Algebra 2 Math Workbook

17 lessons · printable · answer key at the back

Name ______________________ Date ______________________

Contents

Simplifying Radicals

WORKED EXAMPLE

$\sqrt{350}$

$350 = 25 \times 14$

$= \sqrt{25} \times \sqrt{14}$

$= 5\sqrt{14}$

1. Simplifying radicals

Simplify each square root.

1 $\sqrt{63}$	2 $\sqrt{126}$	3 $\sqrt{360}$	4 $\sqrt{224}$
5 $\sqrt{56}$	6 $\sqrt{125}$	7 $\sqrt{8}$	8 $\sqrt{32}$
9 $\sqrt{486}$	10 $\sqrt{24}$	11 $\sqrt{72}$	12 $\sqrt{96}$
13 $\sqrt{12}$	14 $\sqrt{175}$	15 $\sqrt{135}$	16 $\sqrt{112}$
17 $\sqrt{504}$	18 $\sqrt{75}$	19 $\sqrt{52}$	20 $\sqrt{98}$
21 $\sqrt{90}$	22 $\sqrt{300}$	23 $\sqrt{44}$	24 $\sqrt{128}$
25 $\sqrt{48}$	26 $\sqrt{240}$	27 $\sqrt{539}$	28 $\sqrt{99}$

Cube Roots

WORKED EXAMPLE

$\sqrt[3]{270}$

$270 = 27 \times 10$
$= \sqrt[3]{27} \times \sqrt[3]{10}$
$= 3\sqrt[3]{10}$

2. Simplifying radicals

Simplify each cube root.

1 $\sqrt[3]{40}$	2 $\sqrt[3]{48}$	3 $\sqrt[3]{297}$	4 $\sqrt[3]{54}$
5 $\sqrt[3]{96}$	6 $\sqrt[3]{320}$	7 $\sqrt[3]{108}$	8 $\sqrt[3]{250}$
9 $\sqrt[3]{88}$	10 $\sqrt[3]{80}$	11 $\sqrt[3]{243}$	12 $\sqrt[3]{81}$
13 $\sqrt[3]{24}$	14 $\sqrt[3]{324}$	15 $\sqrt[3]{16}$	16 $\sqrt[3]{625}$
17 $\sqrt[3]{135}$	18 $\sqrt[3]{128}$	19 $\sqrt[3]{189}$	20 $\sqrt[3]{56}$
21 $\sqrt[3]{256}$	22 $\sqrt[3]{72}$	23 $\sqrt[3]{32}$	24 $\sqrt[3]{192}$

Rational Exponents

WORKED EXAMPLE

$216^{1/3}$ = 6

$\sqrt[3]{216} = 6$
$= \sqrt[3]{216}$
$= 6$

3. Rational exponents

Work out each one. Take the root first, then raise it.

1 $4^{3/2} =$ ______

2 $64^{5/3} =$ ______

3 $216^{2/3} =$ ______

4 $25^{1/2} =$ ______

5 $25^{5/2} =$ ______

6 $256^{1/4} =$ ______

7 $36^{3/2} =$ ______

8 $1000^{4/3} =$ ______

9 $9^{1/2} =$ ______

10 $100^{1/2} =$ ______

11 $27^{1/3} =$ ______

12 $8^{1/3} =$ ______

13 $16^{3/4} =$ ______

14 $9^{3/2} =$ ______

15 $729^{4/3} =$ ______

16 $16^{1/2} =$ ______

17 $256^{5/4} =$ ______

18 $27^{5/3} =$ ______

19 $4^{1/2} =$ ______

20 $81^{5/4} =$ ______

21 $36^{1/2} =$ ______

22 $6561^{1/4} =$ ______

23 $27^{2/3} =$ ______

24 $100^{3/2} =$ ______

25 $8^{4/3} =$ ______

26 $49^{1/2} =$ ______

27 $125^{1/3} =$ ______

28 $16^{5/2} =$ ______

2-1

Logarithms

WORKED EXAMPLE

$\log_2 4 = \underline{2}$

$2^2 = 4$

$= 2$

4. Logarithms

Work out each logarithm, or find the missing number.

1 $\log_7$ ________ $= 2$

2 $\log_2 \frac{1}{32} =$ ________

3 $\log_8$ ________ $= 3$

4 $\log_8 64 =$ ________

5 $\log_3 59049 =$ ________

6 $\log_3$ ________ $= 2$

7 $\log_4 256 =$ ________

8 $\log_6 \frac{1}{216} =$ ________

9 $\log_9$ ________ $= 2$

10 $\log_9$ ________ $= 4$

11 $\log_6$ ________ $= 3$

12 $\log_5$ ________ $= 7$

13 $\log_4$ ________ $= 3$

14 $\log_8 4096 =$ ________

15 $\log_{10}$ ________ $= 2$

16 $\log_9 729 =$ ________

17 $\log_{10} 100000 =$ ________

18 $\log_8 \frac{1}{512} =$ ________

19 $\log_2$ ________ $= 11$

20 $\log_6$ ________ $= 5$

21 $\log_5$ ________ $= 2$

22 $\log_6 36 =$ ________

23 $\log_{10}$ ________ $= 5$

24 $\log_5 125 =$ ________

25 $\log_2$ ________ $= 14$

26 $\log_5 \frac{1}{25} =$ ________

27 $\log_9 \frac{1}{6561} =$ ________

28 $\log_3 27 =$ ________

The Laws of Logarithms

WORKED EXAMPLE

$\log_5 2 + \log_5 12 = \log_5$ 24

$\log_5 m + \log_5 n = \log_5 (m \times n)$

$2 \times 12 = 24$

$= \log_5 24$

5. Laws of logarithms

Write each of these as one logarithm.

1 $\log_2 4 + \log_2 6 = \log_2$ _____

2 $\log_5 6 + \log_5 7 = \log_5$ _____

3 $\log_{10} 27 - \log_{10} 9 = \log_{10}$ _____

4 $\log_{10} 2 + \log_{10} 4 = \log_{10}$ _____

5 $2 \log_3 11 = \log_3$ _____

6 $\log_5 2 + \log_5 5 = \log_5$ _____

7 $\log_2 6 + \log_2 8 = \log_2$ _____

8 $\log_{10} 3 + \log_{10} 3 = \log_{10}$ _____

9 $2 \log_2 6 = \log_2$ _____

10 $\log_2 12 - \log_2 2 = \log_2$ _____

11 $2 \log_{10} 9 = \log_{10}$ _____

12 $\log_2 22 - \log_2 2 = \log_2$ _____

13 $\log_2 3 + \log_2 11 = \log_2$ _____

14 $\log_5 12 - \log_5 3 = \log_5$ _____

15 $\log_5 24 - \log_5 12 = \log_5$ _____

5. Laws of logarithms (continued)

Write each of these as one logarithm.

16 $\log_{10} 108 - \log_{10} 9 = \log_{10}$ ____

17 $\log_2 42 - \log_2 7 = \log_2$ ____

18 $\log_3 4 + \log_3 4 = \log_3$ ____

19 $\log_3 4 + \log_3 6 = \log_3$ ____

20 $\log_2 44 - \log_2 11 = \log_2$ ____

21 $\log_5 132 - \log_5 12 = \log_5$ ____

22 $\log_2 21 - \log_2 7 = \log_2$ ____

23 $\log_2 30 - \log_2 5 = \log_2$ ____

24 $\log_5 24 - \log_5 2 = \log_5$ ____

Exponential Functions

WORKED EXAMPLE

$f(x) = 12 \cdot 3^x$ **$f(2) =$** <u>108</u>

$f(2) = 12 \cdot 3^2$

$3^2 = 9$

$12 \cdot 9 = 108$

6. Evaluate exponential functions

Evaluate each exponential function at the value given.

1 $f(x) = 7 \cdot 5^x$ **$f(1) =$** ______

2 $f(x) = 7 \cdot 6^x$ **$f(1) =$** ______

3 $f(x) = 7 \cdot 4^x$ **$f(0) =$** ______

4 $f(x) = 3 \cdot 5^x$ **$f(1) =$** ______

5 $f(x) = 7 \cdot 2^x$ **$f(6) =$** ______

6 $f(x) = 5 \cdot 9^x$ **$f(0) =$** ______

7 $f(x) = 10 \cdot 7^x$ **$f(0) =$** ______

8 $f(x) = 8 \cdot 3^x$ **$f(1) =$** ______

9 $f(x) = 8 \cdot 7^x$ **$f(1) =$** ______

10 $f(x) = 2 \cdot 6^x$ **$f(0) =$** ______

11 $f(x) = 6 \cdot 4^x$ **$f(4) =$** ______

12 $f(x) = 4 \cdot 2^x$ **$f(3) =$** ______

13 $f(x) = 10 \cdot 8^x$ **$f(1) =$** ______

14 $f(x) = 10 \cdot 8^x$ **$f(0) =$** ______

15 $f(x) = 4 \cdot 2^x$ **$f(5) =$** ______

16 $f(x) = 3 \cdot 9^x$ **$f(1) =$** ______

17 $f(x) = 6 \cdot 8^x$ **$f(0) =$** ______

18 $f(x) = 3 \cdot 4^x$ **$f(1) =$** ______

19 $f(x) = 6 \cdot 3^x$ **$f(5) =$** ______

20 $f(x) = 6 \cdot 4^x$ **$f(2) =$** ______

3-1

Completing the Square

WORKED EXAMPLE

$x^2 - 4x - 36$

$-4 \div 2 = -2$

$(x - 2)^2 = x^2 - 4x + 4$

$= (x - 2)^2 - 40$

7. Completing the square

Write each expression as $(x + p)^2 + q$.

1 $x^2 + 8x + 11$

2 $x^2 - 16x + 48$

3 $x^2 + 6x + 37$

4 $x^2 - 2x - 8$

5 $x^2 + 22x + 129$

6 $x^2 + 12x + 16$

7 $x^2 - 4x - 12$

8 $x^2 - 4x - 21$

9 $x^2 + 14x + 42$

10 $x^2 + 24x + 139$

11 $x^2 + 2x + 29$

12 $x^2 + 10x + 9$

13 $x^2 + 2x - 2$

14 $x^2 + 20x + 110$

15 $x^2 + 2x + 9$

16 $x^2 - 16x + 66$

17 $x^2 + 22x + 121$

18 $x^2 - 6x + 15$

19 $x^2 + 16x + 62$

20 $x^2 - 2x - 1$

21 $x^2 + 24x + 168$

22 $x^2 - 2x - 4$

How Many Real Solutions

WORKED EXAMPLE

$x^2 + 3x + 157 = 0$

$a = 1, b = 3, c = 157$

$b^2 - 4ac = 3^2 - 4(1)(157)$

$= -619 < 0 \rightarrow$ none

8. How many real solutions?

How many real solutions does each equation have? Circle one.

1 $x^2 + 22x + 149 = 0$

2 $x^2 + 12x - 160 = 0$

3 $x^2 - 3x + 98 = 0$

4 $x^2 + 18x + 107 = 0$

5 $x^2 + 2x + 1 = 0$

6 $x^2 + 16x + 65 = 0$

7 $x^2 - 34x - 34 = 0$

8. How many real solutions? (continued)

How many real solutions does each equation have? Circle one.

8 $x^2 - 28x - 147 = 0$

9 $x^2 - 9x - 178 = 0$

10 $x^2 - 20x + 86 = 0$

11 $x^2 - 18x + 81 = 0$

12 $x^2 + 34x + 56 = 0$

13 $x^2 + 15x + 151 = 0$

14 $x^2 + 102 = 0$

15 $x^2 - 9x + 42 = 0$

16 $x^2 + 23x - 48 = 0$

8. How many real solutions? (continued)

How many real solutions does each equation have? Circle one.

17 $x^2 - 6x + 9 = 0$

18 $x^2 - 8x + 72 = 0$

19 $x^2 - 21x - 137 = 0$

20 $x^2 + 24x - 18 = 0$

21 $x^2 - 4x + 4 = 0$

22 $x^2 - 7x + 152 = 0$

3-3

Adding and Subtracting Complex Numbers

WORKED EXAMPLE

$(2 - 5i) - (-1 - 3i)$

$2 - (-1) = 3$

$-5 - (-3) = -2$

$= 3 - 2i$

9. Complex numbers

Add or subtract each pair of complex numbers.

1 $(4 - 3i) + (-2 - 3i)$

2 $(-5 + i) - (-4 + 5i)$

3 $(-2 + 5i) - (1 + 3i)$

4 $(-3 - 3i) - (5 - 6i)$

5 $(-1 + 3i) - (2 + 3i)$

6 $(-4 - 4i) + (5 + i)$

7 $(-1 + 6i) + (1 - 4i)$

8 $(-5 + 6i) + (-6 - i)$

9 $(1 + 3i) - (-5 + 6i)$

10 $(2 - 4i) + (5 + i)$

11 $(6 - i) + (4 - 5i)$

12 $(-1 - 4i) - (-2 + 4i)$

13 $(3 - 4i) - (5 + i)$

14 $(6 - 4i) + (5 + i)$

15 $(3 + 2i) - (-4 - 2i)$

16 $(-4 + 4i) + (2 - 4i)$

17 $(4 + 3i) + (2 + 3i)$

18 $(5 - 6i) - (-6 - 6i)$

19 $(6 - 3i) + (-2 - 3i)$

20 $(2 + 4i) + (-5 - i)$

21 $(-3 - 6i) + (6 + i)$

22 $(4 - i) + (4 - 5i)$

Multiplying Complex Numbers

WORKED EXAMPLE

$(-2 - 4i)(-2 + 4i)$
$= 4 - 8i + 8i - 16i^2$
$= 4 + 16 - 8i + 8i$
$= 20$

10. Complex numbers

Multiply each pair of complex numbers.

1 $(2 - 4i)(-2 + 4i)$

2 $(3 + 4i)(-5 - i)$

3 $(4 - 3i)(-2 - 3i)$

4 $(-6 + 6i)(-6 - i)$

5 $(1 + 3i)(2 + 3i)$

6 $(6 - 3i)(-2 - 3i)$

7 $(6 - 2i)(-3 + 5i)$

8 $(-2 + 2i)(-4 - 2i)$

9 $(-5 + 2i)(3 - 5i)$

10 $(-4 + i)(3 + 2i)$

11 $(-2 - 6i)(-6 - 6i)$

12 $(-3 - 4i)(5 + i)$

13 $(-4 + 2i)(-4 - 2i)$

14 $(-6 - 5i)(6 - 6i)$

15 $(4 - 4i)(5 + i)$

16 $(-2 + 2i)(3 - 5i)$

17 $(-5 - 5i)(6 - 6i)$

18 $(4 - 4i)(-2 + 4i)$

19 $(-1 - i)(-3 - 2i)$

20 $(-5 - 6i)(6 + i)$

Adding, Subtracting and Multiplying Polynomials

WORKED EXAMPLE

$(4x^2 + 8x + 2) + (3x^2 + 4x + 6)$

$4x^2 + 3x^2 = 7x^2$

$8x + 4x = 12x$

$2 + 6 = 8$

$7x^2 + 12x + 8$

11. Add, subtract and multiply polynomials

Work each one out.

1 $(4x^2 + 7x + 2) + (6x^2 + 8x + 5)$

2 $(7x^2 + 6x + 12) - (5x^2 - 3x + 7)$

3 $(3x^2 + 5x + 7) + (x^2 + 4x + 7)$

4 $(3x + 5)(x + 1)$

5 $(8x^2 + 5x + 10) - (x^2 - 9x + 6)$

6 $(6x^2 + 3x + 5) + (3x^2 + 9x + 6)$

7 $(4x + 3)(x + 2)$

8 $(6x^2 + 7x + 2) + (6x^2 + 8x + 7)$

9 $(6x^2 + 8x + 7) + (2x^2 + 5x + 4)$

10 $(x + 2)(x + 5)$

11 $(2x^2 + 6x + 9) + (2x^2 + 3x + 9)$

12 $(6x^2 + 7x + 12) - (4x^2 - 4x + 2)$

11. Add, subtract and multiply polynomials (continued)

Work each one out.

13 $(2x + 1)(x + 5)$

14 $(2x^2 + x + 6) + (6x^2 + 2x + 1)$

15 $(7x^2 + 4x + 11) - (x^2 - 8x + 3)$

16 $(4x + 2)(x + 3)$

17 $(9x^2 + x + 7) - (7x^2 - 4x + 6)$

18 $(3x + 5)(x + 3)$

Dividing Polynomials

WORKED EXAMPLE

$$\begin{array}{r}
x^2 \quad - x + 3 \\
x+1 \overline{)\, x^3 \qquad + 2x - 1} \\
\underline{-x^3 - x^2} \qquad\qquad \\
-x^2 + 2x - 1 \\
\underline{x^2 \quad + x} \quad\; \\
3x - 1 \\
\underline{-3x - 3} \\
-4
\end{array}$$

12. Dividing polynomials

Divide each polynomial. Write 0 for the remainder when it divides exactly.

1

$$\begin{array}{r}
x^2 + 3x - 2 \\
x+1 \overline{)\, x^3 + 4x^2 \quad + x - 2} \\
-x^3 \quad - x^2 \qquad\qquad \\
3x^2 \quad + x - 2 \\
-3x^2 - 3x \qquad \\
-2x - 2 \\
2x + 2 \\
0
\end{array}$$

2

$$\begin{array}{r}
x^2 \quad - 3x \quad + 2 \\
x-6 \overline{)\, x^3 - 9x^2 + 20x - 12} \\
-x^3 + 6x^2 \qquad\qquad \\
-3x^2 + 20x - 12 \\
3x^2 - 18x \qquad \\
2x - 12 \\
-2x + 12 \\
0
\end{array}$$

3

$$\begin{array}{r}
x^2 + 2x - 2 \\
x-1 \overline{)\, x^3 \quad + x^2 - 4x + 3} \\
-x^3 \quad + x^2 \qquad\qquad \\
2x^2 - 4x + 3 \\
-2x^2 + 2x \qquad \\
-2x + 3 \\
2x - 2 \\
1
\end{array}$$

4

$$\begin{array}{r}
x^2 \quad - 4x + 1 \\
x+6 \overline{)\, x^3 + 2x^2 - 23x + 6} \\
-x^3 - 6x^2 \qquad\qquad \\
-4x^2 - 23x + 6 \\
4x^2 + 24x \qquad \\
x + 6 \\
-x - 6 \\
0
\end{array}$$

5

$$\begin{array}{r}
x^2 \quad + 0 \quad - 3 \\
x-6 \overline{)\, x^3 - 6x^2 - 3x + 19} \\
-x^3 + 6x^2 \qquad\qquad \\
-3x + 19 \\
3x - 18 \\
1
\end{array}$$

6

$$\begin{array}{r}
x^2 \quad + 0 - 3 \\
x-3 \overline{)\, x^3 - 3x^2 - 3x \quad} \\
-x^3 + 3x^2 \qquad\qquad \\
-3x \quad \\
3x - 9 \\
-9
\end{array}$$

7

$$\begin{array}{r}
x^2 \quad + 5x \quad + 6 \\
x+4 \overline{)\, x^3 + 9x^2 + 26x + 20} \\
-x^3 - 4x^2 \qquad\qquad \\
5x^2 + 26x + 20 \\
-5x^2 - 20x \qquad \\
6x + 20 \\
-6x - 24 \\
-4
\end{array}$$

8

$$\begin{array}{r}
x^2 \quad + 0 \quad + 3 \\
x-6 \overline{)\, x^3 - 6x^2 + 3x \quad - 9} \\
-x^3 + 6x^2 \qquad\qquad \\
3x \quad - 9 \\
-3x + 18 \\
9
\end{array}$$

9

$$\begin{array}{r}
x^2 - 4x \quad - 5 \\
x+1 \overline{)\, x^3 - 3x^2 - 9x - 14} \\
-x^3 \quad - x^2 \qquad\qquad \\
-4x^2 - 9x - 14 \\
4x^2 + 4x \qquad \\
-5x - 14 \\
5x \quad + 5 \\
-9
\end{array}$$

10

$$\begin{array}{r}
x^2 \quad + 5x - 1 \\
x-5 \overline{)\, x^3 \qquad - 26x + 1} \\
-x^3 + 5x^2 \qquad\qquad \\
5x^2 - 26x + 1 \\
-5x^2 + 25x \qquad \\
-x + 1 \\
x - 5 \\
-4
\end{array}$$

Simplifying Rational Expressions

WORKED EXAMPLE

$$\frac{x^2 - 2x - 120}{x^2 - 5x - 150}$$

① $x^2 - 2x - 120 = (x + 10)(x - 12)$

② $x^2 - 5x - 150 = (x + 10)(x - 15)$

①/② $\dfrac{(x + 10)(x - 12)}{(x + 10)(x - 15)}$

$= \dfrac{(x - 12)}{(x - 15)}$

13. Simplifying rational expressions

Factor the top and the bottom, then cancel.

1 $\dfrac{x^2 + 5x - 66}{x^2 - 10x + 24}$

2 $\dfrac{x^2 - 22x + 112}{x^2 - 14x}$

3 $\dfrac{x^2 - 7x - 228}{x^2 - 36x + 323}$

4 $\dfrac{x^2 + 3x + 2}{x^2 + 22x + 40}$

5 $\dfrac{x^2 - 19x + 88}{x^2 - x - 110}$

6 $\dfrac{x^2 + 10x - 75}{x^2 + 29x + 210}$

7 $\dfrac{x^2 - 10x + 16}{x^2 - 8x}$

8 $\dfrac{x^2 + x - 30}{x^2 - 12x - 108}$

9 $\dfrac{x^2 - 2x - 323}{x^2 + 16x - 17}$

10 $\dfrac{x^2 - 18x + 72}{x^2 - 4x - 12}$

11 $\dfrac{x^2 - 6x - 112}{x^2 - 7x - 120}$

12 $\dfrac{x^2 + 14x - 51}{x^2 + 31x + 238}$

13 $\dfrac{x^2 + 21x + 80}{x^2 - 2x - 288}$

14 $\dfrac{x^2 + 8x - 33}{x^2 + 3x - 18}$

5-2

Multiplying and Dividing Rational Expressions

WORKED EXAMPLE

$$\frac{(x + 8)}{(x + 6)} \div \frac{(x - 7)}{(x + 6)}$$

(x + 8)/(x + 6) × (x + 6)/(x − 7)

(x + 6) cancels

(x + 8) / (x − 7)

$x \neq -6, 7$

14. Multiplying and dividing rational expressions

Multiply or divide, then cancel. Write each answer as one fraction.

1 $\frac{(x - 1)}{(x - 6)} \div \frac{(x - 5)}{(x - 6)}$

2 $\frac{(x + 8)}{(x - 3)} \div \frac{(x + 4)}{(x - 3)}$

3 $\frac{(x - 9)}{(x - 8)} \div \frac{(x - 6)}{(x - 8)}$

4 $\frac{(x + 3)}{(x + 4)} \times \frac{(x + 4)}{(x + 7)}$

5 $\frac{(x - 6)}{(x + 8)} \times \frac{(x + 8)}{(x + 2)}$

6 $\frac{(x - 6)}{(x + 9)} \times \frac{(x + 9)}{(x + 6)}$

7 $\frac{(x - 4)}{(x + 3)} \times \frac{(x + 3)}{(x - 8)}$

8 $\frac{(x - 8)}{(x - 9)} \times \frac{(x - 9)}{(x - 5)}$

9 $\frac{(x - 5)}{(x + 6)} \times \frac{(x + 6)}{(x - 1)}$

10 $\frac{(x - 5)}{(x + 4)} \div \frac{(x - 9)}{(x + 4)}$

11 $\frac{(x + 8)}{(x + 7)} \div \frac{(x - 3)}{(x + 7)}$

12 $\frac{(x - 7)}{(x - 4)} \times \frac{(x - 4)}{(x - 3)}$

13 $\frac{(x + 7)}{(x - 9)} \div \frac{(x - 1)}{(x - 9)}$

14 $\frac{(x + 4)}{(x - 7)} \div \frac{(x - 8)}{(x - 7)}$

Solving Rational Equations

WORKED EXAMPLE

$$\frac{5}{(x-7)} = \frac{-5}{(x-3)},\ x = \underline{5}$$

$x \neq 3, 7$

$5(x-3) = -5(x-7)$

$x = 5$

$x = 5$ and $x \neq 3, 7$

15. Solving rational equations

Solve each equation for x.

1 $\frac{11}{(x + 4)} = \frac{8}{(x + 1)}$, x = _____

2 $\frac{16}{(x - 4)} = \frac{-8}{(x - 7)}$, x = _____

3 $\frac{28}{(x + 2)} = \frac{64}{(x - 7)}$, x = _____

4 $\frac{3}{(x - 3)} = \frac{-18}{(x + 4)}$, x = _____

5 $\frac{4}{x} = 2$, x = _____

6 $\frac{-54}{(x + 3)} = 9$, x = _____

7 $\frac{32}{(x + 8)} = 8$, x = _____

8 $\frac{44}{(x + 6)} = 4$, x = _____

9 $\frac{7}{(x + 6)} = \frac{-14}{(x - 3)}$, x = _____

10 $\frac{80}{(x + 9)} = \frac{24}{(x + 2)}$, x = _____

11 $\frac{18}{(x + 2)} = 9$, x = _____

12 $\frac{48}{(x + 9)} = 6$, x = _____

13 $\frac{24}{(x - 2)} = \frac{33}{(x - 5)}$, x = _____

14 $\frac{72}{(x + 4)} = 8$, x = _____

15 $\frac{4}{x} = \frac{12}{(x + 4)}$, x = _____

16 $\frac{-64}{(x + 1)} = 8$, x = _____

Arithmetic and Geometric Sequences

WORKED EXAMPLE

$-4, -3, -2, -1, \ldots$ $a_{10} =$ 5

$d = 1$

$a_n = -4 + (n - 1) \times 1$

$a_{10} = -4 + 9 \times 1$

$-4 + 9 = 5$

$a_{10} = 5$

16. Arithmetic and geometric sequences

Find the term shown for each sequence.

1 $5, 9, 13, 17, \ldots$ $a_{17} =$ ______

2 $2, 1, 0, -1, \ldots$ $a_{17} =$ ______

3 $5, 10, 15, 20, \ldots$ $a_{24} =$ ______

4 $12, 9, 6, 3, \ldots$ $a_{20} =$ ______

5 $-7, -16, -25, -34, \ldots$ $a_{22} =$ ______

6 $1, 5, 9, 13, \ldots$ $a_{15} =$ ______

7 $8, 1, -6, -13, \ldots$ $a_{15} =$ ______

8 $10, 6, 2, -2, \ldots$ $a_{24} =$ ______

9 $11, 14, 17, 20, \ldots$ $a_{25} =$ ______

16. Arithmetic and geometric sequences (continued)

Find the term shown for each sequence.

10 $-4, -6, -8, -10, \ldots$ $a_{19} =$ ______

11 $0, 9, 18, 27, \ldots$ $a_{19} =$ ______

12 $12, 19, 26, 33, \ldots$ $a_{12} =$ ______

13 $-2, 6, 14, 22, \ldots$ $a_{23} =$ ______

14 $-1, -10, -19, -28, \ldots$ $a_{14} =$ ______

15 $4, -5, -14, -23, \ldots$ $a_{22} =$ ______

16 $-7, -2, 3, 8, \ldots$ $a_{18} =$ ______

17 $4, 10, 16, 22, \ldots$ $a_{24} =$ ______

18 $5, 8, 11, 14, \ldots$ $a_{10} =$ ______

19 $6, 10, 14, 18, \ldots$ $a_{23} =$ ______

20 $-3, -1, 1, 3, \ldots$ $a_{22} =$ ______

21 $12, 14, 16, 18, \ldots$ $a_{24} =$ ______

22 $1, -5, -11, -17, \ldots$ $a_{23} =$ ______

Sums of Series

WORKED EXAMPLE

5, 10, 20, ... S_5 = 155

$S_5 = 5(2^5 - 1) \div (2 - 1)$
$S_5 = 5 \times 31 \div 1 = 155$

17. Sums of series

S_n is the sum of the first n terms. Does each sequence add or multiply?

1 6, 24, 96, ... S_6 = ________

2 5, 10, 20, ... S_6 = ________

3 2, 11, 20, ... S_8 = ________

4 9, 27, 81, ... S_7 = ________

5 7, 11, 15, ... S_8 = ________

6 8, 32, 128, ... S_6 = ________

7 9, 12, 15, ... S_9 = ________

8 8, 32, 128, ... S_5 = ________

9 1, 4, 16, ... S_7 = ________

10 8, 16, 32, ... S_7 = ________

11 11, 1, −9, ... S_9 = ________

12 11, 17, 23, ... S_9 = ________

13 5, −5, −15, ... S_5 = ________

14 2, −1, −4, ... S_{10} = ________

15 12, 24, 48, ... S_5 = ________

16 9, 5, 1, ... S_7 = ________

Answer Key (1 of 5)

Simplifying Radicals

Simplifying radicals

1. $\sqrt{63} = 3\sqrt{7}$
2. $\sqrt{126} = 3\sqrt{14}$
3. $\sqrt{360} = 6\sqrt{10}$
4. $\sqrt{224} = 4\sqrt{14}$
5. $\sqrt{56} = 2\sqrt{14}$
6. $\sqrt{125} = 5\sqrt{5}$
7. $\sqrt{8} = 2\sqrt{2}$
8. $\sqrt{32} = 4\sqrt{2}$
9. $\sqrt{486} = 9\sqrt{6}$
10. $\sqrt{24} = 2\sqrt{6}$
11. $\sqrt{72} = 6\sqrt{2}$
12. $\sqrt{96} = 4\sqrt{6}$
13. $\sqrt{12} = 2\sqrt{3}$
14. $\sqrt{175} = 5\sqrt{7}$
15. $\sqrt{135} = 3\sqrt{15}$
16. $\sqrt{112} = 4\sqrt{7}$
17. $\sqrt{504} = 6\sqrt{14}$
18. $\sqrt{75} = 5\sqrt{3}$
19. $\sqrt{52} = 2\sqrt{13}$
20. $\sqrt{98} = 7\sqrt{2}$
21. $\sqrt{90} = 3\sqrt{10}$
22. $\sqrt{300} = 10\sqrt{3}$
23. $\sqrt{44} = 2\sqrt{11}$
24. $\sqrt{128} = 8\sqrt{2}$
25. $\sqrt{48} = 4\sqrt{3}$
26. $\sqrt{240} = 4\sqrt{15}$
27. $\sqrt{539} = 7\sqrt{11}$
28. $\sqrt{99} = 3\sqrt{11}$

Cube Roots

Simplifying radicals

1. $\sqrt[3]{40} = 2\sqrt[3]{5}$
2. $\sqrt[3]{48} = 2\sqrt[3]{6}$
3. $\sqrt[3]{297} = 3\sqrt[3]{11}$
4. $\sqrt[3]{54} = 3\sqrt[3]{2}$
5. $\sqrt[3]{96} = 2\sqrt[3]{12}$
6. $\sqrt[3]{320} = 4\sqrt[3]{5}$
7. $\sqrt[3]{108} = 3\sqrt[3]{4}$
8. $\sqrt[3]{250} = 5\sqrt[3]{2}$
9. $\sqrt[3]{88} = 2\sqrt[3]{11}$
10. $\sqrt[3]{80} = 2\sqrt[3]{10}$
11. $\sqrt[3]{243} = 3\sqrt[3]{9}$
12. $\sqrt[3]{81} = 3\sqrt[3]{3}$
13. $\sqrt[3]{24} = 2\sqrt[3]{3}$
14. $\sqrt[3]{324} = 3\sqrt[3]{12}$
15. $\sqrt[3]{16} = 2\sqrt[3]{2}$
16. $\sqrt[3]{625} = 5\sqrt[3]{5}$
17. $\sqrt[3]{135} = 3\sqrt[3]{5}$

Cube Roots (continued)

Simplifying radicals (continued)

18. $\sqrt[3]{128} = 4\sqrt[3]{2}$
19. $\sqrt[3]{189} = 3\sqrt[3]{7}$
20. $\sqrt[3]{56} = 2\sqrt[3]{7}$
21. $\sqrt[3]{256} = 4\sqrt[3]{4}$
22. $\sqrt[3]{72} = 2\sqrt[3]{9}$
23. $\sqrt[3]{32} = 2\sqrt[3]{4}$
24. $\sqrt[3]{192} = 4\sqrt[3]{3}$

Rational Exponents

Rational exponents

1. 4^(3/2) = 8
2. 64^(5/3) = 1024
3. 216^(2/3) = 36
4. 25^(1/2) = 5
5. 25^(5/2) = 3125
6. 256^(1/4) = 4
7. 36^(3/2) = 216
8. 1000^(4/3) = 10000
9. 9^(1/2) = 3
10. 100^(1/2) = 10
11. 27^(1/3) = 3
12. 8^(1/3) = 2
13. 16^(3/4) = 8
14. 9^(3/2) = 27
15. 729^(4/3) = 6561
16. 16^(1/2) = 4
17. 256^(5/4) = 1024
18. 27^(5/3) = 243
19. 4^(1/2) = 2
20. 81^(5/4) = 243
21. 36^(1/2) = 6
22. 6561^(1/4) = 9
23. 27^(2/3) = 9
24. 100^(3/2) = 1000
25. 8^(4/3) = 16
26. 49^(1/2) = 7
27. 125^(1/3) = 5
28. 16^(5/2) = 1024

Logarithms

Logarithms

1. $\log_7 49 = 2$
2. $\log_2 1/32 = -5$
3. $\log_8 512 = 3$
4. $\log_8 64 = 2$
5. $\log_3 59049 = 10$
6. $\log_3 9 = 2$
7. $\log_4 256 = 4$

Logarithms (continued)

Logarithms (continued)

8. $\log_6 1/216 = -3$
9. $\log_9 81 = 2$
10. $\log_9 6561 = 4$
11. $\log_6 216 = 3$
12. $\log_5 78125 = 7$
13. $\log_4 64 = 3$
14. $\log_8 4096 = 4$
15. $\log_{10} 100 = 2$
16. $\log_9 729 = 3$
17. $\log_{10} 100000 = 5$
18. $\log_8 1/512 = -3$
19. $\log_2 2048 = 11$
20. $\log_6 7776 = 5$
21. $\log_5 25 = 2$
22. $\log_6 36 = 2$
23. $\log_{10} 100000 = 5$
24. $\log_5 125 = 3$
25. $\log_2 16384 = 14$
26. $\log_5 1/25 = -2$
27. $\log_9 1/6561 = -4$
28. $\log_3 27 = 3$

The Laws of Logarithms

Laws of logarithms

1. $\log_2 4 + \log_2 6 = \log_2 24$
2. $\log_5 6 + \log_5 7 = \log_5 42$
3. $\log_{10} 27 - \log_{10} 9 = \log_{10} 3$
4. $\log_{10} 2 + \log_{10} 4 = \log_{10} 8$
5. $2 \log_3 11 = \log_3 121$
6. $\log_5 2 + \log_5 5 = \log_5 10$
7. $\log_2 6 + \log_2 8 = \log_2 48$
8. $\log_{10} 3 + \log_{10} 3 = \log_{10} 9$
9. $2 \log_2 6 = \log_2 36$
10. $\log_2 12 - \log_2 2 = \log_2 6$
11. $2 \log_{10} 9 = \log_{10} 81$
12. $\log_2 22 - \log_2 2 = \log_2 11$
13. $\log_2 3 + \log_2 11 = \log_2 33$
14. $\log_5 12 - \log_5 3 = \log_5 4$
15. $\log_5 24 - \log_5 12 = \log_5 2$
16. $\log_{10} 108 - \log_{10} 9 = \log_{10} 12$
17. $\log_2 42 - \log_2 7 = \log_2 6$
18. $\log_3 4 + \log_3 4 = \log_3 16$
19. $\log_3 4 + \log_3 6 = \log_3 24$
20. $\log_2 44 - \log_2 11 = \log_2 4$, and that is 2
21. $\log_5 132 - \log_5 12 = \log_5 11$
22. $\log_2 21 - \log_2 7 = \log_2 3$
23. $\log_2 30 - \log_2 5 = \log_2 6$
24. $\log_5 24 - \log_5 2 = \log_5 12$

Exponential Functions

Evaluate exponential functions

1. $f(x) = 7 \cdot 5^x \rightarrow f(1) = 35$
2. $f(x) = 7 \cdot 6^x \rightarrow f(1) = 42$
3. $f(x) = 7 \cdot 4^x \rightarrow f(0) = 7$
4. $f(x) = 3 \cdot 5^x \rightarrow f(1) = 15$
5. $f(x) = 7 \cdot 2^x \rightarrow f(6) = 448$
6. $f(x) = 5 \cdot 9^x \rightarrow f(0) = 5$
7. $f(x) = 10 \cdot 7^x \rightarrow f(0) = 10$
8. $f(x) = 8 \cdot 3^x \rightarrow f(1) = 24$
9. $f(x) = 8 \cdot 7^x \rightarrow f(1) = 56$
10. $f(x) = 2 \cdot 6^x \rightarrow f(0) = 2$
11. $f(x) = 6 \cdot 4^x \rightarrow f(4) = 1536$
12. $f(x) = 4 \cdot 2^x \rightarrow f(3) = 32$
13. $f(x) = 10 \cdot 8^x \rightarrow f(1) = 80$
14. $f(x) = 10 \cdot 8^x \rightarrow f(0) = 10$
15. $f(x) = 4 \cdot 2^x \rightarrow f(5) = 128$
16. $f(x) = 3 \cdot 9^x \rightarrow f(1) = 27$
17. $f(x) = 6 \cdot 8^x \rightarrow f(0) = 6$
18. $f(x) = 3 \cdot 4^x \rightarrow f(1) = 12$
19. $f(x) = 6 \cdot 3^x \rightarrow f(5) = 1458$
20. $f(x) = 6 \cdot 4^x \rightarrow f(2) = 96$

Completing the Square

Completing the square

1. $x^2 + 8x + 11 = (x + 4)^2 - 5$
2. $x^2 - 16x + 48 = (x - 8)^2 - 16$
3. $x^2 + 6x + 37 = (x + 3)^2 + 28$
4. $x^2 - 2x - 8 = (x - 1)^2 - 9$
5. $x^2 + 22x + 129 = (x + 11)^2 + 8$
6. $x^2 + 12x + 16 = (x + 6)^2 - 20$
7. $x^2 - 4x - 12 = (x - 2)^2 - 16$
8. $x^2 - 4x - 21 = (x - 2)^2 - 25$
9. $x^2 + 14x + 42 = (x + 7)^2 - 7$
10. $x^2 + 24x + 139 = (x + 12)^2 - 5$
11. $x^2 + 2x + 29 = (x + 1)^2 + 28$
12. $x^2 + 10x + 9 = (x + 5)^2 - 16$
13. $x^2 + 2x - 2 = (x + 1)^2 - 3$
14. $x^2 + 20x + 110 = (x + 10)^2 + 10$
15. $x^2 + 2x + 9 = (x + 1)^2 + 8$
16. $x^2 - 16x + 66 = (x - 8)^2 + 2$
17. $x^2 + 22x + 121 = (x + 11)^2$
18. $x^2 - 6x + 15 = (x - 3)^2 + 6$
19. $x^2 + 16x + 62 = (x + 8)^2 - 2$
20. $x^2 - 2x - 1 = (x - 1)^2 - 2$
21. $x^2 + 24x + 168 = (x + 12)^2 + 24$
22. $x^2 - 2x - 4 = (x - 1)^2 - 5$

Answer Key (3 of 5)

How Many Real Solutions

How many real solutions?

1. $x^2 + 22x + 149 = 0$: $22^2 - 4 \times 1 \times 149 = 484 - 596 = -112$ → None
2. $x^2 + 12x - 160 = 0$: $12^2 - 4 \times 1 \times -160 = 144 - -640 = 784$ → Two
3. $x^2 - 3x + 98 = 0$: $-3^2 - 4 \times 1 \times 98 = 9 - 392 = -383$ → None
4. $x^2 + 18x + 107 = 0$: $18^2 - 4 \times 1 \times 107 = 324 - 428 = -104$ → None
5. $x^2 + 2x + 1 = 0$: $2^2 - 4 \times 1 \times 1 = 4 - 4 = 0$ → One
6. $x^2 + 16x + 65 = 0$: $16^2 - 4 \times 1 \times 65 = 256 - 260 = -4$ → None
7. $x^2 - 34x - 34 = 0$: $-34^2 - 4 \times 1 \times -34 = 1156 - -136 = 1292$ → Two
8. $x^2 - 28x - 147 = 0$: $-28^2 - 4 \times 1 \times -147 = 784 - -588 = 1372$ → Two
9. $x^2 - 9x - 178 = 0$: $-9^2 - 4 \times 1 \times -178 = 81 - -712 = 793$ → Two
10. $x^2 - 20x + 86 = 0$: $-20^2 - 4 \times 1 \times 86 = 400 - 344 = 56$ → Two
11. $x^2 - 18x + 81 = 0$: $-18^2 - 4 \times 1 \times 81 = 324 - 324 = 0$ → One
12. $x^2 + 34x + 56 = 0$: $34^2 - 4 \times 1 \times 56 = 1156 - 224 = 932$ → Two
13. $x^2 + 15x + 151 = 0$: $15^2 - 4 \times 1 \times 151 = 225 - 604 = -379$ → None
14. $x^2 + 102 = 0$: $0^2 - 4 \times 1 \times 102 = 0 - 408 = -408$ → None
15. $x^2 - 9x + 42 = 0$: $-9^2 - 4 \times 1 \times 42 = 81 - 168 = -87$ → None
16. $x^2 + 23x - 48 = 0$: $23^2 - 4 \times 1 \times -48 = 529 - -192 = 721$ → Two
17. $x^2 - 6x + 9 = 0$: $-6^2 - 4 \times 1 \times 9 = 36 - 36 = 0$ → One
18. $x^2 - 8x + 72 = 0$: $-8^2 - 4 \times 1 \times 72 = 64 - 288 = -224$ → None
19. $x^2 - 21x - 137 = 0$: $-21^2 - 4 \times 1 \times -137 = 441 - -548 = 989$ → Two
20. $x^2 + 24x - 18 = 0$: $24^2 - 4 \times 1 \times -18 = 576 - -72 = 648$ → Two
21. $x^2 - 4x + 4 = 0$: $-4^2 - 4 \times 1 \times 4 = 16 - 16 = 0$ → One
22. $x^2 - 7x + 152 = 0$: $-7^2 - 4 \times 1 \times 152 = 49 - 608 = -559$ → None

Adding and Subtracting Complex Numbers

Complex numbers

1. $(4 - 3i) + (-2 - 3i) = 2 - 6i$
2. $(-5 + i) - (-4 + 5i) = -1 - 4i$
3. $(-2 + 5i) - (1 + 3i) = -3 + 2i$
4. $(-3 - 3i) - (5 - 6i) = -8 + 3i$
5. $(-1 + 3i) - (2 + 3i) = -3$
6. $(-4 - 4i) + (5 + i) = 1 - 3i$
7. $(-1 + 6i) + (1 - 4i) = 2i$
8. $(-5 + 6i) + (-6 - i) = -11 + 5i$
9. $(1 + 3i) - (-5 + 6i) = 6 - 3i$
10. $(2 - 4i) + (5 + i) = 7 - 3i$
11. $(6 - i) + (4 - 5i) = 10 - 6i$

Adding and Subtracting Complex Numbers (continued)

Complex numbers (continued)

12. $(-1 - 4i) - (-2 + 4i) = 1 - 8i$
13. $(3 - 4i) - (5 + i) = -2 - 5i$
14. $(6 - 4i) + (5 + i) = 11 - 3i$
15. $(3 + 2i) - (-4 - 2i) = 7 + 4i$
16. $(-4 + 4i) + (2 - 4i) = -2$
17. $(4 + 3i) + (2 + 3i) = 6 + 6i$
18. $(5 - 6i) - (-6 - 6i) = 11$
19. $(6 - 3i) + (-2 - 3i) = 4 - 6i$
20. $(2 + 4i) + (-5 - i) = -3 + 3i$
21. $(-3 - 6i) + (6 + i) = 3 - 5i$
22. $(4 - i) + (4 - 5i) = 8 - 6i$

Multiplying Complex Numbers

Complex numbers

1. $(2 - 4i)(-2 + 4i) = 12 + 16i$
2. $(3 + 4i)(-5 - i) = -11 - 23i$
3. $(4 - 3i)(-2 - 3i) = -17 - 6i$
4. $(-6 + 6i)(-6 - i) = 42 - 30i$
5. $(1 + 3i)(2 + 3i) = -7 + 9i$
6. $(6 - 3i)(-2 - 3i) = -21 - 12i$
7. $(6 - 2i)(-3 + 5i) = -8 + 36i$
8. $(-2 + 2i)(-4 - 2i) = 12 - 4i$
9. $(-5 + 2i)(3 - 5i) = -5 + 31i$
10. $(-4 + i)(3 + 2i) = -14 - 5i$
11. $(-2 - 6i)(-6 - 6i) = -24 + 48i$
12. $(-3 - 4i)(5 + i) = -11 - 23i$
13. $(-4 + 2i)(-4 - 2i) = 20$
14. $(-6 - 5i)(6 - 6i) = -66 + 6i$
15. $(4 - 4i)(5 + i) = 24 - 16i$
16. $(-2 + 2i)(3 - 5i) = 4 + 16i$
17. $(-5 - 5i)(6 - 6i) = -60$
18. $(4 - 4i)(-2 + 4i) = 8 + 24i$
19. $(-1 - i)(-3 - 2i) = 1 + 5i$
20. $(-5 - 6i)(6 + i) = -24 - 41i$

Adding, Subtracting and Multiplying Polynomials

Add, subtract and multiply polynomials

1. $(4x^2 + 7x + 2) + (6x^2 + 8x + 5)$ → $10x^2 + 15x + 7$
2. $(7x^2 + 6x + 12) - (5x^2 - 3x + 7)$ → $2x^2 + 9x + 5$
3. $(3x^2 + 5x + 7) + (x^2 + 4x + 7)$ → $4x^2 + 9x + 14$
4. $(3x + 5)(x + 1)$ → $3x^2 + 8x + 5$
5. $(8x^2 + 5x + 10) - (x^2 - 9x + 6)$ → $7x^2 + 14x + 4$
6. $(6x^2 + 3x + 5) + (3x^2 + 9x + 6)$ → $9x^2 + 12x + 11$
7. $(4x + 3)(x + 2)$ → $4x^2 + 11x + 6$
8. $(6x^2 + 7x + 2) + (6x^2 + 8x + 7)$ → $12x^2 + 15x + 9$
9. $(6x^2 + 8x + 7) + (2x^2 + 5x + 4)$ → $8x^2 + 13x + 11$
10. $(x + 2)(x + 5)$ → $1x^2 + 7x + 10$
11. $(2x^2 + 6x + 9) + (2x^2 + 3x + 9)$ → $4x^2 + 9x + 18$

Answer Key (4 of 5)

Adding, Subtracting and Multiplying Polynomials (continued)

Add, subtract and multiply polynomials (continued)

12. $(6x^2 + 7x + 12) - (4x^2 - 4x + 2) \rightarrow 2x^2 + 11x + 10$
13. $(2x + 1)(x + 5) \rightarrow 2x^2 + 11x + 5$
14. $(2x^2 + x + 6) + (6x^2 + 2x + 1) \rightarrow 8x^2 + 3x + 7$
15. $(7x^2 + 4x + 11) - (x^2 - 8x + 3) \rightarrow 6x^2 + 12x + 8$
16. $(4x + 2)(x + 3) \rightarrow 4x^2 + 14x + 6$
17. $(9x^2 + x + 7) - (7x^2 - 4x + 6) \rightarrow 2x^2 + 5x + 1$
18. $(3x + 5)(x + 3) \rightarrow 3x^2 + 14x + 15$

Dividing Polynomials

Dividing polynomials

1. $(x^3 + 4x^2 + x - 2) \div (x + 1) = x^2 + 3x - 2$
2. $(x^3 - 9x^2 + 20x - 12) \div (x - 6) = x^2 - 3x + 2$
3. $(x^3 + x^2 - 4x + 3) \div (x - 1) = x^2 + 2x - 2$ remainder 1
4. $(x^3 + 2x^2 - 23x + 6) \div (x + 6) = x^2 - 4x + 1$
5. $(x^3 - 6x^2 - 3x + 19) \div (x - 6) = x^2 - 3$ remainder 1
6. $(x^3 - 3x^2 - 3x) \div (x - 3) = x^2 - 3$ remainder -9
7. $(x^3 + 9x^2 + 26x + 20) \div (x + 4) = x^2 + 5x + 6$ remainder -4
8. $(x^3 - 6x^2 + 3x - 9) \div (x - 6) = x^2 + 3$ remainder 9
9. $(x^3 - 3x^2 - 9x - 14) \div (x + 1) = x^2 - 4x - 5$ remainder -9
10. $(x^3 - 26x + 1) \div (x - 5) = x^2 + 5x - 1$ remainder -4

Simplifying Rational Expressions

Simplifying rational expressions

1. $(x^2 + 5x - 66) / (x^2 - 10x + 24) = (x + 11) / (x - 4)$
2. $(x^2 - 22x + 112) / (x^2 - 14x) = (x - 8) / x$
3. $(x^2 - 7x - 228) / (x^2 - 36x + 323) = (x + 12) / (x - 17)$
4. $(x^2 + 3x + 2) / (x^2 + 22x + 40) = (x + 1) / (x + 20)$
5. $(x^2 - 19x + 88) / (x^2 - x - 110) = (x - 8) / (x + 10)$
6. $(x^2 + 10x - 75) / (x^2 + 29x + 210) = (x - 5) / (x + 14)$
7. $(x^2 - 10x + 16) / (x^2 - 8x) = (x - 2) / x$
8. $(x^2 + x - 30) / (x^2 - 12x - 108) = (x - 5) / (x - 18)$
9. $(x^2 - 2x - 323) / (x^2 + 16x - 17) = (x - 19) / (x - 1)$
10. $(x^2 - 18x + 72) / (x^2 - 4x - 12) = (x - 12) / (x + 2)$
11. $(x^2 - 6x - 112) / (x^2 - 7x - 120) = (x - 14) / (x - 15)$
12. $(x^2 + 14x - 51) / (x^2 + 31x + 238) = (x - 3) / (x + 14)$
13. $(x^2 + 21x + 80) / (x^2 - 2x - 288) = (x + 5) / (x - 18)$
14. $(x^2 + 8x - 33) / (x^2 + 3x - 18) = (x + 11) / (x + 6)$

Multiplying and Dividing Rational Expressions

Multiplying and dividing rational expressions

1. $(x - 1)/(x - 6) \div (x - 5)/(x - 6) = (x - 1) / (x - 5)$ $(x \neq 5, 6)$
2. $(x + 8)/(x - 3) \div (x + 4)/(x - 3) = (x + 8) / (x + 4)$ $(x \neq -4, 3)$
3. $(x - 9)/(x - 8) \div (x - 6)/(x - 8) = (x - 9) / (x - 6)$ $(x \neq 6, 8)$
4. $(x + 3)/(x + 4) \times (x + 4)/(x + 7) = (x + 3) / (x + 7)$ $(x \neq -7, -4)$
5. $(x - 6)/(x + 8) \times (x + 8)/(x + 2) = (x - 6) / (x + 2)$ $(x \neq -8, -2)$
6. $(x - 6)/(x + 9) \times (x + 9)/(x + 6) = (x - 6) / (x + 6)$ $(x \neq -9, -6)$
7. $(x - 4)/(x + 3) \times (x + 3)/(x - 8) = (x - 4) / (x - 8)$ $(x \neq -3, 8)$
8. $(x - 8)/(x - 9) \times (x - 9)/(x - 5) = (x - 8) / (x - 5)$ $(x \neq 5, 9)$

Multiplying and Dividing Rational Expressions (continued)

Multiplying and dividing rational expressions (continued)

9. $(x - 5)/(x + 6) \times (x + 6)/(x - 1) = (x - 5) / (x - 1)$ $(x \neq -6, 1)$
10. $(x - 5)/(x + 4) \div (x - 9)/(x + 4) = (x - 5) / (x - 9)$ $(x \neq -4, 9)$
11. $(x + 8)/(x + 7) \div (x - 3)/(x + 7) = (x + 8) / (x - 3)$ $(x \neq -7, 3)$
12. $(x - 7)/(x - 4) \times (x - 4)/(x - 3) = (x - 7) / (x - 3)$ $(x \neq 3, 4)$
13. $(x + 7)/(x - 9) \div (x - 1)/(x - 9) = (x + 7) / (x - 1)$ $(x \neq 1, 9)$
14. $(x + 4)/(x - 7) \div (x - 8)/(x - 7) = (x + 4) / (x - 8)$ $(x \neq 7, 8)$

Solving Rational Equations

Solving rational equations

1. $11/(x + 4) = 8/(x + 1) \rightarrow x = 7$ $(x \neq -4, -1)$
2. $16/(x - 4) = -8/(x - 7) \rightarrow x = 6$ $(x \neq 4, 7)$
3. $28/(x + 2) = 64/(x - 7) \rightarrow x = -9$ $(x \neq -2, 7)$
4. $3/(x - 3) = -18/(x + 4) \rightarrow x = 2$ $(x \neq -4, 3)$
5. $4/x = 2 \rightarrow x = 2$ $(x \neq 0)$
6. $-54/(x + 3) = 9 \rightarrow x = -9$ $(x \neq -3)$
7. $32/(x + 8) = 8 \rightarrow x = -4$ $(x \neq -8)$
8. $44/(x + 6) = 4 \rightarrow x = 5$ $(x \neq -6)$
9. $7/(x + 6) = -14/(x - 3) \rightarrow x = -3$ $(x \neq -6, 3)$
10. $80/(x + 9) = 24/(x + 2) \rightarrow x = 1$ $(x \neq -9, -2)$
11. $18/(x + 2) = 9 \rightarrow x = 0$ $(x \neq -2)$
12. $48/(x + 9) = 6 \rightarrow x = -1$ $(x \neq -9)$
13. $24/(x - 2) = 33/(x - 5) \rightarrow x = -6$ $(x \neq 2, 5)$
14. $72/(x + 4) = 8 \rightarrow x = 5$ $(x \neq -4)$
15. $4/x = 12/(x + 4) \rightarrow x = 2$ $(x \neq -4, 0)$
16. $-64/(x + 1) = 8 \rightarrow x = -9$ $(x \neq -1)$

Arithmetic and Geometric Sequences

Arithmetic and geometric sequences

1. 5, 9, 13, 17, ... the 17th term = 69
2. 2, 1, 0, −1, ... the 17th term = −14
3. 5, 10, 15, 20, ... the 24th term = 120
4. 12, 9, 6, 3, ... the 20th term = −45
5. −7, −16, −25, −34, ... the 22nd term = −196
6. 1, 5, 9, 13, ... the 15th term = 57
7. 8, 1, −6, −13, ... the 15th term = −90
8. 10, 6, 2, −2, ... the 24th term = −82
9. 11, 14, 17, 20, ... the 25th term = 83
10. −4, −6, −8, −10, ... the 19th term = −40
11. 0, 9, 18, 27, ... the 19th term = 162
12. 12, 19, 26, 33, ... the 12th term = 89
13. −2, 6, 14, 22, ... the 23rd term = 174
14. −1, −10, −19, −28, ... the 14th term = −118
15. 4, −5, −14, −23, ... the 22nd term = −185
16. −7, −2, 3, 8, ... the 18th term = 78
17. 4, 10, 16, 22, ... the 24th term = 142
18. 5, 8, 11, 14, ... the 10th term = 32
19. 6, 10, 14, 18, ... the 23rd term = 94
20. −3, −1, 1, 3, ... the 22nd term = 39

Answer Key (5 of 5)

Arithmetic and Geometric Sequences (continued)

Arithmetic and geometric sequences (continued)

21. 12, 14, 16, 18, … the 24th term = 58
22. 1, −5, −11, −17, … the 23rd term = −131

Sums of Series

Sums of series

1. S_6 for 6, 24, 96, … = 8190
2. S_6 for 5, 10, 20, … = 315
3. S_8 for 2, 11, 20, … = 268
4. S_7 for 9, 27, 81, … = 9837
5. S_8 for 7, 11, 15, … = 168
6. S_6 for 8, 32, 128, … = 10920
7. S_9 for 9, 12, 15, … = 189
8. S_5 for 8, 32, 128, … = 2728
9. S_7 for 1, 4, 16, … = 5461
10. S_7 for 8, 16, 32, … = 1016
11. S_9 for 11, 1, −9, … = −261
12. S_9 for 11, 17, 23, … = 315
13. S_5 for 5, −5, −15, … = −75
14. S_{10} for 2, −1, −4, … = −115
15. S_5 for 12, 24, 48, … = 372
16. S_7 for 9, 5, 1, … = −21

Simplifying Radicals

Simplifying radicals

1. $\sqrt{63}$

$\sqrt{63} = 3\sqrt{7}$

Split 63 into a perfect square times whatever is left: 63 = 9 × 7. 9 is 3 squared, and 7 has no square factor of its own — that last part is what makes this the LARGEST perfect square inside, and it is where the answer stops.

A root splits across a product, so the one root becomes two.

√9 is 3, so it comes out in front. √7 stays under the sign, because 7 has nothing more to give.

2. $\sqrt{126}$

$\sqrt{126} = 3\sqrt{14}$

Split 126 into a perfect square times whatever is left: 126 = 9 × 14. 9 is 3 squared, and 14 has no square factor of its own — that last part is what makes this the LARGEST perfect square inside, and it is where the answer stops.

A root splits across a product, so the one root becomes two.

√9 is 3, so it comes out in front. √14 stays under the sign, because 14 has nothing more to give.

Rational Exponents

Rational exponents

1. $4^{3/2}$ = ________

4^(3/2) = 8

The BOTTOM of the exponent says which root to take, so the square root of 4 comes first. Root before power, always — the other order works and makes the numbers enormous.

The TOP says what to raise it to. 2 to the power 3 is 8.

So 4^(3/2) is 8.

Rational Exponents (continued)

Rational exponents (continued)

2. $64^{5/3}$ = ________

$64^{(5/3)} = 1024$

The BOTTOM of the exponent says which root to take, so the cube root of 64 comes first. Root before power, always — the other order works and makes the numbers enormous.

The TOP says what to raise it to. 4 to the power 5 is 1024.

So $64^{(5/3)}$ is 1024.

Logarithms

Logarithms

1. $\log_7$ __________ = 2

$7^2 = 49$

A logarithm names a power. $\log_7$ of a number is 2 when that number IS the 2nd power of 7, so there is nothing to search for — just work the power out.

2. $\log_2 \frac{1}{32}$ = _____

$\log_2 1/32 = -5$

$\log_2 1/32$ asks a question: which power of 2 lands on 1/32? It is not an instruction to divide by 2 — that is the mistake this notation collects.

1/32 is 1 over 32, and a negative exponent is what writes a reciprocal: 2^{-5} means $1 \div 2^5$. So the power that lands on 1/32 is the same one, turned over.

So $\log_2 1/32$ is −5. A logarithm is negative exactly when the number is between 0 and 1 — nothing has gone wrong.

The Laws of Logarithms

Laws of logarithms

1. $\log_2 4 + \log_2 6 = \log_2$ ____

$$\log_2 4 + \log_2 6$$

$$\log_2 m + \log_2 n = \log_2 (m \times n)$$

$$4 \times 6 = 24$$

$$\log_2 4 + \log_2 6 \quad = \quad \log_2 24$$

Both logarithms are to the same base, 2. That is what makes a law available at all — two logarithms to different bases cannot be combined, and checking the bases match is the first thing to do rather than the last.

A SUM of two logarithms with the same base is the logarithm of the two arguments MULTIPLIED. Adding the logs multiplies the numbers — that is the law, and it is the one worth saying out loud, because the operation changes.

So the two arguments become one: $4 \times 6 = 24$. The answer is $\log_2 24$.

2. $\log_5 6 + \log_5 7 = \log_5$ ____

$$\log_5 6 + \log_5 7$$

$$\log_5 m + \log_5 n = \log_5 (m \times n)$$

$$6 \times 7 = 42$$

$$\log_5 6 + \log_5 7 \quad = \quad \log_5 42$$

Both logarithms are to the same base, 5. That is what makes a law available at all — two logarithms to different bases cannot be combined, and checking the bases match is the first thing to do rather than the last.

A SUM of two logarithms with the same base is the logarithm of the two arguments MULTIPLIED. Adding the logs multiplies the numbers — that is the law, and it is the one worth saying out loud, because the operation changes.

So the two arguments become one: $6 \times 7 = 42$. The answer is $\log_5 42$.

Solutions (4 of 13)

Exponential Functions

Evaluate exponential functions

1. $f(x) = 7 \cdot 5^x$ $f(1) =$ ___

$$f(1) = 7 \cdot 5^1$$
$$5^1 = 5$$
$$7 \cdot 5 = 35$$

Substitute $x = 1$: $f(1) = 7 \cdot 5^1$.

Do the power first: $5^1 = 5$.

Then multiply by the coefficient: $7 \cdot 5 = 35$.

2. $f(x) = 7 \cdot 6^x$ $f(1) =$ ___

$$f(1) = 7 \cdot 6^1$$
$$6^1 = 6$$
$$7 \cdot 6 = 42$$

Substitute $x = 1$: $f(1) = 7 \cdot 6^1$.

Do the power first: $6^1 = 6$.

Then multiply by the coefficient: $7 \cdot 6 = 42$.

Completing the Square

Completing the square

1. $x^2 + 8x + 11$

$$x^2 + 8x + 11 = (x + 4)^2 - 5$$

Halve the middle coefficient. Half of 8 is 4, and that number goes straight into the bracket — sign and all.

But $(x + 4)^2$ is not the expression yet. Multiply it out and it brings a 16 along with it, and the original has 11 there instead.

So give the 16 back: $11 - 16 = -5$. That is the number that goes after the bracket.

Completing the Square (continued)

Completing the square (continued)

2. $x^2 - 16x + 48$

$$x^2 - 16x + 48 = (x - 8)^2 - 16$$

Halve the middle coefficient. Half of −16 is −8, and that number goes straight into the bracket — sign and all.

But $(x - 8)^2$ is not the expression yet. Multiply it out and it brings a 64 along with it, and the original has 48 there instead.

So give the 64 back: 48 − 64 = −16. That is the number that goes after the bracket.

How Many Real Solutions

How many real solutions?

1. $x^2 + 22x + 149 = 0$

$= -112 < 0 \rightarrow$ **none**

You do not have to solve this to answer it. One number decides how many real solutions a quadratic has, and it is $b^2 - 4ac$ — the part that ends up under the square root sign when you do solve it.

Read off a, b and c from $x^2 + 22x + 149 = 0$: a = 1, b = 22, c = 149. Write all three in, even the a — on this sheet it is 1, and forgetting it is the mistake that only shows up when it is not 1.

Work it out: 22^2 is 484, and 4 × 1 × 149 is 596. That leaves 484 − 596 = −112.

−112 is below zero, and no real number squares to a negative, so there is nothing to take the square root of. So there are NO real solutions.

How Many Real Solutions (continued)

How many real solutions? (continued)

2. $x^2 + 12x - 160 = 0$

= 784 > 0 → **two**

You do not have to solve this to answer it. One number decides how many real solutions a quadratic has, and it is $b^2 - 4ac$ — the part that ends up under the square root sign when you do solve it.

Read off a, b and c from $x^2 + 12x - 160 = 0$: a = 1, b = 12, c = −160. Write all three in, even the a — on this sheet it is 1, and forgetting it is the mistake that only shows up when it is not 1.

Work it out: 12^2 is 144, and 4 × 1 × −160 is −640. That leaves 144 − −640 = 784.

784 is above zero, and a positive number has two square roots — one either way from the ±. So there are TWO real solutions.

Adding and Subtracting Complex Numbers

Complex numbers

1. $(4 - 3i) + (-2 - 3i)$

$(4 - 3i) + (-2 - 3i) = 2 - 6i$

Add the two REAL parts, and leave the i alone. The real parts are 4 and −2.

Now the two imaginary parts, the same way — this works because i is a number like any other here, so 3 of them plus 3 of them is what it sounds like.

Put the two parts back together: 2 − 6i.

2. $(-5 + i) - (-4 + 5i)$

$(-5 + i) - (-4 + 5i) = -1 - 4i$

Subtract the two REAL parts, and leave the i alone. The real parts are −5 and −4.

Now the two imaginary parts, the same way — this works because i is a number like any other here, so 1 of them minus 5 of them is what it sounds like.

Put the two parts back together: −1 − 4i.

Solutions (7 of 13)

Adding, Subtracting and Multiplying Polynomials

Add, subtract and multiply polynomials

1. $(4x^2 + 7x + 2) + (6x^2 + 8x + 5)$

$10x^2 + 15x + 7$

Only matching terms combine: x^2 with x^2, x with x, numbers with numbers. Line the two brackets up by power before doing anything, and nothing can drift into the wrong column.

Adding brackets changes no signs at all, so the terms can be gathered as they stand.

Now take the columns one at a time, adding the rewritten signs. x^2: $4 + 6 = 10$. x: $7 + 8 = 15$. Numbers: $2 + 5 = 7$.

So the answer is $10x^2 + 15x + 7$. Read it back and count the terms — three went in on each side, and three come out.

2. $(7x^2 + 6x + 12) - (5x^2 - 3x + 7)$

$2x^2 + 9x + 5$

Only matching terms combine: x^2 with x^2, x with x, numbers with numbers. Line the two brackets up by power before doing anything, and nothing can drift into the wrong column.

Subtracting a bracket flips the sign of EVERY term inside it, not just the first: taking away $5x^2 - 3x + 7$ is the same as adding $-5x^2 + 3x - 7$. Rewrite it that way before adding — it is slower and it is the step that gets marks.

Now take the columns one at a time, adding the rewritten signs. x^2: $7 + (-5) = 2$. x: $6 + 3 = 9$. Numbers: $12 + (-7) = 5$.

So the answer is $2x^2 + 9x + 5$. Read it back and count the terms — three went in on each side, and three come out.

Solutions (8 of 13)

Dividing Polynomials

Dividing polynomials

1.

$$
\begin{array}{r|l}
 & x^2 + 3x - 2 \\
\hline
x + 1 & x^3 + 4x^2 + x - 2 \\
 & -x^3 - x^2 \\
\hline
 & 3x^2 + x - 2 \\
 & -3x^2 - 3x \\
\hline
 & -2x - 2 \\
 & 2x + 2 \\
\hline
 & 0
\end{array}
$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x + 1)$ is $x^3 + x^2$ — and take that away, leaving $3x^2 + x - 2$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$3x^2 \div x$ is $3x$, so $3x$ goes in the x place. Multiply the WHOLE divisor by it — $3x(x + 1)$ is $3x^2 + 3x$ — and take that away, leaving $-2x - 2$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$-2x \div x$ is -2, so -2 goes in the constant place. Multiply the WHOLE divisor by it — $-2(x + 1)$ is $-2x - 2$ — and take that away, leaving 0. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

Nothing is left, so $x + 1$ divides it exactly: $(x^3 + 4x^2 + x - 2) \div (x + 1) = x^2 + 3x - 2$.

2.

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x - 6 & x^3 - 9x^2 + 20x - 12 \\
 & -x^3 + 6x^2 \\
\hline
 & -3x^2 + 20x - 12 \\
 & 3x^2 - 18x \\
\hline
 & 2x - 12 \\
 & -2x + 12 \\
\hline
 & 0
\end{array}
$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x - 6)$ is $x^3 - 6x^2$ — and take that away, leaving $-3x^2 + 20x - 12$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$-3x^2 \div x$ is $-3x$, so $-3x$ goes in the x place. Multiply the WHOLE divisor by it — $-3x(x - 6)$ is $-3x^2 + 18x$ — and take that away, leaving $2x - 12$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$2x \div x$ is 2, so 2 goes in the constant place. Multiply the WHOLE divisor by it — $2(x - 6)$ is $2x - 12$ — and take that away, leaving 0. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

Nothing is left, so $x - 6$ divides it exactly: $(x^3 - 9x^2 + 20x - 12) \div (x - 6) = x^2 - 3x + 2$.

Solutions (9 of 13)

Simplifying Rational Expressions

Simplifying rational expressions

1. $\dfrac{x^2 + 5x - 66}{x^2 - 10x + 24}$

①/② $\dfrac{(x - 6)(x + 11)}{(x - 6)(x - 4)}$

$= \dfrac{(x + 11)}{(x - 4)}$

Nothing can cancel while these are sums. Factor the top first — two numbers that multiply to −66 and add to 5. Call it ①.

Now the bottom, the same way — call it ②. $(x - 6)$ turns up again, and that is not a coincidence: it is the whole reason this expression simplifies.

The question is ① over ②. $(x - 6)$ is a FACTOR of both, so it divides out and leaves 1. Only a factor can go this way — striking the x^2 off the top and the bottom of the sums above is the mistake this page exists to catch. And the cancelling changed something quietly: x may not be 4 or 6, because the original has no value there.

2. $\dfrac{x^2 - 22x + 112}{x^2 - 14x}$

①/② $\dfrac{(x - 14)(x - 8)}{x(x - 14)}$

$= \dfrac{(x - 8)}{x}$

Nothing can cancel while these are sums. Factor the top first — two numbers that multiply to 112 and add to −22. Call it ①.

Now the bottom, the same way — call it ②. $(x - 14)$ turns up again, and that is not a coincidence: it is the whole reason this expression simplifies.

The question is ① over ②. $(x - 14)$ is a FACTOR of both, so it divides out and leaves 1. Only a factor can go this way — striking the x^2 off the top and the bottom of the sums above is the mistake this page exists to catch. And the cancelling changed something quietly: x may not be 0 or 14, because the original has no value there.

Multiplying and Dividing Rational Expressions

Multiplying and dividing rational expressions

1. $\frac{(x-1)}{(x-6)} \div \frac{(x-5)}{(x-6)}$

$(x - 1)/(x - 6) \div (x - 5)/(x - 6)$

$(x - 1)/(x - 6) \times (x - 6)/(x - 5)$

$(x - 1)/(x - 6) \times (x - 6)/(x - 5)$

$(x - 6)$ cancels

$(x - 1) / (x - 5)$

$x \neq 5, 6$

Dividing by a fraction is multiplying by it upside down, and that has to happen FIRST. Turn (x − 5)/(x − 6) over and change the ÷ to a ×. Cancelling before the flip is cancelling across a division sign, which is not a move.

Now it is one product, so anything on top may cancel with anything underneath — the two fractions are no longer separate. (x − 6) appears in both, so it goes.

What is left is (x − 1) / (x − 5).

One more thing, and it is the half that gets dropped: x may not be 5 or 6. Those are the numbers that made a denominator zero in the ORIGINAL — including (x − 6), which cancelled. A bracket does not stop mattering because it left the page.

2. $\frac{(x+8)}{(x-3)} \div \frac{(x+4)}{(x-3)}$

$(x + 8)/(x - 3) \div (x + 4)/(x - 3)$

$(x + 8)/(x - 3) \times (x - 3)/(x + 4)$

$(x + 8)/(x - 3) \times (x - 3)/(x + 4)$

$(x - 3)$ cancels

$(x + 8) / (x + 4)$

$x \neq -4, 3$

Dividing by a fraction is multiplying by it upside down, and that has to happen FIRST. Turn (x + 4)/(x − 3) over and change the ÷ to a ×. Cancelling before the flip is cancelling across a division sign, which is not a move.

Now it is one product, so anything on top may cancel with anything underneath — the two fractions are no longer separate. (x − 3) appears in both, so it goes.

What is left is (x + 8) / (x + 4).

One more thing, and it is the half that gets dropped: x may not be −4 or 3. Those are the numbers that made a denominator zero in the ORIGINAL — including (x − 3), which cancelled. A bracket does not stop mattering because it left the page.

Solving Rational Equations

Solving rational equations

1. $\dfrac{11}{(x + 4)} = \dfrac{8}{(x + 1)}$, $x =$ ____

$x \neq -4, -1$

$11/(x + 4) = 8/(x + 1)$

$11(x + 1) = 8(x + 4)$

$x = 7$

$x = 7$ and $x \neq -4, -1$

Before anything else, note what x cannot be. (x + 4) is a denominator, so x may not be −4, and the same for the other one: not −1. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Two fractions, one equals sign — so cross-multiply. Each numerator meets the OTHER denominator, which is the same "multiply both sides by both denominators" done in one move.

Multiply out, gather the x terms on one side and the numbers on the other, and x = 7.

Last, check it against the first line: x may not be −4 or −1, and 7 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

2. $\dfrac{16}{(x - 4)} = \dfrac{-8}{(x - 7)}$, $x =$ ____

$x \neq 4, 7$

$16/(x - 4) = -8/(x - 7)$

$16(x - 7) = -8(x - 4)$

$x = 6$

$x = 6$ and $x \neq 4, 7$

Before anything else, note what x cannot be. (x − 4) is a denominator, so x may not be 4, and the same for the other one: not 7. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Two fractions, one equals sign — so cross-multiply. Each numerator meets the OTHER denominator, which is the same "multiply both sides by both denominators" done in one move.

Multiply out, gather the x terms on one side and the numbers on the other, and x = 6.

Last, check it against the first line: x may not be 4 or 7, and 6 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

Arithmetic and Geometric Sequences

Arithmetic and geometric sequences

1. 5, 9, 13, 17, ... a_{17} = ____

$$d = 4$$
$$a_n = 5 + (n - 1) \times 4$$
$$a_{17} = 5 + 16 \times 4$$
$$5 + 64 = 69$$
$$a_{17} = 69$$

This is an arithmetic sequence: it changes by 4 each step (9 – 5 = 4).

The first term is already there, so to reach the 17th term you take the step only 16 times — that is (n – 1), not n.

16 steps of 4 is 64, and 5 + 64 = 69.

So the 17th term is 69.

2. 2, 1, 0, −1, ... a_{17} = ____

$$d = -1$$
$$a_n = 2 + (n - 1) \times -1$$
$$a_{17} = 2 + 16 \times -1$$
$$2 + -16 = -14$$
$$a_{17} = -14$$

This is an arithmetic sequence: it changes by −1 each step (1 – 2 = −1).

The first term is already there, so to reach the 17th term you take the step only 16 times — that is (n – 1), not n.

16 steps of −1 is −16, and 2 + −16 = −14.

So the 17th term is −14.

Solutions (13 of 13)

Sums of Series

Sums of series

1. **6, 24, 96, ... S_6 = __________**

$$S_6 = 6 \times 4095 \div 3 = 8190$$

Each term is the one before times 4, so this is a geometric series and the sum is $a(r^n - 1) \div (r - 1)$. Here a is 6, r is 4 and n is 6 — substituting the wrong one of the three is the whole difficulty.

4 to the power 6 is 4096, so the top is 6 × 4095 and the bottom is 3.

2. **5, 10, 20, ... S_6 = __________**

$$S_6 = 5 \times 63 \div 1 = 315$$

Each term is the one before times 2, so this is a geometric series and the sum is $a(r^n - 1) \div (r - 1)$. Here a is 5, r is 2 and n is 6 — substituting the wrong one of the three is the whole difficulty.

2 to the power 6 is 64, so the top is 5 × 63 and the bottom is 1.