sqrt[3]{32}$

21 $\sqrt[3]{256}$

22 $\sqrt[3]{162}$

23 $\sqrt[3]{192}$

24 $\sqrt[3]{250}$

Rational Exponents

WORKED EXAMPLE

$216^{1/3} = 6$

$\sqrt[3]{216} = 6$

$= \sqrt[3]{216}$

$= 6$

3. Rational exponents

Work out each one. Take the root first, then raise it.

1 $4^{3/2} =$ _____

2 $64^{5/3} =$ _____

3 $216^{2/3} =$ _____

4 $25^{1/2} =$ _____

5 $25^{5/2} =$ _____

6 $36^{3/2} =$ _____

7 $256^{1/4} =$ _____

8 $9^{1/2} =$ _____

9 $27^{1/3} =$ _____

10 $8^{1/3} =$ _____

11 $100^{1/2} =$ _____

12 $9^{3/2} =$ _____

13 $16^{1/2} =$ _____

14 $16^{3/4} =$ _____

15 $27^{5/3} =$ _____

16 $4^{1/2} =$ _____

17 $36^{1/2} =$ _____

18 $256^{5/4} =$ _____

19 $27^{2/3} =$ _____

20 $8^{4/3} =$ _____

21 $81^{5/4} =$ _____

22 $49^{1/2} =$ _____

23 $16^{5/2} =$ _____

24 $4^{5/2} =$ _____

25 $100^{3/2} =$ _____

26 $49^{3/2} =$ _____

27 $64^{4/3} =$ _____

28 $125^{1/3} =$ _____

Logarithms

WORKED EXAMPLE

$\log_4 64 = 3$

$4^3 = 64$

4. Logarithms

Work out each logarithm, or find the missing number.

1 $\log_4 \_\_\_\_ = 2$

2 $\log_3 \frac{1}{9} = \_\_\_\_$

3 $\log_9 \_\_\_\_ = 4$

4 $\log_2 4 = \_\_\_\_$

5 $\log_{10} \frac{1}{1000} = \_\_\_\_$

6 $\log_8 \_\_\_\_ = 3$

7 $\log_2 \frac{1}{128} = \_\_\_\_$

8 $\log_2 \frac{1}{8} = \_\_\_\_$

9 $\log_7 \frac{1}{49} = \_\_\_\_$

10 $\log_6 36 = \_\_\_\_$

11 $\log_5 \_\_\_\_ = 4$

12 $\log_9 729 = \_\_\_\_$

13 $\log_7 \_\_\_\_ = 2$

14 $\log_8 \_\_\_\_ = 5$

15 $\log_9 \_\_\_\_ = 2$

16 $\log_2 16 = \_\_\_\_$

17 $\log_8 \_\_\_\_ = 2$

18 $\log_5 \_\_\_\_ = 6$

19 $\log_9 \_\_\_\_ = 3$

20 $\log_3 \_\_\_\_ = 3$

21 $\log_6 1296 = \_\_\_\_$

22 $\log_9 \frac{1}{81} = \_\_\_\_$

23 $\log_3 27 = \_\_\_\_$

24 $\log_2 \_\_\_\_ = 3$

25 $\log_7 \_\_\_\_ = 4$

26 $\log_5 125 = \_\_\_\_$

27 $\log_8 \frac{1}{512} = \_\_\_\_$

28 $\log_5 \_\_\_\_ = 5$

5. Simplifying radicals

Simplify each square root.

29 $\sqrt{224}$

30 $\sqrt{52}$

31 $\sqrt{128}$

32 $\sqrt{40}$

33 $\sqrt{80}$

34 $\sqrt{50}$

35 $\sqrt{216}$

36 $\sqrt{99}$

37 $\sqrt{126}$

38 $\sqrt{135}$

The Laws of Logarithms

WORKED EXAMPLE

$\log_5 2 + \log_5 12 = \log_5 \underline{24}$

$\log_5 m + \log_5 n = \log_5 (m \times n)$

$2 \times 12 = 24$

$= \log_5 24$

6. Laws of logarithms

Write each of these as one logarithm.

1 $\log_2 4 + \log_2 6 = \log_2 \_\_\_\_$

2 $\log_{10} 27 - \log_{10} 9 = \log_{10} \_\_\_\_$

3 $\log_5 6 + \log_5 7 = \log_5 \_\_\_\_$

4 $\log_{10} 2 + \log_{10} 4 = \log_{10} \_\_\_\_$

5 $\log_5 2 + \log_5 5 = \log_5 \_\_\_\_$

6 $\log_{10} 3 + \log_{10} 3 = \log_{10} \_\_\_\_$

7 $\log_2 6 + \log_2 8 = \log_2 \_\_\_\_$

8 $\log_2 12 - \log_2 2 = \log_2 \_\_\_\_$

9 $\log_2 22 - \log_2 2 = \log_2 \_\_\_\_$

10 $\log_5 12 - \log_5 3 = \log_5 \_\_\_\_$

11 $2 \log_2 6 = \log_2 \_\_\_\_$

12 $\log_5 24 - \log_5 12 = \log_5 \_\_\_\_$

13 $\log_3 4 + \log_3 4 = \log_3 \_\_\_\_$

14 $\log_2 3 + \log_2 11 = \log_2 \_\_\_\_$

15 $\log_3 4 + \log_3 6 = \log_3 \_\_\_\_$

16 $\log_2 21 - \log_2 7 = \log_2 \_\_\_\_$

6. Laws of logarithms (continued)

Write each of these as one logarithm.

17 $\log_5 24 - \log_5 2 = \log_5$ _____

18 $\log_2 42 - \log_2 7 = \log_2$ _____

19 $\log_{10} 3 + \log_{10} 9 = \log_{10}$ _____

20 $\log_3 25 - \log_3 5 = \log_3$ _____

21 $\log_2 44 - \log_2 11 = \log_2$ _____

22 $\log_3 22 - \log_3 2 = \log_3$ _____

23 $\log_5 2 + \log_5 6 = \log_5$ _____

24 $\log_{10} 16 - \log_{10} 4 = \log_{10}$ _____

7. Simplifying radicals

Simplify each cube root.

25 $\sqrt[3]{96}$

26 $\sqrt[3]{162}$

27 $\sqrt[3]{128}$

28 $\sqrt[3]{81}$

29 $\sqrt[3]{72}$

30 $\sqrt[3]{56}$

31 $\sqrt[3]{192}$

32 $\sqrt[3]{24}$

33 $\sqrt[3]{135}$

34 $\sqrt[3]{80}$

Exponential Functions

WORKED EXAMPLE

$f(x) = 12 \cdot 3^x$ **f(2) =** 108

$f(2) = 12 \cdot 3^2$

$3^2 = 9$

$12 \cdot 9 = 108$

8. Evaluate exponential functions

Evaluate each exponential function at the value given.

1 $f(x) = 2 \cdot 6^x$ **f(1) =** _____

2 $f(x) = 10 \cdot 5^x$ **f(1) =** _____

3 $f(x) = 12 \cdot 3^x$ **f(3) =** _____

4 $f(x) = 5 \cdot 7^x$ **f(0) =** _____

5 $f(x) = 3 \cdot 2^x$ **f(2) =** _____

6 $f(x) = 10 \cdot 4^x$ **f(1) =** _____

7 $f(x) = 2 \cdot 5^x$ **f(3) =** _____

8 $f(x) = 7 \cdot 3^x$ **f(1) =** _____

9 $f(x) = 3 \cdot 2^x$ **f(0) =** _____

10 $f(x) = 2 \cdot 5^x$ **f(1) =** _____

11 $f(x) = 2 \cdot 6^x$ **f(2) =** _____

12 $f(x) = 7 \cdot 5^x$ **f(1) =** _____

13 $f(x) = 8 \cdot 8^x$ **f(0) =** _____

14 $f(x) = 5 \cdot 7^x$ **f(2) =** _____

15 $f(x) = 3 \cdot 6^x$ **f(1) =** _____

16 $f(x) = 2 \cdot 7^x$ **f(0) =** _____

8. Evaluate exponential functions (continued)

Evaluate each exponential function at the value given.

17 $f(x) = 4 \cdot 3^x$ $f(1) =$ _____

18 $f(x) = 4 \cdot 9^x$ $f(2) =$ _____

19 $f(x) = 3 \cdot 8^x$ $f(0) =$ _____

20 $f(x) = 7 \cdot 2^x$ $f(1) =$ _____

21 $f(x) = 2 \cdot 7^x$ $f(3) =$ _____

22 $f(x) = 10 \cdot 5^x$ $f(0) =$ _____

23 $f(x) = 3 \cdot 4^x$ $f(0) =$ _____

24 $f(x) = 6 \cdot 5^x$ $f(1) =$ _____

9. Rational exponents

Work out each one. Take the root first, then raise it.

25 $4^{3/2} =$ _____

26 $27^{1/3} =$ _____

27 $125^{1/3} =$ _____

28 $64^{4/3} =$ _____

29 $64^{5/3} =$ _____

30 $8^{4/3} =$ _____

31 $64^{1/2} =$ _____

32 $8^{1/3} =$ _____

33 $25^{1/2} =$ _____

34 $4^{5/2} =$ _____

Completing the Square

WORKED EXAMPLE

$x^2 - 2x - 8$

$-2 \div 2 = -1$

$(x - 1)^2 = x^2 - 2x + 1$

$= (x - 1)^2 - 9$

10. Completing the square

Write each expression as $(x + p)^2 + q$.

1 $x^2 + 2x - 2$

2 $x^2 - 12x + 33$

3 $x^2 + 14x + 61$

4 $x^2 + 10x + 33$

5 $x^2 - 12x + 40$

6 $x^2 + 10x + 18$

7 $x^2 + 10x + 9$

8 $x^2 - 4x - 12$

9 $x^2 - 6x + 15$

10 $x^2 + 10x + 27$

11 $x^2 + 20x + 101$

12 $x^2 + 4x + 2$

13 $x^2 + 6x + 2$

14 $x^2 + 12x + 27$

15 $x^2 - 2x + 16$

16 $x^2 - 2x + 7$

17 $x^2 + 10x + 37$

18 $x^2 + 10x$

19 $x^2 + 12x + 34$

20 $x^2 + 6x + 11$

21 $x^2 - 10x - 7$

22 $x^2 + 2x + 3$

11. Logarithms

Work out each logarithm, or find the missing number.

23 $\log_{10} \_\_\_\_\_ = 2$

24 $\log_2 \frac{1}{8} = \_\_\_\_\_$

25 $\log_5 \frac{1}{625} = \_\_\_\_\_$

26 $\log_3 \frac{1}{9} = \_\_\_\_\_$

27 $\log_7 343 = \_\_\_\_\_$

28 $\log_9 \frac{1}{81} = \_\_\_\_\_$

29 $\log_9 \_\_\_\_\_ = 4$

30 $\log_{10} \_\_\_\_\_ = 3$

31 $\log_7 \frac{1}{49} = \_\_\_\_\_$

32 $\log_5 \_\_\_\_\_ = 2$

How Many Real Solutions

WORKED EXAMPLE

$x^2 + 24x + 144 = 0$

$a = 1, b = 24, c = 144$

$b^2 - 4ac = 24^2 - 4(1)(144)$

$= 0 \rightarrow$ one

12. How many real solutions?

How many real solutions does each equation have? Circle one.

1 $x^2 + 22x + 149 = 0$

2 $x^2 + 12x - 160 = 0$

3 $x^2 - 3x + 98 = 0$

4 $x^2 + 18x + 107 = 0$

5 $x^2 + 2x + 1 = 0$

6 $x^2 + 16x + 65 = 0$

12. How many real solutions? (continued)

How many real solutions does each equation have? Circle one.

7 $x^2 - 34x - 34 = 0$

8 $x^2 - 28x - 147 = 0$

9 $x^2 - 9x - 178 = 0$

10 $x^2 - 20x + 86 = 0$

11 $x^2 - 18x + 81 = 0$

12 $x^2 + 34x + 56 = 0$

13 $x^2 + 15x + 151 = 0$

14 $x^2 + 102 = 0$

15 $x^2 - 9x + 42 = 0$

12. How many real solutions? (continued)

How many real solutions does each equation have? Circle one.

16 $x^2 + 23x - 48 = 0$

17 $x^2 - 6x + 9 = 0$

18 $x^2 - 8x + 72 = 0$

19 $x^2 - 21x - 137 = 0$

20 $x^2 + 24x - 18 = 0$

21 $x^2 - 4x + 4 = 0$

22 $x^2 - 7x + 152 = 0$

23 $x^2 + 17x + 172 = 0$

24 $x^2 + 26x + 90 = 0$

13. Laws of logarithms

Write each of these as one logarithm.

25 $\log_5 2 + \log_5 12 = \log_5$ ____

26 $\log_2 4 + \log_2 5 = \log_2$ ____

27 $\log_3 48 - \log_3 6 = \log_3$ ____

28 $\log_3 14 - \log_3 2 = \log_3$ ____

29 $\log_{10} 2 + \log_{10} 6 = \log_{10}$ ____

30 $\log_3 24 - \log_3 3 = \log_3$ ____

31 $2 \log_3 6 = \log_3$ ____

32 $\log_3 8 - \log_3 2 = \log_3$ ____

33 $4 \log_{10} 2 = \log_{10}$ ____

34 $\log_{10} 18 - \log_{10} 3 = \log_{10}$ ____

Adding and Subtracting Complex Numbers

WORKED EXAMPLE

(2 − 5i) − (−1 − 3i)

2 − (−1) = 3
−5 − (−3) = −2
= 3 − 2i

14. Complex numbers

Add or subtract each pair of complex numbers.

1 **(2 + 5i) − (1 + 3i)**

2 **(1 + i) − (3 + 2i)**

3 **(3 − i) − (4 − 5i)**

4 **(−6 + i) + (3 + 2i)**

5 **(2 + 4i) + (2 − 4i)**

6 **(−3 − 3i) − (−2 − 3i)**

7 **(4 − 4i) + (5 + i)**

8 **(−1 − 5i) + (−1 − 3i)**

9 **(1 + 6i) + (1 − 4i)**

10 **(1 + 5i) + (1 + 3i)**

11 **(1 − 3i) − (5 − 6i)**

12 **(2 + 2i) + (3 − 5i)**

13 **(2 − i) + (−3 − 2i)**

14 **(3 − 4i) − (5 + i)**

15 **(2 − 2i) + (4 + 2i)**

16 **(−3 + i) − (3 + 2i)**

17 **(−3 − i) − (−3 − 2i)**

18 **(3 + i) − (−4 + 5i)**

19 **(6 + i) + (3 + 2i)**

20 **(1 − 4i) − (5 + i)**

21 **(5 − 2i) − (4 + 2i)**

22 **(3 − i) − (−3 − 2i)**

15. Evaluate exponential functions

Evaluate each exponential function at the value given.

23 $f(x) = 7 \cdot 7^x$ **f(0) =** ___

24 $f(x) = 2 \cdot 6^x$ **f(1) =** ___

25 $f(x) = 6 \cdot 4^x$ **f(2) =** ___

26 $f(x) = 4 \cdot 2^x$ **f(0) =** ___

27 $f(x) = 2 \cdot 8^x$ **f(1) =** ___

28 $f(x) = 6 \cdot 4^x$ **f(1) =** ___

29 $f(x) = 3 \cdot 2^x$ **f(4) =** ___

30 $f(x) = 12 \cdot 7^x$ **f(0) =** ___

31 $f(x) = 5 \cdot 6^x$ **f(0) =** ___

32 $f(x) = 4 \cdot 6^x$ **f(0) =** ___

Multiplying Complex Numbers

WORKED EXAMPLE

$(-2 - 4i)(-2 + 4i)$

$= 4 - 8i + 8i - 16i^2$

$= 4 + 16 - 8i + 8i$

$= 20$

16. Complex numbers

Multiply each pair of complex numbers.

1 $(2 - 4i)(-2 + 4i)$

2 $(4 - 3i)(-2 - 3i)$

3 $(3 + 4i)(-5 - i)$

4 $(1 + 3i)(2 + 3i)$

5 $(-2 + 2i)(-4 - 2i)$

6 $(-4 + i)(3 + 2i)$

7 $(6 - 3i)(-2 - 3i)$

8 $(-4 + 2i)(-4 - 2i)$

9 $(-2 + 2i)(3 - 5i)$

10 $(-1 - i)(-3 - 2i)$

11 $(-5 + 2i)(3 - 5i)$

12 $(1 - 3i)(-2 - 3i)$

13 $(-2 - 2i)(4 + 2i)$

14 $(-3 - 4i)(5 + i)$

15 $(-2 - i)(4 - 5i)$

16 $(-1 - 5i)(-1 - 3i)$

17 $(-1 + 3i)(2 + 3i)$

18 $(4 - 4i)(5 + i)$

19 $(-1 + 2i)(-4 - 2i)$

20 $(-4 - 3i)(-2 - 3i)$

21 $(4 - 4i)(-2 + 4i)$

22 $(1 + 5i)(1 + 3i)$

23 $(3 - 5i)(-1 - 3i)$

17. Completing the square

Write each expression as $(x + p)^2 + q$.

24 $x^2 + 8x + 14$

25 $x^2 - 4x - 21$

26 $x^2 + 10x - 15$

27 $x^2 - 2x + 25$

28 $x^2 + 4x + 19$

29 $x^2 - 6x + 10$

30 $x^2 + 14x + 57$

31 $x^2 + 10x + 22$

32 $x^2 + 4x - 5$

33 $x^2 - 2x + 2$

Adding, Subtracting and Multiplying Polynomials

WORKED EXAMPLE

$(4x^2 + 8x + 2) + (3x^2 + 4x + 6)$

$4x^2 + 3x^2 = 7x^2$
$8x + 4x = 12x$
$2 + 6 = 8$
$7x^2 + 12x + 8$

18. Add, subtract and multiply polynomials

Work each one out.

1 **$(4x^2 + 7x + 2) + (6x^2 + 8x + 5)$**

2 **$(3x^2 + 5x + 7) + (x^2 + 4x + 7)$**

3 **$(7x^2 + 6x + 12) - (5x^2 - 3x + 7)$**

4 **$(6x^2 + 3x + 5) + (3x^2 + 9x + 6)$**

5 **$(6x^2 + 8x + 7) + (2x^2 + 5x + 4)$**

6 **$(2x^2 + 6x + 9) + (2x^2 + 3x + 9)$**

7 **$(8x^2 + 5x + 10) - (x^2 - 9x + 6)$**

8 **$(2x^2 + x + 6) + (6x^2 + 2x + 1)$**

9 **$(9x^2 + x + 7) - (7x^2 - 4x + 6)$**

10 **$(6x^2 + x + 10) - (2x^2 - 2x + 8)$**

11 **$(6x^2 + 7x + 2) + (6x^2 + 8x + 7)$**

12 **$(2x^2 + 6x + 3) + (3x^2 + 2x + 3)$**

18. Add, subtract and multiply polynomials (continued)

Work each one out.

13 $(3x^2 + 7x + 5) + (x^2 + 6x + 7)$

14 $(6x^2 + 7x + 12) - (4x^2 - 4x + 2)$

15 $(3x^2 + 2x + 8) + (4x^2 + 6x + 5)$

16 $(3x^2 + 7x + 2) + (3x^2 + 3x + 4)$

17 $(x^2 + 4x + 7) + (4x^2 + 8x + 4)$

18 $(7x^2 + 4x + 11) - (x^2 - 8x + 3)$

19 $(3x^2 + 6x + 9) + (5x^2 + 8x + 1)$

20 $(2x^2 + 8x + 4) + (x^2 + 7x + 6)$

21 $(9x^2 + 7x + 6) - (8x^2 - 8x + 1)$

22 $(5x^2 + 5x + 7) + (x^2 + 4x + 9)$

23 $(6x^2 + 8x + 5) + (4x^2 + 2x + 2)$

24 $(4x^2 + 4x + 8) - (3x^2 - 5x + 6)$

19. How many real solutions?

How many real solutions does each equation have? Circle one.

25 $x^2 - 5x + 130 = 0$

26 $x^2 - 34x + 63 = 0$

19. How many real solutions? (continued)

How many real solutions does each equation have? Circle one.

27 $x^2 + 32x - 83 = 0$

28 $x^2 + 2x + 1 = 0$

29 $x^2 + 16x + 65 = 0$

30 $x^2 + 17x + 32 = 0$

31 $x^2 - 20x + 22 = 0$

32 $x^2 + 32x + 70 = 0$

33 $x^2 + 35x + 102 = 0$

34 $x^2 - 18x + 81 = 0$

Dividing Polynomials

WORKED EXAMPLE

$$
\begin{array}{r|l}
x+1 & x^3 + 2x - 1 \\
\multicolumn{2}{r}{x^2 - x + 3} \\
\multicolumn{2}{r}{-x^3 - x^2} \\
\multicolumn{2}{r}{-x^2 + 2x - 1} \\
\multicolumn{2}{r}{x^2 + x} \\
\multicolumn{2}{r}{3x - 1} \\
\multicolumn{2}{r}{-3x - 3} \\
\multicolumn{2}{r}{-4}
\end{array}
$$

20. Dividing polynomials

Divide each polynomial. Write 0 for the remainder when it divides exactly.

1

$$
\begin{array}{r|l}
x-1 & x^3 - 7x^2 + 6x + 9 \\
\multicolumn{2}{r}{x^2 - 6x + 0} \\
\multicolumn{2}{r}{-x^3 + x^2} \\
\multicolumn{2}{r}{-6x^2 + 6x + 9} \\
\multicolumn{2}{r}{6x^2 - 6x} \\
\multicolumn{2}{r}{9}
\end{array}
$$

2

$$
\begin{array}{r|l}
x-1 & x^3 + x^2 - 4x + 3 \\
\multicolumn{2}{r}{x^2 + 2x - 2} \\
\multicolumn{2}{r}{-x^3 + x^2} \\
\multicolumn{2}{r}{2x^2 - 4x + 3} \\
\multicolumn{2}{r}{-2x^2 + 2x} \\
\multicolumn{2}{r}{-2x + 3} \\
\multicolumn{2}{r}{2x - 2} \\
\multicolumn{2}{r}{1}
\end{array}
$$

3

$$
\begin{array}{r|l}
x+2 & x^3 + 8x^2 + 12x \\
\multicolumn{2}{r}{x^2 + 6x + 0} \\
\multicolumn{2}{r}{-x^3 - 2x^2} \\
\multicolumn{2}{r}{6x^2 + 12x} \\
\multicolumn{2}{r}{-6x^2 - 12x} \\
\multicolumn{2}{r}{0}
\end{array}
$$

4

$$
\begin{array}{r|l}
x-3 & x^3 - 2x^2 - 6x + 9 \\
\multicolumn{2}{r}{x^2 + x - 3} \\
\multicolumn{2}{r}{-x^3 + 3x^2} \\
\multicolumn{2}{r}{x^2 - 6x + 9} \\
\multicolumn{2}{r}{-x^2 + 3x} \\
\multicolumn{2}{r}{-3x + 9} \\
\multicolumn{2}{r}{3x - 9} \\
\multicolumn{2}{r}{0}
\end{array}
$$

5

$$
\begin{array}{r|l}
x-2 & x^3 - 6x^2 + 3x + 10 \\
\multicolumn{2}{r}{x^2 - 4x - 5} \\
\multicolumn{2}{r}{-x^3 + 2x^2} \\
\multicolumn{2}{r}{-4x^2 + 3x + 10} \\
\multicolumn{2}{r}{4x^2 - 8x} \\
\multicolumn{2}{r}{-5x + 10} \\
\multicolumn{2}{r}{5x - 10} \\
\multicolumn{2}{r}{0}
\end{array}
$$

6

$$
\begin{array}{r|l}
x+4 & x^3 + x^2 - 10x + 8 \\
\multicolumn{2}{r}{x^2 - 3x + 2} \\
\multicolumn{2}{r}{-x^3 - 4x^2} \\
\multicolumn{2}{r}{-3x^2 - 10x + 8} \\
\multicolumn{2}{r}{3x^2 + 12x} \\
\multicolumn{2}{r}{2x + 8} \\
\multicolumn{2}{r}{-2x - 8} \\
\multicolumn{2}{r}{0}
\end{array}
$$

7

$$
\begin{array}{r|l}
x+3 & x^3 - 14x - 15 \\
\multicolumn{2}{r}{x^2 - 3x - 5} \\
\multicolumn{2}{r}{-x^3 - 3x^2} \\
\multicolumn{2}{r}{-3x^2 - 14x - 15} \\
\multicolumn{2}{r}{3x^2 + 9x} \\
\multicolumn{2}{r}{-5x - 15} \\
\multicolumn{2}{r}{5x + 15} \\
\multicolumn{2}{r}{0}
\end{array}
$$

8

$$
\begin{array}{r|l}
x-1 & x^3 - 2x^2 - 3x + 8 \\
\multicolumn{2}{r}{x^2 - x - 4} \\
\multicolumn{2}{r}{-x^3 + x^2} \\
\multicolumn{2}{r}{-x^2 - 3x + 8} \\
\multicolumn{2}{r}{x^2 - x} \\
\multicolumn{2}{r}{-4x + 8} \\
\multicolumn{2}{r}{4x - 4} \\
\multicolumn{2}{r}{4}
\end{array}
$$

9

$$
\begin{array}{r|l}
x-1 & x^3 + 2x^2 + 2x - 9 \\
\multicolumn{2}{r}{x^2 + 3x + 5} \\
\multicolumn{2}{r}{-x^3 + x^2} \\
\multicolumn{2}{r}{3x^2 + 2x - 9} \\
\multicolumn{2}{r}{-3x^2 + 3x} \\
\multicolumn{2}{r}{5x - 9} \\
\multicolumn{2}{r}{-5x + 5} \\
\multicolumn{2}{r}{-4}
\end{array}
$$

10

$$
\begin{array}{r|l}
x-3 & x^3 - x^2 - 8x + 7 \\
\multicolumn{2}{r}{x^2 + 2x - 2} \\
\multicolumn{2}{r}{-x^3 + 3x^2} \\
\multicolumn{2}{r}{2x^2 - 8x + 7} \\
\multicolumn{2}{r}{-2x^2 + 6x} \\
\multicolumn{2}{r}{-2x + 7} \\
\multicolumn{2}{r}{2x - 6} \\
\multicolumn{2}{r}{1}
\end{array}
$$

21. Complex numbers

Add or subtract each pair of complex numbers.

11 $(-1 - i) - (-3 - 2i)$

12 $(-3 + 3i) - (2 + 3i)$

13 $(5 + 5i) + (1 + 3i)$

14 $(1 + 2i) - (3 - 5i)$

15 $(-3 - 2i) - (4 + 2i)$

16 $(-4 + 2i) + (-4 - 2i)$

17 $(4 + 4i) + (2 - 4i)$

18 $(5 - i) - (-3 - 2i)$

19 $(2 - 3i) + (-2 - 3i)$

20 $(2 - 4i) + (5 + i)$

Simplifying Rational Expressions

WORKED EXAMPLE

$$\frac{x^2 - 2x - 120}{x^2 - 5x - 150}$$

① $x^2 - 2x - 120 = (x + 10)(x - 12)$

② $x^2 - 5x - 150 = (x + 10)(x - 15)$

①/② $\dfrac{(x + 10)(x - 12)}{(x + 10)(x - 15)}$

$= \dfrac{(x - 12)}{(x - 15)}$

22. Simplifying rational expressions

Factor the top and the bottom, then cancel.

1 $\dfrac{x^2 + 5x - 66}{x^2 - 10x + 24}$

2 $\dfrac{x^2 + 3x + 2}{x^2 + 22x + 40}$

3 $\dfrac{x^2 - 22x + 112}{x^2 - 14x}$

4 $\dfrac{x^2 - 10x + 16}{x^2 - 8x}$

5 $\dfrac{x^2 - 18x + 72}{x^2 - 4x - 12}$

6 $\dfrac{x^2 + 8x - 33}{x^2 + 3x - 18}$

7 $\dfrac{x^2 - 19x + 88}{x^2 - x - 110}$

8 $\dfrac{x^2 - 20x + 51}{x^2 - 10x + 21}$

22. Simplifying rational expressions (continued)

Factor the top and the bottom, then cancel.

9 $\dfrac{x^2 - 2x - 8}{x^2 - 5x + 4}$

10 $\dfrac{x^2 - 14x - 51}{x^2 + 18x + 45}$

11 $\dfrac{x^2 + x - 30}{x^2 - 12x - 108}$

12 $\dfrac{x^2 + 18x - 19}{x^2 - 16x + 15}$

13 $\dfrac{x^2 + 15x + 36}{x^2 - 10x - 39}$

14 $\dfrac{x^2 - 6x - 112}{x^2 - 7x - 120}$

15 $\dfrac{x^2 + 7x + 12}{x^2 + 24x + 80}$

16 $\dfrac{x^2 + 18x + 72}{x^2 + 12x}$

17 $\dfrac{x^2 - 9x + 18}{x^2 - 13x + 30}$

18 $\dfrac{x^2 - 13x - 114}{x^2 - 15x - 76}$

19 $\dfrac{x^2 + 9x - 52}{x^2 - 9x + 20}$

20 $\dfrac{x^2 + 11x}{x^2 + 18x}$

21 $\dfrac{x^2 - 8x - 180}{x^2 + 16x + 60}$

23. Simplifying radicals

Simplify each cube root.

22 $\sqrt[3]{72}$

23 $\sqrt[3]{32}$

24 $\sqrt[3]{324}$

25 $\sqrt[3]{56}$

26 $\sqrt[3]{135}$

27 $\sqrt[3]{81}$

28 $\sqrt[3]{320}$

29 $\sqrt[3]{108}$

30 $\sqrt[3]{189}$

31 $\sqrt[3]{24}$

Multiplying and Dividing Rational Expressions

WORKED EXAMPLE

$$\frac{(x+8)}{(x+6)} \div \frac{(x-7)}{(x+6)}$$

$(x + 8)/(x + 6) \times (x + 6)/(x - 7)$
$(x + 6)$ cancels
$(x + 8) / (x - 7)$
$x \neq -6, 7$

24. Multiplying and dividing rational expressions

Multiply or divide, then cancel. Write each answer as one fraction.

1 $\frac{(x-1)}{(x-6)} \div \frac{(x-5)}{(x-6)}$

2 $\frac{(x+3)}{(x+4)} \times \frac{(x+4)}{(x+7)}$

3 $\frac{(x+8)}{(x-3)} \div \frac{(x+4)}{(x-3)}$

4 $\frac{(x-5)}{(x+6)} \times \frac{(x+6)}{(x-1)}$

5 $\frac{(x-7)}{(x-4)} \times \frac{(x-4)}{(x-3)}$

6 $\frac{(x+2)}{(x-7)} \times \frac{(x-7)}{(x+5)}$

7 $\frac{(x-6)}{(x+8)} \times \frac{(x+8)}{(x+2)}$

8 $\frac{(x-1)}{(x-3)} \div \frac{(x+6)}{(x-3)}$

9 $\frac{(x+3)}{(x+1)} \times \frac{(x+1)}{(x-5)}$

10 $\frac{(x+4)}{(x+7)} \times \frac{(x+7)}{(x+1)}$

11 $\frac{(x-4)}{(x+3)} \times \frac{(x+3)}{(x-8)}$

12 $\frac{(x+3)}{(x-4)} \div \frac{(x-1)}{(x-4)}$

13 $\frac{(x+7)}{(x-2)} \times \frac{(x-2)}{(x+3)}$

14 $\frac{(x+8)}{(x+7)} \div \frac{(x-3)}{(x+7)}$

15 $\frac{x}{(x-4)} \times \frac{(x-4)}{(x+7)}$

16 $\frac{(x-5)}{(x-4)} \div \frac{(x+6)}{(x-4)}$

17 $\frac{(x+7)}{(x-3)} \times \frac{(x-3)}{(x-1)}$

18 $\frac{(x+4)}{(x-7)} \div \frac{(x-8)}{(x-7)}$

25. Add, subtract and multiply polynomials

Work each one out.

19 $(6x^2 + 5x + 8) - (5x^2 - x + 4)$

20 $(2x^2 + 5x + 3) + (6x^2 + 6x + 2)$

21 $(9x^2 + 7x + 11) - (7x^2 - 9x + 9)$

22 $(5x^2 + x + 12) - (4x^2 - 5x + 9)$

23 $(3x^2 + 5x + 7) + (x^2 + 4x + 7)$

24 $(4x^2 + 7x + 2) + (6x^2 + 8x + 5)$

25 $(9x^2 + 8x + 11) - (4x^2 - 5x + 7)$

26 $(3x^2 + 5x + 4) + (3x^2 + x + 4)$

27 $(4x^2 + x + 3) + (2x^2 + 9x + 9)$

28 $(3x^2 + 5x + 1) + (2x^2 + 3x + 1)$

Solving Rational Equations

WORKED EXAMPLE

$\frac{5}{(x-7)} = \frac{-5}{(x-3)}$, x = 5

$x \neq 3, 7$

$5(x-3) = -5(x-7)$

$x = 5$

$x = 5$ and $x \neq 3, 7$

26. Solving rational equations

Solve each equation for x.

1 $\frac{6}{(x-7)} = 6$, x = ____

2 $\frac{4}{(x-9)} = \frac{-12}{(x-1)}$, x = ____

3 $\frac{30}{(x+7)} = 2$, x = ____

4 $\frac{3}{(x-6)} = \frac{-9}{(x+2)}$, x = ____

5 $\frac{15}{(x-8)} = \frac{-3}{(x-2)}$, x = ____

6 $\frac{14}{(x-6)} = \frac{15}{(x-7)}$, x = ____

7 $\frac{-20}{(x-7)} = 5$, x = ____

8 $\frac{8}{(x-1)} = \frac{4}{x}$, x = ____

9 $\frac{6}{(x+6)} = \frac{3}{(x-1)}$, x = ____

10 $\frac{-5}{(x-5)} = 5$, x = ____

26. Solving rational equations (continued)

Solve each equation for x.

11 $\frac{30}{(x+6)} = \frac{-5}{(x-8)}$, x = ____

12 $\frac{14}{(x+4)} = \frac{-8}{(x-7)}$, x = ____

13 $\frac{8}{(x+6)} = 8$, x = ____

14 $\frac{18}{(x-4)} = \frac{14}{(x-2)}$, x = ____

15 $\frac{12}{(x+8)} = \frac{-9}{(x-6)}$, x = ____

16 $\frac{9}{(x+7)} = \frac{12}{(x+8)}$, x = ____

17 $\frac{2}{(x+3)} = \frac{-6}{(x+7)}$, x = ____

18 $\frac{16}{(x+5)} = \frac{8}{(x+7)}$, x = ____

19 $\frac{2}{(x-8)} = \frac{-10}{(x+4)}$, x = ____

20 $\frac{8}{(x+4)} = 2$, x = ____

21 $\frac{18}{(x-4)} = 6$, x = ____

22 $\frac{6}{(x+6)} = \frac{9}{(x+7)}$, x = ____

23 $\frac{6}{(x-3)} = 6$, x = ____

24 $\frac{7}{(x-2)} = \frac{-14}{(x-5)}$, x = ____

27. Dividing polynomials

Divide each polynomial. Write 0 for the remainder when it divides exactly.

25

$$x - 1 \overline{)\, x^3 + 4x^2 + x - 6}$$

$x^2 + 5x + 6$

$-x^3 + x^2$

$5x^2 + x - 6$

$-5x^2 + 5x$

$6x - 6$

$-6x + 6$

0

26

$$x - 3 \overline{)\, x^3 - 3x^2 - 3x}$$

$x^2 + 0 - 3$

$-x^3 + 3x^2$

$-3x$

$3x - 9$

-9

27

$$x + 4 \overline{)\, x^3 + 3x^2 - 8x - 20}$$

$x^2 - x - 4$

$-x^3 - 4x^2$

$-x^2 - 8x - 20$

$x^2 + 4x$

$-4x - 20$

$4x + 16$

-4

28

$$x - 2 \overline{)\, x^3 - 7x^2 + 11x - 2}$$

$x^2 - 5x + 1$

$-x^3 + 2x^2$

$-5x^2 + 11x - 2$

$5x^2 - 10x$

$x - 2$

$-x + 2$

0

29

$$x - 3 \overline{)\, x^3 - 6x^2 + 11x - 2}$$

$x^2 - 3x + 2$

$-x^3 + 3x^2$

$-3x^2 + 11x - 2$

$3x^2 - 9x$

$2x - 2$

$-2x + 6$

4

30

$$x + 2 \overline{)\, x^3 + 2x^2 - 3x - 5}$$

$x^2 + 0 - 3$

$-x^3 - 2x^2$

$-3x - 5$

$3x + 6$

1

31

$$x - 4 \overline{)\, x^3 + x^2 - 21x + 8}$$

$x^2 + 5x - 1$

$-x^3 + 4x^2$

$5x^2 - 21x + 8$

$-5x^2 + 20x$

$-x + 8$

$x - 4$

4

32

$$x + 5 \overline{)\, x^3 + 7x^2 + 8x - 9}$$

$x^2 + 2x - 2$

$-x^3 - 5x^2$

$2x^2 + 8x - 9$

$-2x^2 - 10x$

$-2x - 9$

$2x + 10$

1

Arithmetic and Geometric Sequences

WORKED EXAMPLE

$-2, 2, 6, 10, \ldots \quad a_{19} =$ 70

$d = 4$

$a_n = -2 + (n - 1) \times 4$

$a_{19} = -2 + 18 \times 4$

$-2 + 72 = 70$

$a_{19} = 70$

28. Arithmetic and geometric sequences

Find the term shown for each sequence.

1 $2, 1, 0, -1, \ldots \quad a_{17} =$ _____

2 $1, 5, 9, 13, \ldots \quad a_{15} =$ _____

3 $5, 9, 13, 17, \ldots \quad a_{17} =$ _____

4 $12, 19, 26, 33, \ldots \quad a_{12} =$ _____

5 $5, 8, 11, 14, \ldots \quad a_{10} =$ _____

6 $4, 8, 12, 16, \ldots \quad a_{11} =$ _____

7 $12, 9, 6, 3, \ldots \quad a_{20} =$ _____

8 $4, 0, -4, -8, \ldots \quad a_{10} =$ _____

9 $-3, 1, 5, 9, \ldots \quad a_{13} =$ _____

10 $11, 8, 5, 2, \ldots \quad a_{14} =$ _____

28. Arithmetic and geometric sequences (continued)

Find the term shown for each sequence.

11 8, 1, −6, −13, … a_{15} = _____

12 1, −3, −7, −11, … a_{14} = _____

13 12, 11, 10, 9, … a_{11} = _____

14 −4, −6, −8, −10, … a_{19} = _____

15 8, 12, 16, 20, … a_{13} = _____

16 3, 0, −3, −6, … a_{10} = _____

17 −7, −11, −15, −19, … a_{10} = _____

18 −1, −10, −19, −28, … a_{14} = _____

19 −7, −3, 1, 5, … a_{11} = _____

20 9, 15, 21, 27, … a_{10} = _____

21 −7, −2, 3, 8, … a_{18} = _____

22 −4, −3, −2, −1, … a_{10} = _____

23 6, 1, −4, −9, … a_{15} = _____

24 −7, −6, −5, −4, … a_{14} = _____

29. Simplifying rational expressions

Factor the top and the bottom, then cancel.

25 $\dfrac{x^2 + 16x + 15}{x^2 + 7x + 6}$

26 $\dfrac{x^2 - 5x + 6}{x^2 + 18x - 40}$

27 $\dfrac{x^2 - 12x - 160}{x^2 + 14x + 48}$

28 $\dfrac{x^2 - 13x - 30}{x^2 + 7x + 10}$

29 $\dfrac{x^2 - 3x}{x^2 + 7x - 30}$

30 $\dfrac{x^2 - 18x - 40}{x^2 - 12x - 28}$

31 $\dfrac{x^2 + 24x + 95}{x^2 + 11x + 30}$

32 $\dfrac{x^2 - 6x}{x^2 + 2x}$

33 $\dfrac{x^2 - 7x - 8}{x^2 + 14x + 13}$

34 $\dfrac{x^2 + 9x - 52}{x^2 - 9x + 20}$

Sums of Series

WORKED EXAMPLE

5, 10, 20, … S_5 = 155

$S_5 = 5(2^5 - 1) \div (2 - 1)$

$S_5 = 5 \times 31 \div 1 = 155$

30. Sums of series

S_n is the sum of the first n terms. Does each sequence add or multiply?

1 **5, 10, 20, … S_6 = _____**

2 **8, 32, 128, … S_5 = _____**

3 **6, 24, 96, … S_6 = _____**

4 **12, 24, 48, … S_5 = _____**

5 **4, 12, 36, … S_6 = _____**

6 **2, 4, 8, … S_5 = _____**

7 **9, 27, 81, … S_7 = _____**

8 **3, −2, −7, … S_5 = _____**

9 **11, 33, 99, … S_5 = _____**

10 **3, 12, 48, … S_5 = _____**

11 **8, 32, 128, … S_6 = _____**

12 **1, 3, 9, … S_5 = _____**

13 **7, 2, −3, … S_5 = _____**

14 **8, 16, 32, … S_7 = _____**

15 **2, 4, 8, … S_6 = _____**

16 **8, 16, 32, … S_6 = _____**

17 **6, 24, 96, … S_5 = _____**

18 **5, −5, −15, … S_5 = _____**

19 **11, 44, 176, … S_5 = _____**

20 **1, 4, 16, … S_5 = _____**

21 **2, 8, 32, … S_6 = _____**

31. Multiplying and dividing rational expressions

Multiply or divide, then cancel. Write each answer as one fraction.

22 $\dfrac{(x-5)}{(x-4)} \div \dfrac{(x+6)}{(x-4)}$

23 $\dfrac{(x-1)}{(x+2)} \times \dfrac{(x+2)}{(x+3)}$

24 $\dfrac{(x+8)}{(x+2)} \div \dfrac{(x+1)}{(x+2)}$

25 $\dfrac{(x+5)}{(x+6)} \div \dfrac{(x+2)}{(x+6)}$

26 $\dfrac{(x+3)}{(x+1)} \div \dfrac{(x-5)}{(x+1)}$

27 $\dfrac{(x-3)}{(x-7)} \div \dfrac{(x+4)}{(x-7)}$

28 $\dfrac{(x-7)}{(x+5)} \div \dfrac{(x+8)}{(x+5)}$

29 $\dfrac{(x+7)}{(x-2)} \times \dfrac{(x-2)}{(x+3)}$

30 $\dfrac{(x-7)}{(x+3)} \times \dfrac{(x+3)}{(x+1)}$

31 $\dfrac{(x-5)}{(x+7)} \div \dfrac{(x+3)}{(x+7)}$

Check the Log Laws

32. Check the log laws

Read each line of working and choose the answer that is correct.

1 **Ben wrote log(7x) − log 7 = log x ÷ log 7. Which is correct?**

log x | 7 log x | log x ÷ log 7 | log 7 − log x

Why? .....

2 **Check this line: log(2x) − log 2 = log x. Which is correct?**

log x ÷ log 2 | log x | log 2 − log x | 2 log x

Why? .....

3 **Check this line: log(13x) − log 13 = 13 log x. Which is correct?**

13 log x | log 13 − log x | log x | log x ÷ log 13

Why? .....

4 **Priya says log(17x) − log 17 comes to log x ÷ log 17. Which is correct?**

17 log x | log 17 − log x | log x ÷ log 17 | log x

Why? .....

5 **Check this line: log(x^2) + log(x^5) = 7 log x. Which is correct?**

log($x^2 + x^5$) | 10 log x | 7 log x | 10(log x)^2

Why? .....

6 **Check this line: log(14x) − log 14 = 14 log x. Which is correct?**

log x ÷ log 14 | log x | 14 log x | log 14 − log x

Why? .....

7 **Imani wrote log(5x) − log 5 = log x. Which is correct?**

log x | log 5 − log x | log x ÷ log 5 | 5 log x

Why? .....

8 **Sam wrote log(23x) − log 23 = log x. Which is correct?**

log x ÷ log 23 | log x | 23 log x | log 23 − log x

Why? .....

Answer Key (1 of 7)

Simplifying Radicals

Simplifying radicals

1. $\sqrt{63} = 3\sqrt{7}$
2. $\sqrt{126} = 3\sqrt{14}$
3. $\sqrt{360} = 6\sqrt{10}$
4. $\sqrt{224} = 4\sqrt{14}$
5. $\sqrt{56} = 2\sqrt{14}$
6. $\sqrt{8} = 2\sqrt{2}$
7. $\sqrt{125} = 5\sqrt{5}$
8. $\sqrt{32} = 4\sqrt{2}$
9. $\sqrt{24} = 2\sqrt{6}$
10. $\sqrt{96} = 4\sqrt{6}$
11. $\sqrt{72} = 6\sqrt{2}$
12. $\sqrt{12} = 2\sqrt{3}$
13. $\sqrt{135} = 3\sqrt{15}$
14. $\sqrt{175} = 5\sqrt{7}$
15. $\sqrt{112} = 4\sqrt{7}$
16. $\sqrt{75} = 5\sqrt{3}$
17. $\sqrt{52} = 2\sqrt{13}$
18. $\sqrt{504} = 6\sqrt{14}$
19. $\sqrt{90} = 3\sqrt{10}$
20. $\sqrt{44} = 2\sqrt{11}$
21. $\sqrt{98} = 7\sqrt{2}$
22. $\sqrt{48} = 4\sqrt{3}$
23. $\sqrt{240} = 4\sqrt{15}$
24. $\sqrt{99} = 3\sqrt{11}$
25. $\sqrt{128} = 8\sqrt{2}$
26. $\sqrt{45} = 3\sqrt{5}$
27. $\sqrt{28} = 2\sqrt{7}$
28. $\sqrt{539} = 7\sqrt{11}$

Cube Roots

Simplifying radicals

1. $\sqrt[3]{40} = 2\sqrt[3]{5}$
2. $\sqrt[3]{48} = 2\sqrt[3]{6}$
3. $\sqrt[3]{297} = 3\sqrt[3]{11}$
4. $\sqrt[3]{54} = 3\sqrt[3]{2}$
5. $\sqrt[3]{96} = 2\sqrt[3]{12}$
6. $\sqrt[3]{108} = 3\sqrt[3]{4}$
7. $\sqrt[3]{320} = 4\sqrt[3]{5}$
8. $\sqrt[3]{88} = 2\sqrt[3]{11}$
9. $\sqrt[3]{80} = 2\sqrt[3]{10}$
10. $\sqrt[3]{81} = 3\sqrt[3]{3}$
11. $\sqrt[3]{243} = 3\sqrt[3]{9}$
12. $\sqrt[3]{24} = 2\sqrt[3]{3}$
13. $\sqrt[3]{16} = 2\sqrt[3]{2}$
14. $\sqrt[3]{324} = 3\sqrt[3]{12}$
15. $\sqrt[3]{135} = 3\sqrt[3]{5}$
16. $\sqrt[3]{189} = 3\sqrt[3]{7}$
17. $\sqrt[3]{56} = 2\sqrt[3]{7}$

Cube Roots (continued)

Simplifying radicals (continued)

18. $\sqrt[3]{128} = 4\sqrt[3]{2}$
19. $\sqrt[3]{72} = 2\sqrt[3]{9}$
20. $\sqrt[3]{32} = 2\sqrt[3]{4}$
21. $\sqrt[3]{256} = 4\sqrt[3]{4}$
22. $\sqrt[3]{162} = 3\sqrt[3]{6}$
23. $\sqrt[3]{192} = 4\sqrt[3]{3}$
24. $\sqrt[3]{250} = 5\sqrt[3]{2}$

Rational Exponents

Rational exponents

1. 4^(3/2) = 8
2. 64^(5/3) = 1024
3. 216^(2/3) = 36
4. 25^(1/2) = 5
5. 25^(5/2) = 3125
6. 36^(3/2) = 216
7. 256^(1/4) = 4
8. 9^(1/2) = 3
9. 27^(1/3) = 3
10. 8^(1/3) = 2
11. 100^(1/2) = 10
12. 9^(3/2) = 27
13. 16^(1/2) = 4
14. 16^(3/4) = 8
15. 27^(5/3) = 243
16. 4^(1/2) = 2
17. 36^(1/2) = 6
18. 256^(5/4) = 1024
19. 27^(2/3) = 9
20. 8^(4/3) = 16
21. 81^(5/4) = 243
22. 49^(1/2) = 7
23. 16^(5/2) = 1024
24. 4^(5/2) = 32
25. 100^(3/2) = 1000
26. 49^(3/2) = 343
27. 64^(4/3) = 256
28. 125^(1/3) = 5

Logarithms

Logarithms

1. $\log_4 16 = 2$
2. $\log_3 1/9 = -2$
3. $\log_9 6561 = 4$
4. $\log_2 4 = 2$
5. $\log_{10} 1/1000 = -3$
6. $\log_8 512 = 3$
7. $\log_2 1/128 = -7$

Answer Key (2 of 7)

Logarithms (continued)

Logarithms (continued)

8. $\log_2 1/8 = -3$
9. $\log_7 1/49 = -2$
10. $\log_6 36 = 2$
11. $\log_5 625 = 4$
12. $\log_9 729 = 3$
13. $\log_7 49 = 2$
14. $\log_8 32768 = 5$
15. $\log_9 81 = 2$
16. $\log_2 16 = 4$
17. $\log_8 64 = 2$
18. $\log_5 15625 = 6$
19. $\log_9 729 = 3$
20. $\log_3 27 = 3$
21. $\log_6 1296 = 4$
22. $\log_9 1/81 = -2$
23. $\log_3 27 = 3$
24. $\log_2 8 = 3$
25. $\log_7 2401 = 4$
26. $\log_5 125 = 3$
27. $\log_8 1/512 = -3$
28. $\log_5 3125 = 5$

Simplifying radicals

29. $\sqrt{224} = 4\sqrt{14}$
30. $\sqrt{52} = 2\sqrt{13}$
31. $\sqrt{128} = 8\sqrt{2}$
32. $\sqrt{40} = 2\sqrt{10}$
33. $\sqrt{80} = 4\sqrt{5}$
34. $\sqrt{50} = 5\sqrt{2}$
35. $\sqrt{216} = 6\sqrt{6}$
36. $\sqrt{99} = 3\sqrt{11}$
37. $\sqrt{126} = 3\sqrt{14}$
38. $\sqrt{135} = 3\sqrt{15}$

The Laws of Logarithms

Laws of logarithms

1. $\log_2 4 + \log_2 6 = \log_2 24$
2. $\log_{10} 27 - \log_{10} 9 = \log_{10} 3$
3. $\log_5 6 + \log_5 7 = \log_5 42$
4. $\log_{10} 2 + \log_{10} 4 = \log_{10} 8$
5. $\log_5 2 + \log_5 5 = \log_5 10$
6. $\log_{10} 3 + \log_{10} 3 = \log_{10} 9$
7. $\log_2 6 + \log_2 8 = \log_2 48$
8. $\log_2 12 - \log_2 2 = \log_2 6$
9. $\log_2 22 - \log_2 2 = \log_2 11$
10. $\log_5 12 - \log_5 3 = \log_5 4$
11. $2 \log_2 6 = \log_2 36$
12. $\log_5 24 - \log_5 12 = \log_5 2$
13. $\log_3 4 + \log_3 4 = \log_3 16$

The Laws of Logarithms (continued)

Laws of logarithms (continued)

14. $\log_2 3 + \log_2 11 = \log_2 33$
15. $\log_3 4 + \log_3 6 = \log_3 24$
16. $\log_2 21 - \log_2 7 = \log_2 3$
17. $\log_5 24 - \log_5 2 = \log_5 12$
18. $\log_2 42 - \log_2 7 = \log_2 6$
19. $\log_{10} 3 + \log_{10} 9 = \log_{10} 27$
20. $\log_3 25 - \log_3 5 = \log_3 5$
21. $\log_2 44 - \log_2 11 = \log_2 4$, and that is 2
22. $\log_3 22 - \log_3 2 = \log_3 11$
23. $\log_5 2 + \log_5 6 = \log_5 12$
24. $\log_{10} 16 - \log_{10} 4 = \log_{10} 4$

Simplifying radicals

25. $\sqrt[3]{96} = 2\sqrt[3]{12}$
26. $\sqrt[3]{162} = 3\sqrt[3]{6}$
27. $\sqrt[3]{128} = 4\sqrt[3]{2}$
28. $\sqrt[3]{81} = 3\sqrt[3]{3}$
29. $\sqrt[3]{72} = 2\sqrt[3]{9}$
30. $\sqrt[3]{56} = 2\sqrt[3]{7}$
31. $\sqrt[3]{192} = 4\sqrt[3]{3}$
32. $\sqrt[3]{24} = 2\sqrt[3]{3}$
33. $\sqrt[3]{135} = 3\sqrt[3]{5}$
34. $\sqrt[3]{80} = 2\sqrt[3]{10}$

Exponential Functions

Evaluate exponential functions

1. $f(x) = 2 \cdot 6^x \rightarrow f(1) = 12$
2. $f(x) = 10 \cdot 5^x \rightarrow f(1) = 50$
3. $f(x) = 12 \cdot 3^x \rightarrow f(3) = 324$
4. $f(x) = 5 \cdot 7^x \rightarrow f(0) = 5$
5. $f(x) = 3 \cdot 2^x \rightarrow f(2) = 12$
6. $f(x) = 10 \cdot 4^x \rightarrow f(1) = 40$
7. $f(x) = 2 \cdot 5^x \rightarrow f(3) = 250$
8. $f(x) = 7 \cdot 3^x \rightarrow f(1) = 21$
9. $f(x) = 3 \cdot 2^x \rightarrow f(0) = 3$
10. $f(x) = 2 \cdot 5^x \rightarrow f(1) = 10$
11. $f(x) = 2 \cdot 6^x \rightarrow f(2) = 72$
12. $f(x) = 7 \cdot 5^x \rightarrow f(1) = 35$
13. $f(x) = 8 \cdot 8^x \rightarrow f(0) = 8$
14. $f(x) = 5 \cdot 7^x \rightarrow f(2) = 245$
15. $f(x) = 3 \cdot 6^x \rightarrow f(1) = 18$
16. $f(x) = 2 \cdot 7^x \rightarrow f(0) = 2$
17. $f(x) = 4 \cdot 3^x \rightarrow f(1) = 12$
18. $f(x) = 4 \cdot 9^x \rightarrow f(2) = 324$
19. $f(x) = 3 \cdot 8^x \rightarrow f(0) = 3$
20. $f(x) = 7 \cdot 2^x \rightarrow f(1) = 14$
21. $f(x) = 2 \cdot 7^x \rightarrow f(3) = 686$
22. $f(x) = 10 \cdot 5^x \rightarrow f(0) = 10$
23. $f(x) = 3 \cdot 4^x \rightarrow f(0) = 3$

Answer Key (3 of 7)

Exponential Functions (continued)

Evaluate exponential functions (continued)

24. $f(x) = 6 \cdot 5^x \rightarrow f(1) = 30$

Rational exponents

25. 4^(3/2) = 8
26. 27^(1/3) = 3
27. 125^(1/3) = 5
28. 64^(4/3) = 256
29. 64^(5/3) = 1024
30. 8^(4/3) = 16
31. 64^(1/2) = 8
32. 8^(1/3) = 2
33. 25^(1/2) = 5
34. 4^(5/2) = 32

Completing the Square

Completing the square

1. $x^2 + 2x - 2 = (x + 1)^2 - 3$
2. $x^2 - 12x + 33 = (x - 6)^2 - 3$
3. $x^2 + 14x + 61 = (x + 7)^2 + 12$
4. $x^2 + 10x + 33 = (x + 5)^2 + 8$
5. $x^2 - 12x + 40 = (x - 6)^2 + 4$
6. $x^2 + 10x + 18 = (x + 5)^2 - 7$
7. $x^2 + 10x + 9 = (x + 5)^2 - 16$
8. $x^2 - 4x - 12 = (x - 2)^2 - 16$
9. $x^2 - 6x + 15 = (x - 3)^2 + 6$
10. $x^2 + 10x + 27 = (x + 5)^2 + 2$
11. $x^2 + 20x + 101 = (x + 10)^2 + 1$
12. $x^2 + 4x + 2 = (x + 2)^2 - 2$
13. $x^2 + 6x + 2 = (x + 3)^2 - 7$
14. $x^2 + 12x + 27 = (x + 6)^2 - 9$
15. $x^2 - 2x + 16 = (x - 1)^2 + 15$
16. $x^2 - 2x + 7 = (x - 1)^2 + 6$
17. $x^2 + 10x + 37 = (x + 5)^2 + 12$
18. $x^2 + 10x = (x + 5)^2 - 25$
19. $x^2 + 12x + 34 = (x + 6)^2 - 2$
20. $x^2 + 6x + 11 = (x + 3)^2 + 2$
21. $x^2 - 10x - 7 = (x - 5)^2 - 32$
22. $x^2 + 2x + 3 = (x + 1)^2 + 2$

Logarithms

23. $\log_{10} 100 = 2$
24. $\log_2 1/8 = -3$
25. $\log_5 1/625 = -4$
26. $\log_3 1/9 = -2$
27. $\log_7 343 = 3$
28. $\log_9 1/81 = -2$
29. $\log_9 6561 = 4$
30. $\log_{10} 1000 = 3$
31. $\log_7 1/49 = -2$
32. $\log_5 25 = 2$

How Many Real Solutions

How many real solutions?

1. $x^2 + 22x + 149 = 0$: $22^2 - 4 \times 1 \times 149 = 484 - 596 = -112$ → None
2. $x^2 + 12x - 160 = 0$: $12^2 - 4 \times 1 \times -160 = 144 - -640 = 784$ → Two
3. $x^2 - 3x + 98 = 0$: $-3^2 - 4 \times 1 \times 98 = 9 - 392 = -383$ → None
4. $x^2 + 18x + 107 = 0$: $18^2 - 4 \times 1 \times 107 = 324 - 428 = -104$ → None
5. $x^2 + 2x + 1 = 0$: $2^2 - 4 \times 1 \times 1 = 4 - 4 = 0$ → One
6. $x^2 + 16x + 65 = 0$: $16^2 - 4 \times 1 \times 65 = 256 - 260 = -4$ → None
7. $x^2 - 34x - 34 = 0$: $-34^2 - 4 \times 1 \times -34 = 1156 - -136 = 1292$ → Two
8. $x^2 - 28x - 147 = 0$: $-28^2 - 4 \times 1 \times -147 = 784 - -588 = 1372$ → Two
9. $x^2 - 9x - 178 = 0$: $-9^2 - 4 \times 1 \times -178 = 81 - -712 = 793$ → Two
10. $x^2 - 20x + 86 = 0$: $-20^2 - 4 \times 1 \times 86 = 400 - 344 = 56$ → Two
11. $x^2 - 18x + 81 = 0$: $-18^2 - 4 \times 1 \times 81 = 324 - 324 = 0$ → One
12. $x^2 + 34x + 56 = 0$: $34^2 - 4 \times 1 \times 56 = 1156 - 224 = 932$ → Two
13. $x^2 + 15x + 151 = 0$: $15^2 - 4 \times 1 \times 151 = 225 - 604 = -379$ → None
14. $x^2 + 102 = 0$: $0^2 - 4 \times 1 \times 102 = 0 - 408 = -408$ → None
15. $x^2 - 9x + 42 = 0$: $-9^2 - 4 \times 1 \times 42 = 81 - 168 = -87$ → None
16. $x^2 + 23x - 48 = 0$: $23^2 - 4 \times 1 \times -48 = 529 - -192 = 721$ → Two
17. $x^2 - 6x + 9 = 0$: $-6^2 - 4 \times 1 \times 9 = 36 - 36 = 0$ → One
18. $x^2 - 8x + 72 = 0$: $-8^2 - 4 \times 1 \times 72 = 64 - 288 = -224$ → None
19. $x^2 - 21x - 137 = 0$: $-21^2 - 4 \times 1 \times -137 = 441 - -548 = 989$ → Two
20. $x^2 + 24x - 18 = 0$: $24^2 - 4 \times 1 \times -18 = 576 - -72 = 648$ → Two
21. $x^2 - 4x + 4 = 0$: $-4^2 - 4 \times 1 \times 4 = 16 - 16 = 0$ → One
22. $x^2 - 7x + 152 = 0$: $-7^2 - 4 \times 1 \times 152 = 49 - 608 = -559$ → None
23. $x^2 + 17x + 172 = 0$: $17^2 - 4 \times 1 \times 172 = 289 - 688 = -399$ → None
24. $x^2 + 26x + 90 = 0$: $26^2 - 4 \times 1 \times 90 = 676 - 360 = 316$ → Two

Laws of logarithms

25. $\log_5 2 + \log_5 12 = \log_5 24$
26. $\log_2 4 + \log_2 5 = \log_2 20$
27. $\log_3 48 - \log_3 6 = \log_3 8$
28. $\log_3 14 - \log_3 2 = \log_3 7$
29. $\log_{10} 2 + \log_{10} 6 = \log_{10} 12$
30. $\log_3 24 - \log_3 3 = \log_3 8$
31. $2 \log_3 6 = \log_3 36$
32. $\log_3 8 - \log_3 2 = \log_3 4$
33. $4 \log_{10} 2 = \log_{10} 16$

Answer Key (4 of 7)

How Many Real Solutions (continued)

Laws of logarithms (continued)

34. $\log_{10} 18 - \log_{10} 3 = \log_{10} 6$

Adding and Subtracting Complex Numbers

Complex numbers

1. $(2 + 5i) - (1 + 3i) = 1 + 2i$
2. $(1 + i) - (3 + 2i) = -2 - i$
3. $(3 - i) - (4 - 5i) = -1 + 4i$
4. $(-6 + i) + (3 + 2i) = -3 + 3i$
5. $(2 + 4i) + (2 - 4i) = 4$
6. $(-3 - 3i) - (-2 - 3i) = -1$
7. $(4 - 4i) + (5 + i) = 9 - 3i$
8. $(-1 - 5i) + (-1 - 3i) = -2 - 8i$
9. $(1 + 6i) + (1 - 4i) = 2 + 2i$
10. $(1 + 5i) + (1 + 3i) = 2 + 8i$
11. $(1 - 3i) - (5 - 6i) = -4 + 3i$
12. $(2 + 2i) + (3 - 5i) = 5 - 3i$
13. $(2 - i) + (-3 - 2i) = -1 - 3i$
14. $(3 - 4i) - (5 + i) = -2 - 5i$
15. $(2 - 2i) + (4 + 2i) = 6$
16. $(-3 + i) - (3 + 2i) = -6 - i$
17. $(-3 - i) - (-3 - 2i) = i$
18. $(3 + i) - (-4 + 5i) = 7 - 4i$
19. $(6 + i) + (3 + 2i) = 9 + 3i$
20. $(1 - 4i) - (5 + i) = -4 - 5i$
21. $(5 - 2i) - (4 + 2i) = 1 - 4i$
22. $(3 - i) - (-3 - 2i) = 6 + i$

Evaluate exponential functions

23. $f(x) = 7 \cdot 7^x \rightarrow f(0) = 7$
24. $f(x) = 2 \cdot 6^x \rightarrow f(1) = 12$
25. $f(x) = 6 \cdot 4^x \rightarrow f(2) = 96$
26. $f(x) = 4 \cdot 2^x \rightarrow f(0) = 4$
27. $f(x) = 2 \cdot 8^x \rightarrow f(1) = 16$
28. $f(x) = 6 \cdot 4^x \rightarrow f(1) = 24$
29. $f(x) = 3 \cdot 2^x \rightarrow f(4) = 48$
30. $f(x) = 12 \cdot 7^x \rightarrow f(0) = 12$
31. $f(x) = 5 \cdot 6^x \rightarrow f(0) = 5$
32. $f(x) = 4 \cdot 6^x \rightarrow f(0) = 4$

Multiplying Complex Numbers

Complex numbers

1. $(2 - 4i)(-2 + 4i) = 12 + 16i$
2. $(4 - 3i)(-2 - 3i) = -17 - 6i$
3. $(3 + 4i)(-5 - i) = -11 - 23i$
4. $(1 + 3i)(2 + 3i) = -7 + 9i$
5. $(-2 + 2i)(-4 - 2i) = 12 - 4i$
6. $(-4 + i)(3 + 2i) = -14 - 5i$
7. $(6 - 3i)(-2 - 3i) = -21 - 12i$
8. $(-4 + 2i)(-4 - 2i) = 20$

Multiplying Complex Numbers (continued)

Complex numbers (continued)

9. $(-2 + 2i)(3 - 5i) = 4 + 16i$
10. $(-1 - i)(-3 - 2i) = 1 + 5i$
11. $(-5 + 2i)(3 - 5i) = -5 + 31i$
12. $(1 - 3i)(-2 - 3i) = -11 + 3i$
13. $(-2 - 2i)(4 + 2i) = -4 - 12i$
14. $(-3 - 4i)(5 + i) = -11 - 23i$
15. $(-2 - i)(4 - 5i) = -13 + 6i$
16. $(-1 - 5i)(-1 - 3i) = -14 + 8i$
17. $(-1 + 3i)(2 + 3i) = -11 + 3i$
18. $(4 - 4i)(5 + i) = 24 - 16i$
19. $(-1 + 2i)(-4 - 2i) = 8 - 6i$
20. $(-4 - 3i)(-2 - 3i) = -1 + 18i$
21. $(4 - 4i)(-2 + 4i) = 8 + 24i$
22. $(1 + 5i)(1 + 3i) = -14 + 8i$
23. $(3 - 5i)(-1 - 3i) = -18 - 4i$

Completing the square

24. $x^2 + 8x + 14 = (x + 4)^2 - 2$
25. $x^2 - 4x - 21 = (x - 2)^2 - 25$
26. $x^2 + 10x - 15 = (x + 5)^2 - 40$
27. $x^2 - 2x + 25 = (x - 1)^2 + 24$
28. $x^2 + 4x + 19 = (x + 2)^2 + 15$
29. $x^2 - 6x + 10 = (x - 3)^2 + 1$
30. $x^2 + 14x + 57 = (x + 7)^2 + 8$
31. $x^2 + 10x + 22 = (x + 5)^2 - 3$
32. $x^2 + 4x - 5 = (x + 2)^2 - 9$
33. $x^2 - 2x + 2 = (x - 1)^2 + 1$

Adding, Subtracting and Multiplying Polynomials

Add, subtract and multiply polynomials

1. $(4x^2 + 7x + 2) + (6x^2 + 8x + 5) \rightarrow 10x^2 + 15x + 7$
2. $(3x^2 + 5x + 7) + (x^2 + 4x + 7) \rightarrow 4x^2 + 9x + 14$
3. $(7x^2 + 6x + 12) - (5x^2 - 3x + 7) \rightarrow 2x^2 + 9x + 5$
4. $(6x^2 + 3x + 5) + (3x^2 + 9x + 6) \rightarrow 9x^2 + 12x + 11$
5. $(6x^2 + 8x + 7) + (2x^2 + 5x + 4) \rightarrow 8x^2 + 13x + 11$
6. $(2x^2 + 6x + 9) + (2x^2 + 3x + 9) \rightarrow 4x^2 + 9x + 18$
7. $(8x^2 + 5x + 10) - (x^2 - 9x + 6) \rightarrow 7x^2 + 14x + 4$
8. $(2x^2 + x + 6) + (6x^2 + 2x + 1) \rightarrow 8x^2 + 3x + 7$
9. $(9x^2 + x + 7) - (7x^2 - 4x + 6) \rightarrow 2x^2 + 5x + 1$
10. $(6x^2 + x + 10) - (2x^2 - 2x + 8) \rightarrow 4x^2 + 3x + 2$
11. $(6x^2 + 7x + 2) + (6x^2 + 8x + 7) \rightarrow 12x^2 + 15x + 9$
12. $(2x^2 + 6x + 3) + (3x^2 + 2x + 3) \rightarrow 5x^2 + 8x + 6$
13. $(3x^2 + 7x + 5) + (x^2 + 6x + 7) \rightarrow 4x^2 + 13x + 12$
14. $(6x^2 + 7x + 12) - (4x^2 - 4x + 2) \rightarrow 2x^2 + 11x + 10$
15. $(3x^2 + 2x + 8) + (4x^2 + 6x + 5) \rightarrow 7x^2 + 8x + 13$
16. $(3x^2 + 7x + 2) + (3x^2 + 3x + 4) \rightarrow 6x^2 + 10x + 6$
17. $(x^2 + 4x + 7) + (4x^2 + 8x + 4) \rightarrow 5x^2 + 12x + 11$
18. $(7x^2 + 4x + 11) - (x^2 - 8x + 3) \rightarrow 6x^2 + 12x + 8$
19. $(3x^2 + 6x + 9) + (5x^2 + 8x + 1) \rightarrow 8x^2 + 14x + 10$

Answer Key (5 of 7)

Adding, Subtracting and Multiplying Polynomials (continued)

Add, subtract and multiply polynomials (continued)

20. $(2x^2 + 8x + 4) + (x^2 + 7x + 6) \rightarrow 3x^2 + 15x + 10$
21. $(9x^2 + 7x + 6) - (8x^2 - 8x + 1) \rightarrow 1x^2 + 15x + 5$
22. $(5x^2 + 5x + 7) + (x^2 + 4x + 9) \rightarrow 6x^2 + 9x + 16$
23. $(6x^2 + 8x + 5) + (4x^2 + 2x + 2) \rightarrow 10x^2 + 10x + 7$
24. $(4x^2 + 4x + 8) - (3x^2 - 5x + 6) \rightarrow 1x^2 + 9x + 2$

How many real solutions?

25. $x^2 - 5x + 130 = 0$: $-5^2 - 4 \times 1 \times 130 = 25 - 520 = -495 \rightarrow$ None
26. $x^2 - 34x + 63 = 0$: $-34^2 - 4 \times 1 \times 63 = 1156 - 252 = 904 \rightarrow$ Two
27. $x^2 + 32x - 83 = 0$: $32^2 - 4 \times 1 \times -83 = 1024 - -332 = 1356 \rightarrow$ Two
28. $x^2 + 2x + 1 = 0$: $2^2 - 4 \times 1 \times 1 = 4 - 4 = 0 \rightarrow$ One
29. $x^2 + 16x + 65 = 0$: $16^2 - 4 \times 1 \times 65 = 256 - 260 = -4 \rightarrow$ None
30. $x^2 + 17x + 32 = 0$: $17^2 - 4 \times 1 \times 32 = 289 - 128 = 161 \rightarrow$ Two
31. $x^2 - 20x + 22 = 0$: $-20^2 - 4 \times 1 \times 22 = 400 - 88 = 312 \rightarrow$ Two
32. $x^2 + 32x + 70 = 0$: $32^2 - 4 \times 1 \times 70 = 1024 - 280 = 744 \rightarrow$ Two
33. $x^2 + 35x + 102 = 0$: $35^2 - 4 \times 1 \times 102 = 1225 - 408 = 817 \rightarrow$ Two
34. $x^2 - 18x + 81 = 0$: $-18^2 - 4 \times 1 \times 81 = 324 - 324 = 0 \rightarrow$ One

Dividing Polynomials

Dividing polynomials

1. $(x^3 - 7x^2 + 6x + 9) \div (x - 1) = x^2 - 6x$ remainder 9
2. $(x^3 + x^2 - 4x + 3) \div (x - 1) = x^2 + 2x - 2$ remainder 1
3. $(x^3 + 8x^2 + 12x) \div (x + 2) = x^2 + 6x$
4. $(x^3 - 2x^2 - 6x + 9) \div (x - 3) = x^2 + x - 3$
5. $(x^3 - 6x^2 + 3x + 10) \div (x - 2) = x^2 - 4x - 5$
6. $(x^3 + x^2 - 10x + 8) \div (x + 4) = x^2 - 3x + 2$
7. $(x^3 - 14x - 15) \div (x + 3) = x^2 - 3x - 5$
8. $(x^3 - 2x^2 - 3x + 8) \div (x - 1) = x^2 - x - 4$ remainder 4
9. $(x^3 + 2x^2 + 2x - 9) \div (x - 1) = x^2 + 3x + 5$ remainder -4
10. $(x^3 - x^2 - 8x + 7) \div (x - 3) = x^2 + 2x - 2$ remainder 1

Complex numbers

11. $(-1 - i) - (-3 - 2i) = 2 + i$
12. $(-3 + 3i) - (2 + 3i) = -5$
13. $(5 + 5i) + (1 + 3i) = 6 + 8i$
14. $(1 + 2i) - (3 - 5i) = -2 + 7i$
15. $(-3 - 2i) - (4 + 2i) = -7 - 4i$
16. $(-4 + 2i) + (-4 - 2i) = -8$
17. $(4 + 4i) + (2 - 4i) = 6$
18. $(5 - i) - (-3 - 2i) = 8 + i$
19. $(2 - 3i) + (-2 - 3i) = -6i$
20. $(2 - 4i) + (5 + i) = 7 - 3i$

Simplifying Rational Expressions

Simplifying rational expressions

1. $(x^2 + 5x - 66) / (x^2 - 10x + 24) = (x + 11) / (x - 4)$
2. $(x^2 + 3x + 2) / (x^2 + 22x + 40) = (x + 1) / (x + 20)$
3. $(x^2 - 22x + 112) / (x^2 - 14x) = (x - 8) / x$
4. $(x^2 - 10x + 16) / (x^2 - 8x) = (x - 2) / x$
5. $(x^2 - 18x + 72) / (x^2 - 4x - 12) = (x - 12) / (x + 2)$
6. $(x^2 + 8x - 33) / (x^2 + 3x - 18) = (x + 11) / (x + 6)$
7. $(x^2 - 19x + 88) / (x^2 - x - 110) = (x - 8) / (x + 10)$
8. $(x^2 - 20x + 51) / (x^2 - 10x + 21) = (x - 17) / (x - 7)$
9. $(x^2 - 2x - 8) / (x^2 - 5x + 4) = (x + 2) / (x - 1)$
10. $(x^2 - 14x - 51) / (x^2 + 18x + 45) = (x - 17) / (x + 15)$
11. $(x^2 + x - 30) / (x^2 - 12x - 108) = (x - 5) / (x - 18)$
12. $(x^2 + 18x - 19) / (x^2 - 16x + 15) = (x + 19) / (x - 15)$
13. $(x^2 + 15x + 36) / (x^2 - 10x - 39) = (x + 12) / (x - 13)$
14. $(x^2 - 6x - 112) / (x^2 - 7x - 120) = (x - 14) / (x - 15)$
15. $(x^2 + 7x + 12) / (x^2 + 24x + 80) = (x + 3) / (x + 20)$
16. $(x^2 + 18x + 72) / (x^2 + 12x) = (x + 6) / x$
17. $(x^2 - 9x + 18) / (x^2 - 13x + 30) = (x - 6) / (x - 10)$
18. $(x^2 - 13x - 114) / (x^2 - 15x - 76) = (x + 6) / (x + 4)$
19. $(x^2 + 9x - 52) / (x^2 - 9x + 20) = (x + 13) / (x - 5)$
20. $(x^2 + 11x) / (x^2 + 18x) = (x + 11) / (x + 18)$
21. $(x^2 - 8x - 180) / (x^2 + 16x + 60) = (x - 18) / (x + 6)$

Simplifying radicals

22. $\sqrt[3]{72} = 2\sqrt[3]{9}$
23. $\sqrt[3]{32} = 2\sqrt[3]{4}$
24. $\sqrt[3]{324} = 3\sqrt[3]{12}$
25. $\sqrt[3]{56} = 2\sqrt[3]{7}$
26. $\sqrt[3]{135} = 3\sqrt[3]{5}$
27. $\sqrt[3]{81} = 3\sqrt[3]{3}$
28. $\sqrt[3]{320} = 4\sqrt[3]{5}$
29. $\sqrt[3]{108} = 3\sqrt[3]{4}$
30. $\sqrt[3]{189} = 3\sqrt[3]{7}$
31. $\sqrt[3]{24} = 2\sqrt[3]{3}$

Multiplying and Dividing Rational Expressions

Multiplying and dividing rational expressions

1. $(x - 1)/(x - 6) \div (x - 5)/(x - 6) = (x - 1) / (x - 5)$ $(x \neq 5, 6)$
2. $(x + 3)/(x + 4) \times (x + 4)/(x + 7) = (x + 3) / (x + 7)$ $(x \neq -7, -4)$
3. $(x + 8)/(x - 3) \div (x + 4)/(x - 3) = (x + 8) / (x + 4)$ $(x \neq -4, 3)$
4. $(x - 5)/(x + 6) \times (x + 6)/(x - 1) = (x - 5) / (x - 1)$ $(x \neq -6, 1)$
5. $(x - 7)/(x - 4) \times (x - 4)/(x - 3) = (x - 7) / (x - 3)$ $(x \neq 3, 4)$
6. $(x + 2)/(x - 7) \times (x - 7)/(x + 5) = (x + 2) / (x + 5)$ $(x \neq -5, 7)$
7. $(x - 6)/(x + 8) \times (x + 8)/(x + 2) = (x - 6) / (x + 2)$ $(x \neq -8, -2)$
8. $(x - 1)/(x - 3) \div (x + 6)/(x - 3) = (x - 1) / (x + 6)$ $(x \neq -6, 3)$
9. $(x + 3)/(x + 1) \times (x + 1)/(x - 5) = (x + 3) / (x - 5)$ $(x \neq -1, 5)$
10. $(x + 4)/(x + 7) \times (x + 7)/(x + 1) = (x + 4) / (x + 1)$ $(x \neq -7, -1)$
11. $(x - 4)/(x + 3) \times (x + 3)/(x - 8) = (x - 4) / (x - 8)$ $(x \neq -3, 8)$
12. $(x + 3)/(x - 4) \div (x - 1)/(x - 4) = (x + 3) / (x - 1)$ $(x \neq 1, 4)$
13. $(x + 7)/(x - 2) \times (x - 2)/(x + 3) = (x + 7) / (x + 3)$ $(x \neq -3, 2)$

Answer Key (6 of 7)

Multiplying and Dividing Rational Expressions (continued)

Multiplying and dividing rational expressions (continued)

14. $(x + 8)/(x + 7) \div (x - 3)/(x + 7) = (x + 8) / (x - 3)$ $(x \neq -7, 3)$
15. $x/(x - 4) \times (x - 4)/(x + 7) = x / (x + 7)$ $(x \neq -7, 4)$
16. $(x - 5)/(x - 4) \div (x + 6)/(x - 4) = (x - 5) / (x + 6)$ $(x \neq -6, 4)$
17. $(x + 7)/(x - 3) \times (x - 3)/(x - 1) = (x + 7) / (x - 1)$ $(x \neq 1, 3)$
18. $(x + 4)/(x - 7) \div (x - 8)/(x - 7) = (x + 4) / (x - 8)$ $(x \neq 7, 8)$

Add, subtract and multiply polynomials

19. $(6x^2 + 5x + 8) - (5x^2 - x + 4) \rightarrow 1x^2 + 6x + 4$
20. $(2x^2 + 5x + 3) + (6x^2 + 6x + 2) \rightarrow 8x^2 + 11x + 5$
21. $(9x^2 + 7x + 11) - (7x^2 - 9x + 9) \rightarrow 2x^2 + 16x + 2$
22. $(5x^2 + x + 12) - (4x^2 - 5x + 9) \rightarrow 1x^2 + 6x + 3$
23. $(3x^2 + 5x + 7) + (x^2 + 4x + 7) \rightarrow 4x^2 + 9x + 14$
24. $(4x^2 + 7x + 2) + (6x^2 + 8x + 5) \rightarrow 10x^2 + 15x + 7$
25. $(9x^2 + 8x + 11) - (4x^2 - 5x + 7) \rightarrow 5x^2 + 13x + 4$
26. $(3x^2 + 5x + 4) + (3x^2 + x + 4) \rightarrow 6x^2 + 6x + 8$
27. $(4x^2 + x + 3) + (2x^2 + 9x + 9) \rightarrow 6x^2 + 10x + 12$
28. $(3x^2 + 5x + 1) + (2x^2 + 3x + 1) \rightarrow 5x^2 + 8x + 2$

Solving Rational Equations

Solving rational equations

1. $6/(x - 7) = 6 \rightarrow x = 8$ $(x \neq 7)$
2. $4/(x - 9) = -12/(x - 1) \rightarrow x = 7$ $(x \neq 1, 9)$
3. $30/(x + 7) = 2 \rightarrow x = 8$ $(x \neq -7)$
4. $3/(x - 6) = -9/(x + 2) \rightarrow x = 4$ $(x \neq -2, 6)$
5. $15/(x - 8) = -3/(x - 2) \rightarrow x = 3$ $(x \neq 2, 8)$
6. $14/(x - 6) = 15/(x - 7) \rightarrow x = -8$ $(x \neq 6, 7)$
7. $-20/(x - 7) = 5 \rightarrow x = 3$ $(x \neq 7)$
8. $8/(x - 1) = 4/x \rightarrow x = -1$ $(x \neq 0, 1)$
9. $6/(x + 6) = 3/(x - 1) \rightarrow x = 8$ $(x \neq -6, 1)$
10. $-5/(x - 5) = 5 \rightarrow x = 4$ $(x \neq 5)$
11. $30/(x + 6) = -5/(x - 8) \rightarrow x = 6$ $(x \neq -6, 8)$
12. $14/(x + 4) = -8/(x - 7) \rightarrow x = 3$ $(x \neq -4, 7)$
13. $8/(x + 6) = 8 \rightarrow x = -5$ $(x \neq -6)$
14. $18/(x - 4) = 14/(x - 2) \rightarrow x = -5$ $(x \neq 2, 4)$
15. $12/(x + 8) = -9/(x - 6) \rightarrow x = 0$ $(x \neq -8, 6)$
16. $9/(x + 7) = 12/(x + 8) \rightarrow x = -4$ $(x \neq -8, -7)$
17. $2/(x + 3) = -6/(x + 7) \rightarrow x = -4$ $(x \neq -7, -3)$
18. $16/(x + 5) = 8/(x + 7) \rightarrow x = -9$ $(x \neq -7, -5)$
19. $2/(x - 8) = -10/(x + 4) \rightarrow x = 6$ $(x \neq -4, 8)$
20. $8/(x + 4) = 2 \rightarrow x = 0$ $(x \neq -4)$
21. $18/(x - 4) = 6 \rightarrow x = 7$ $(x \neq 4)$
22. $6/(x + 6) = 9/(x + 7) \rightarrow x = -4$ $(x \neq -7, -6)$
23. $6/(x - 3) = 6 \rightarrow x = 4$ $(x \neq 3)$
24. $7/(x - 2) = -14/(x - 5) \rightarrow x = 3$ $(x \neq 2, 5)$

Dividing polynomials

25. $(x^3 + 4x^2 + x - 6) \div (x - 1) = x^2 + 5x + 6$
26. $(x^3 - 3x^2 - 3x) \div (x - 3) = x^2 - 3$ remainder -9
27. $(x^3 + 3x^2 - 8x - 20) \div (x + 4) = x^2 - x - 4$ remainder -4
28. $(x^3 - 7x^2 + 11x - 2) \div (x - 2) = x^2 - 5x + 1$

Solving Rational Equations (continued)

Dividing polynomials (continued)

29. $(x^3 - 6x^2 + 11x - 2) \div (x - 3) = x^2 - 3x + 2$ remainder 4
30. $(x^3 + 2x^2 - 3x - 5) \div (x + 2) = x^2 - 3$ remainder 1
31. $(x^3 + x^2 - 21x + 8) \div (x - 4) = x^2 + 5x - 1$ remainder 4
32. $(x^3 + 7x^2 + 8x - 9) \div (x + 5) = x^2 + 2x - 2$ remainder 1

Arithmetic and Geometric Sequences

Arithmetic and geometric sequences

1. 2, 1, 0, −1, ... the 17th term = −14
2. 1, 5, 9, 13, ... the 15th term = 57
3. 5, 9, 13, 17, ... the 17th term = 69
4. 12, 19, 26, 33, ... the 12th term = 89
5. 5, 8, 11, 14, ... the 10th term = 32
6. 4, 8, 12, 16, ... the 11th term = 44
7. 12, 9, 6, 3, ... the 20th term = −45
8. 4, 0, −4, −8, ... the 10th term = −32
9. −3, 1, 5, 9, ... the 13th term = 45
10. 11, 8, 5, 2, ... the 14th term = −28
11. 8, 1, −6, −13, ... the 15th term = −90
12. 1, −3, −7, −11, ... the 14th term = −51
13. 12, 11, 10, 9, ... the 11th term = 2
14. −4, −6, −8, −10, ... the 19th term = −40
15. 8, 12, 16, 20, ... the 13th term = 56
16. 3, 0, −3, −6, ... the 10th term = −24
17. −7, −11, −15, −19, ... the 10th term = −43
18. −1, −10, −19, −28, ... the 14th term = −118
19. −7, −3, 1, 5, ... the 11th term = 33
20. 9, 15, 21, 27, ... the 10th term = 63
21. −7, −2, 3, 8, ... the 18th term = 78
22. −4, −3, −2, −1, ... the 10th term = 5
23. 6, 1, −4, −9, ... the 15th term = −64
24. −7, −6, −5, −4, ... the 14th term = 6

Simplifying rational expressions

25. $(x^2 + 16x + 15) / (x^2 + 7x + 6) = (x + 15) / (x + 6)$
26. $(x^2 - 5x + 6) / (x^2 + 18x - 40) = (x - 3) / (x + 20)$
27. $(x^2 - 12x - 160) / (x^2 + 14x + 48) = (x - 20) / (x + 6)$
28. $(x^2 - 13x - 30) / (x^2 + 7x + 10) = (x - 15) / (x + 5)$
29. $(x^2 - 3x) / (x^2 + 7x - 30) = x / (x + 10)$
30. $(x^2 - 18x - 40) / (x^2 - 12x - 28) = (x - 20) / (x - 14)$
31. $(x^2 + 24x + 95) / (x^2 + 11x + 30) = (x + 19) / (x + 6)$
32. $(x^2 - 6x) / (x^2 + 2x) = (x - 6) / (x + 2)$
33. $(x^2 - 7x - 8) / (x^2 + 14x + 13) = (x - 8) / (x + 13)$
34. $(x^2 + 9x - 52) / (x^2 - 9x + 20) = (x + 13) / (x - 5)$

Sums of Series

Sums of series

1. S_6 for 5, 10, 20, ... = 315
2. S_5 for 8, 32, 128, ... = 2728
3. S_6 for 6, 24, 96, ... = 8190

Answer Key (7 of 7)

Sums of Series (continued)

Sums of series (continued)

4. S_5 for 12, 24, 48, ... = 372
5. S_6 for 4, 12, 36, ... = 1456
6. S_5 for 2, 4, 8, ... = 62
7. S_7 for 9, 27, 81, ... = 9837
8. S_5 for 3, −2, −7, ... = −35
9. S_5 for 11, 33, 99, ... = 1331
10. S_5 for 3, 12, 48, ... = 1023
11. S_6 for 8, 32, 128, ... = 10920
12. S_5 for 1, 3, 9, ... = 121
13. S_5 for 7, 2, −3, ... = −15
14. S_7 for 8, 16, 32, ... = 1016
15. S_6 for 2, 4, 8, ... = 126
16. S_6 for 8, 16, 32, ... = 504
17. S_5 for 6, 24, 96, ... = 2046
18. S_5 for 5, −5, −15, ... = −75
19. S_5 for 11, 44, 176, ... = 3751
20. S_5 for 1, 4, 16, ... = 341
21. S_6 for 2, 8, 32, ... = 2730

Multiplying and dividing rational expressions

22. $(x - 5)/(x - 4) \div (x + 6)/(x - 4) = (x - 5) / (x + 6)$ $(x \neq -6, 4)$
23. $(x - 1)/(x + 2) \times (x + 2)/(x + 3) = (x - 1) / (x + 3)$ $(x \neq -3, -2)$
24. $(x + 8)/(x + 2) \div (x + 1)/(x + 2) = (x + 8) / (x + 1)$ $(x \neq -2, -1)$
25. $(x + 5)/(x + 6) \div (x + 2)/(x + 6) = (x + 5) / (x + 2)$ $(x \neq -6, -2)$
26. $(x + 3)/(x + 1) \div (x - 5)/(x + 1) = (x + 3) / (x - 5)$ $(x \neq -1, 5)$
27. $(x - 3)/(x - 7) \div (x + 4)/(x - 7) = (x - 3) / (x + 4)$ $(x \neq -4, 7)$
28. $(x - 7)/(x + 5) \div (x + 8)/(x + 5) = (x - 7) / (x + 8)$ $(x \neq -8, -5)$
29. $(x + 7)/(x - 2) \times (x - 2)/(x + 3) = (x + 7) / (x + 3)$ $(x \neq -3, 2)$
30. $(x - 7)/(x + 3) \times (x + 3)/(x + 1) = (x - 7) / (x + 1)$ $(x \neq -3, -1)$
31. $(x - 5)/(x + 7) \div (x + 3)/(x + 7) = (x - 5) / (x + 3)$ $(x \neq -7, -3)$

Check the Log Laws

Check the log laws

1. $\log(7x) - \log 7 = \log x$, not $\log x \div \log 7$ (the logs were divided because one was being taken from the other)
2. $\log(2x) - \log 2 = \log x$, which is what was written — this line is correct
3. $\log(13x) - \log 13 = \log x$, not $13 \log x$ (the multiplier inside the log was brought down as a power)
4. $\log(17x) - \log 17 = \log x$, not $\log x \div \log 17$ (the logs were divided because one was being taken from the other)
5. $\log(x^2) + \log(x^5) = 7 \log x$, which is what was written — this line is correct
6. $\log(14x) - \log 14 = \log x$, not $14 \log x$ (the multiplier inside the log was brought down as a power)
7. $\log(5x) - \log 5 = \log x$, which is what was written — this line is correct
8. $\log(23x) - \log 23 = \log x$, which is what was written — this line is correct

Solutions (1 of 14)

Simplifying Radicals

Simplifying radicals

1. $\sqrt{63}$

$\sqrt{63} = 3\sqrt{7}$

Split 63 into a perfect square times whatever is left: 63 = 9 × 7. 9 is 3 squared, and 7 has no square factor of its own — that last part is what makes this the LARGEST perfect square inside, and it is where the answer stops.

A root splits across a product, so the one root becomes two.

√9 is 3, so it comes out in front. √7 stays under the sign, because 7 has nothing more to give.

2. $\sqrt{126}$

$\sqrt{126} = 3\sqrt{14}$

Split 126 into a perfect square times whatever is left: 126 = 9 × 14. 9 is 3 squared, and 14 has no square factor of its own — that last part is what makes this the LARGEST perfect square inside, and it is where the answer stops.

A root splits across a product, so the one root becomes two.

√9 is 3, so it comes out in front. √14 stays under the sign, because 14 has nothing more to give.

Rational Exponents

Rational exponents

1. $4^{3/2}$ = _____

4^(3/2) = 8

The BOTTOM of the exponent says which root to take, so the square root of 4 comes first. Root before power, always — the other order works and makes the numbers enormous.

The TOP says what to raise it to. 2 to the power 3 is 8.

So 4^(3/2) is 8.

Solutions (2 of 14)

Rational Exponents (continued)

Rational exponents (continued)

2. $64^{5/3}$ = _____

64^(5/3) = 1024

The BOTTOM of the exponent says which root to take, so the cube root of 64 comes first. Root before power, always — the other order works and makes the numbers enormous.

The TOP says what to raise it to. 4 to the power 5 is 1024.

So 64^(5/3) is 1024.

Logarithms

Logarithms

1. $\log_4$ _____ = 2

$4^2 = 16$

A logarithm names a power. $\log_4$ of a number is 2 when that number IS the 2nd power of 4, so there is nothing to search for — just work the power out.

2. $\log_3 \frac{1}{9}$ = _____

$\log_3 1/9 = -2$

$\log_3$ 1/9 asks a question: which power of 3 lands on 1/9? It is not an instruction to divide by 3 — that is the mistake this notation collects.

1/9 is 1 over 9, and a negative exponent is what writes a reciprocal: 3^{-2} means $1 \div 3^2$. So the power that lands on 1/9 is the same one, turned over.

So $\log_3$ 1/9 is −2. A logarithm is negative exactly when the number is between 0 and 1 — nothing has gone wrong.

Solutions (3 of 14)

The Laws of Logarithms

Laws of logarithms

1. $\log_2 4 + \log_2 6 = \log_2$ ____

$\log_2 4 + \log_2 6$

$\log_2 m + \log_2 n = \log_2 (m \times n)$

$4 \times 6 = 24$

$\log_2 4 + \log_2 6 = \log_2 24$

Both logarithms are to the same base, 2. That is what makes a law available at all — two logarithms to different bases cannot be combined, and checking the bases match is the first thing to do rather than the last.

A SUM of two logarithms with the same base is the logarithm of the two arguments MULTIPLIED. Adding the logs multiplies the numbers — that is the law, and it is the one worth saying out loud, because the operation changes.

So the two arguments become one: $4 \times 6 = 24$. The answer is $\log_2 24$.

2. $\log_{10} 27 - \log_{10} 9 = \log_{10}$ ____

$\log_{10} 27 - \log_{10} 9$

$\log_{10} m - \log_{10} n = \log_{10} (m \div n)$

$27 \div 9 = 3$

$\log_{10} 27 - \log_{10} 9 = \log_{10} 3$

Both logarithms are to the same base, 10. That is what makes a law available at all — two logarithms to different bases cannot be combined, and checking the bases match is the first thing to do rather than the last.

A DIFFERENCE of two logarithms with the same base is the logarithm of the first argument DIVIDED by the second. Subtracting the logs divides the numbers, and the order matters: it is the first over the second, never the other way round.

So the two arguments become one: $27 \div 9 = 3$. The answer is $\log_{10} 3$.

Solutions (4 of 14)

Exponential Functions

Evaluate exponential functions

1. $f(x) = 2 \cdot 6^x$ **$f(1) =$** ___

$$f(1) = 2 \cdot 6^1$$
$$6^1 = 6$$
$$2 \cdot 6 = 12$$

Substitute $x = 1$: $f(1) = 2 \cdot 6^1$.

Do the power first: $6^1 = 6$.

Then multiply by the coefficient: $2 \cdot 6 = 12$.

2. $f(x) = 10 \cdot 5^x$ **$f(1) =$** ___

$$f(1) = 10 \cdot 5^1$$
$$5^1 = 5$$
$$10 \cdot 5 = 50$$

Substitute $x = 1$: $f(1) = 10 \cdot 5^1$.

Do the power first: $5^1 = 5$.

Then multiply by the coefficient: $10 \cdot 5 = 50$.

Completing the Square

Completing the square

1. $x^2 + 2x - 2$

$$x^2 + 2x - 2 = (x + 1)^2 - 3$$

Halve the middle coefficient. Half of 2 is 1, and that number goes straight into the bracket — sign and all.

But $(x + 1)^2$ is not the expression yet. Multiply it out and it brings a 1 along with it, and the original has -2 there instead.

So give the 1 back: $-2 - 1 = -3$. That is the number that goes after the bracket.

Solutions (5 of 14)

Completing the Square (continued)

Completing the square (continued)

2. $x^2 - 12x + 33$

$$x^2 - 12x + 33 = (x - 6)^2 - 3$$

Halve the middle coefficient. Half of −12 is −6, and that number goes straight into the bracket — sign and all.

But $(x - 6)^2$ is not the expression yet. Multiply it out and it brings a 36 along with it, and the original has 33 there instead.

So give the 36 back: 33 − 36 = −3. That is the number that goes after the bracket.

How Many Real Solutions

How many real solutions?

1. $x^2 + 22x + 149 = 0$

$= -112 < 0 \quad \rightarrow$ **none**

You do not have to solve this to answer it. One number decides how many real solutions a quadratic has, and it is $b^2 - 4ac$ — the part that ends up under the square root sign when you do solve it.

Read off a, b and c from $x^2 + 22x + 149 = 0$: a = 1, b = 22, c = 149. Write all three in, even the a — on this sheet it is 1, and forgetting it is the mistake that only shows up when it is not 1.

Work it out: 22^2 is 484, and 4 × 1 × 149 is 596. That leaves 484 − 596 = −112.

−112 is below zero, and no real number squares to a negative, so there is nothing to take the square root of. So there are NO real solutions.

Solutions (6 of 14)

How Many Real Solutions (continued)

How many real solutions? (continued)

2. $x^2 + 12x - 160 = 0$

$= 784 > 0 \quad \rightarrow$ **two**

You do not have to solve this to answer it. One number decides how many real solutions a quadratic has, and it is $b^2 - 4ac$ — the part that ends up under the square root sign when you do solve it.

Read off a, b and c from $x^2 + 12x - 160 = 0$: $a = 1$, $b = 12$, $c = -160$. Write all three in, even the a — on this sheet it is 1, and forgetting it is the mistake that only shows up when it is not 1.

Work it out: 12^2 is 144, and $4 \times 1 \times -160$ is -640. That leaves $144 - -640 = 784$.

784 is above zero, and a positive number has two square roots — one either way from the ±. So there are TWO real solutions.

Adding and Subtracting Complex Numbers

Complex numbers

1. $(2 + 5i) - (1 + 3i)$

$(2 + 5i) - (1 + 3i) = 1 + 2i$

Subtract the two REAL parts, and leave the i alone. The real parts are 2 and 1.

Now the two imaginary parts, the same way — this works because i is a number like any other here, so 5 of them minus 3 of them is what it sounds like.

Put the two parts back together: $1 + 2i$.

2. $(1 + i) - (3 + 2i)$

$(1 + i) - (3 + 2i) = -2 - i$

Subtract the two REAL parts, and leave the i alone. The real parts are 1 and 3.

Now the two imaginary parts, the same way — this works because i is a number like any other here, so 1 of them minus 2 of them is what it sounds like.

Put the two parts back together: $-2 - i$.

Solutions (7 of 14)

Adding, Subtracting and Multiplying Polynomials

Add, subtract and multiply polynomials

1. **$(4x^2 + 7x + 2) + (6x^2 + 8x + 5)$**

$10x^2 + 15x + 7$

Only matching terms combine: x^2 with x^2, x with x, numbers with numbers. Line the two brackets up by power before doing anything, and nothing can drift into the wrong column.

Adding brackets changes no signs at all, so the terms can be gathered as they stand.

Now take the columns one at a time, adding the rewritten signs. x^2: 4 + 6 = 10. x: 7 + 8 = 15. Numbers: 2 + 5 = 7.

So the answer is $10x^2 + 15x + 7$. Read it back and count the terms — three went in on each side, and three come out.

2. **$(3x^2 + 5x + 7) + (x^2 + 4x + 7)$**

$4x^2 + 9x + 14$

Only matching terms combine: x^2 with x^2, x with x, numbers with numbers. Line the two brackets up by power before doing anything, and nothing can drift into the wrong column.

Adding brackets changes no signs at all, so the terms can be gathered as they stand.

Now take the columns one at a time, adding the rewritten signs. x^2: 3 + 1 = 4. x: 5 + 4 = 9. Numbers: 7 + 7 = 14.

So the answer is $4x^2 + 9x + 14$. Read it back and count the terms — three went in on each side, and three come out.

Solutions (8 of 14)

Dividing Polynomials

Dividing polynomials

1.

$$\begin{array}{r} \quad x^2 - 6x + 0 \\ x - 1 \;\overline{)\; x^3 - 7x^2 + 6x + 9} \\ \underline{-x^3 + x^2} \\ \quad -6x^2 + 6x + 9 \\ \quad \underline{6x^2 - 6x} \\ \quad\quad\quad 9 \end{array}$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x - 1)$ is $x^3 - x^2$ — and take that away, leaving $-6x^2 + 6x + 9$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$-6x^2 \div x$ is $-6x$, so $-6x$ goes in the x place. Multiply the WHOLE divisor by it — $-6x(x - 1)$ is $-6x^2 + 6x$ — and take that away, leaving 9. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

9 has no x term, so a 0 goes in the constant place. Write it — leaving it out slides every coefficient after it up a place.

9 is left and there is no x in it to divide, so that is the remainder. $(x^3 - 7x^2 + 6x + 9) \div (x - 1) = x^2 - 6x$ remainder 9.

2.

$$\begin{array}{r} \quad x^2 + 2x - 2 \\ x - 1 \;\overline{)\; x^3 + x^2 - 4x + 3} \\ \underline{-x^3 + x^2} \\ \quad 2x^2 - 4x + 3 \\ \quad \underline{-2x^2 + 2x} \\ \quad\quad -2x + 3 \\ \quad\quad \underline{2x - 2} \\ \quad\quad\quad 1 \end{array}$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x - 1)$ is $x^3 - x^2$ — and take that away, leaving $2x^2 - 4x + 3$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$2x^2 \div x$ is $2x$, so $2x$ goes in the x place. Multiply the WHOLE divisor by it — $2x(x - 1)$ is $2x^2 - 2x$ — and take that away, leaving $-2x + 3$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$-2x \div x$ is -2, so -2 goes in the constant place. Multiply the WHOLE divisor by it — $-2(x - 1)$ is $-2x + 2$ — and take that away, leaving 1. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

1 is left and there is no x in it to divide, so that is the remainder. $(x^3 + x^2 - 4x + 3) \div (x - 1) = x^2 + 2x - 2$ remainder 1.

Solutions (9 of 14)

Simplifying Rational Expressions

Simplifying rational expressions

1. $\dfrac{x^2 + 5x - 66}{x^2 - 10x + 24}$

①/② $\dfrac{(x - 6)(x + 11)}{(x - 6)(x - 4)}$

$= \dfrac{(x + 11)}{(x - 4)}$

Nothing can cancel while these are sums. Factor the top first — two numbers that multiply to −66 and add to 5. Call it ①.

Now the bottom, the same way — call it ②. (x − 6) turns up again, and that is not a coincidence: it is the whole reason this expression simplifies.

The question is ① over ②. (x − 6) is a FACTOR of both, so it divides out and leaves 1. Only a factor can go this way — striking the x^2 off the top and the bottom of the sums above is the mistake this page exists to catch. And the cancelling changed something quietly: x may not be 4 or 6, because the original has no value there.

2. $\dfrac{x^2 + 3x + 2}{x^2 + 22x + 40}$

①/② $\dfrac{(x + 2)(x + 1)}{(x + 2)(x + 20)}$

$= \dfrac{(x + 1)}{(x + 20)}$

Nothing can cancel while these are sums. Factor the top first — two numbers that multiply to 2 and add to 3. Call it ①.

Now the bottom, the same way — call it ②. (x + 2) turns up again, and that is not a coincidence: it is the whole reason this expression simplifies.

The question is ① over ②. (x + 2) is a FACTOR of both, so it divides out and leaves 1. Only a factor can go this way — striking the x^2 off the top and the bottom of the sums above is the mistake this page exists to catch. And the cancelling changed something quietly: x may not be −20 or −2, because the original has no value there.

Solutions (10 of 14)

Multiplying and Dividing Rational Expressions

Multiplying and dividing rational expressions

1. $\frac{(x-1)}{(x-6)} \div \frac{(x-5)}{(x-6)}$

$(x-1)/(x-6) \div (x-5)/(x-6)$

$(x-1)/(x-6) \times (x-6)/(x-5)$

$(x-1)/(x-6) \times (x-6)/(x-5)$

$(x-6)$ cancels

$(x-1) / (x-5)$

$x \neq 5, 6$

Dividing by a fraction is multiplying by it upside down, and that has to happen FIRST. Turn $(x-5)/(x-6)$ over and change the $\div$ to a $\times$. Cancelling before the flip is cancelling across a division sign, which is not a move.

Now it is one product, so anything on top may cancel with anything underneath — the two fractions are no longer separate. $(x-6)$ appears in both, so it goes.

What is left is $(x-1) / (x-5)$.

One more thing, and it is the half that gets dropped: x may not be 5 or 6. Those are the numbers that made a denominator zero in the ORIGINAL — including $(x-6)$, which cancelled. A bracket does not stop mattering because it left the page.

2. $\frac{(x+3)}{(x+4)} \times \frac{(x+4)}{(x+7)}$

$(x+3)/(x+4) \times (x+4)/(x+7)$

$(x+4)$ cancels

$(x+3) / (x+7)$

$x \neq -7, -4$

Now it is one product, so anything on top may cancel with anything underneath — the two fractions are no longer separate. $(x+4)$ appears in both, so it goes.

What is left is $(x+3) / (x+7)$.

One more thing, and it is the half that gets dropped: x may not be -7 or -4. Those are the numbers that made a denominator zero in the ORIGINAL — including $(x+4)$, which cancelled. A bracket does not stop mattering because it left the page.

Solutions (11 of 14)

Solving Rational Equations

Solving rational equations

1. $\frac{6}{(x-7)} = 6$, x = ____

$x \neq 7$

$6/(x-7) = 6$

$6 = 6(x-7)$

$x = 8$

$x = 8$ and $x \neq 7$

Before anything else, note what x cannot be. (x – 7) is a denominator, so x may not be 7. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Now clear the fraction: multiply both sides by (x – 7). The left side is left with 6 and the right becomes 6((x – 7)).

Multiply out, gather the x terms on one side and the numbers on the other, and x = 8.

Last, check it against the first line: x may not be 7, and 8 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

2. $\frac{4}{(x-9)} = \frac{-12}{(x-1)}$, x = ____

$x \neq 1, 9$

$4/(x-9) = -12/(x-1)$

$4(x-1) = -12(x-9)$

$x = 7$

$x = 7$ and $x \neq 1, 9$

Before anything else, note what x cannot be. (x – 9) is a denominator, so x may not be 9, and the same for the other one: not 1. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Two fractions, one equals sign — so cross-multiply. Each numerator meets the OTHER denominator, which is the same "multiply both sides by both denominators" done in one move.

Multiply out, gather the x terms on one side and the numbers on the other, and x = 7.

Last, check it against the first line: x may not be 1 or 9, and 7 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

Solutions (12 of 14)

Arithmetic and Geometric Sequences

Arithmetic and geometric sequences

1. 2, 1, 0, −1, ... a_{17} = ____

$$d = -1$$

$$a_n = 2 + (n - 1) \times -1$$

$$a_{17} = 2 + 16 \times -1$$

$$2 + -16 = -14$$

$$a_{17} = -14$$

This is an arithmetic sequence: it changes by −1 each step (1 − 2 = −1).

The first term is already there, so to reach the 17th term you take the step only 16 times — that is (n − 1), not n.

16 steps of −1 is −16, and 2 + −16 = −14.

So the 17th term is −14.

2. 1, 5, 9, 13, ... a_{15} = ____

$$d = 4$$

$$a_n = 1 + (n - 1) \times 4$$

$$a_{15} = 1 + 14 \times 4$$

$$1 + 56 = 57$$

$$a_{15} = 57$$

This is an arithmetic sequence: it changes by 4 each step (5 − 1 = 4).

The first term is already there, so to reach the 15th term you take the step only 14 times — that is (n − 1), not n.

14 steps of 4 is 56, and 1 + 56 = 57.

So the 15th term is 57.

Solutions (13 of 14)

Sums of Series

Sums of series

1. **5, 10, 20, … S_6 = _____**

$S_6 = 5 \times 63 \div 1 = 315$

Each term is the one before times 2, so this is a geometric series and the sum is $a(r^n - 1) \div (r - 1)$. Here a is 5, r is 2 and n is 6 — substituting the wrong one of the three is the whole difficulty.

2 to the power 6 is 64, so the top is 5 × 63 and the bottom is 1.

2. **8, 32, 128, … S_5 = _____**

$S_5 = 8 \times 1023 \div 3 = 2728$

Each term is the one before times 4, so this is a geometric series and the sum is $a(r^n - 1) \div (r - 1)$. Here a is 8, r is 4 and n is 5 — substituting the wrong one of the three is the whole difficulty.

4 to the power 5 is 1024, so the top is 8 × 1023 and the bottom is 3.

Check the Log Laws

Check the log laws

1. **Ben wrote log(7x) − log 7 = log x ÷ log 7. Which is correct?**

log x | 7 log x | log x ÷ log 7 | log 7 − log x

Why?

log(7x) − log 7 = **log x**

they wrote: **log x ÷ log 7**

Do not check by reading their line. Somebody else's working is the most persuasive thing on the page, and rereading it is how you end up agreeing with it — do the step yourself first, then compare.

On your own, and writing the step nobody writes down: log(7x) − log 7 becomes log 7 + log x − log 7.

Tidy that up and it is log x.

Now compare. They wrote log x ÷ log 7, and the logs were divided because one was being taken from the other — so the correct answer is log x.

Solutions (14 of 14)

Check the Log Laws (continued)

Check the log laws (continued)

2. **Check this line: log(2x) − log 2 = log x. Which is correct?**

log x ÷ log 2 | log x | log 2 − log x | 2 log x

Why?

log(2x) − log 2 = **log x**

they wrote: **log x**

Do not check by reading their line. Somebody else's working is the most persuasive thing on the page, and rereading it is how you end up agreeing with it — do the step yourself first, then compare.

On your own, and writing the step nobody writes down: log(2x) − log 2 becomes log 2 + log x − log 2.

Tidy that up and it is log x.

Now compare — and this time the two agree. The line on the page was already right, which is a finding, not a missing mistake. Some of them are.