Algebra 2 Rational Expressions Workbook

3 lessons · printable · answer key at the back

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Simplifying Rational Expressions

WORKED EXAMPLE

$$\frac{x^2 - 2x - 120}{x^2 - 5x - 150}$$

① $x^2 - 2x - 120 = (x + 10)(x - 12)$

② $x^2 - 5x - 150 = (x + 10)(x - 15)$

①/② $\frac{(x + 10)(x - 12)}{(x + 10)(x - 15)}$

$= \frac{(x - 12)}{(x - 15)}$

1. Simplifying rational expressions

Factor the top and the bottom, then cancel.

1 $\frac{x^2 + 5x - 66}{x^2 - 10x + 24}$

2 $\frac{x^2 + 3x + 2}{x^2 + 22x + 40}$

3 $\frac{x^2 - 22x + 112}{x^2 - 14x}$

4 $\frac{x^2 - 10x + 16}{x^2 - 8x}$

5 $\frac{x^2 - 18x + 72}{x^2 - 4x - 12}$

6 $\frac{x^2 - 19x + 88}{x^2 - x - 110}$

7 $\frac{x^2 + 8x - 33}{x^2 + 3x - 18}$

8 $\frac{x^2 - 20x + 51}{x^2 - 10x + 21}$

1. Simplifying rational expressions (continued)

Factor the top and the bottom, then cancel.

9 $\dfrac{x^2 - 7x - 228}{x^2 - 36x + 323}$

10 $\dfrac{x^2 - 2x - 8}{x^2 - 5x + 4}$

11 $\dfrac{x^2 + x - 30}{x^2 - 12x - 108}$

12 $\dfrac{x^2 - 14x - 51}{x^2 + 18x + 45}$

13 $\dfrac{x^2 + 18x - 19}{x^2 - 16x + 15}$

14 $\dfrac{x^2 - 6x - 112}{x^2 - 7x - 120}$

15 $\dfrac{x^2 + 15x + 36}{x^2 - 10x - 39}$

16 $\dfrac{x^2 + 7x + 12}{x^2 + 24x + 80}$

17 $\dfrac{x^2 - 13x - 114}{x^2 - 15x - 76}$

18 $\dfrac{x^2 + 18x + 72}{x^2 + 12x}$

19 $\dfrac{x^2 - 9x + 18}{x^2 - 13x + 30}$

20 $\dfrac{x^2 - 8x - 180}{x^2 + 16x + 60}$

21 $\dfrac{x^2 + 9x - 52}{x^2 - 9x + 20}$

2. Simplifying radicals

Simplify each cube root.

22 $\sqrt[3]{72}$

23 $\sqrt[3]{32}$

24 $\sqrt[3]{324}$

25 $\sqrt[3]{56}$

26 $\sqrt[3]{135}$

27 $\sqrt[3]{320}$

28 $\sqrt[3]{81}$

29 $\sqrt[3]{108}$

30 $\sqrt[3]{500}$

31 $\sqrt[3]{189}$

Multiplying and Dividing Rational Expressions

WORKED EXAMPLE

$$\frac{(x + 8)}{(x + 6)} \div \frac{(x - 7)}{(x + 6)}$$

(x + 8)/(x + 6) × (x + 6)/(x − 7)

(x + 6) cancels

(x + 8) / (x − 7)

x ≠ −6, 7

3. Multiplying and dividing rational expressions

Multiply or divide, then cancel. Write each answer as one fraction.

1 $\frac{(x - 1)}{(x - 6)} \div \frac{(x - 5)}{(x - 6)}$

2 $\frac{(x + 8)}{(x - 3)} \div \frac{(x + 4)}{(x - 3)}$

3 $\frac{(x + 3)}{(x + 4)} \times \frac{(x + 4)}{(x + 7)}$

4 $\frac{(x - 9)}{(x - 8)} \div \frac{(x - 6)}{(x - 8)}$

5 $\frac{(x - 5)}{(x + 6)} \times \frac{(x + 6)}{(x - 1)}$

6 $\frac{(x - 6)}{(x + 8)} \times \frac{(x + 8)}{(x + 2)}$

7 $\frac{(x - 7)}{(x - 4)} \times \frac{(x - 4)}{(x - 3)}$

8 $\frac{(x - 6)}{(x + 9)} \times \frac{(x + 9)}{(x + 6)}$

9 $\frac{(x - 4)}{(x + 3)} \times \frac{(x + 3)}{(x - 8)}$

10 $\frac{(x + 2)}{(x - 7)} \times \frac{(x - 7)}{(x + 5)}$

11 $\frac{(x - 8)}{(x - 9)} \times \frac{(x - 9)}{(x - 5)}$

12 $\frac{(x - 1)}{(x - 3)} \div \frac{(x + 6)}{(x - 3)}$

13 $\frac{(x + 8)}{(x + 7)} \div \frac{(x - 3)}{(x + 7)}$

14 $\frac{(x - 5)}{(x + 4)} \div \frac{(x - 9)}{(x + 4)}$

15 $\frac{(x + 3)}{(x + 1)} \times \frac{(x + 1)}{(x - 5)}$

16 $\frac{(x + 4)}{(x - 7)} \div \frac{(x - 8)}{(x - 7)}$

17 $\frac{(x + 4)}{(x + 7)} \times \frac{(x + 7)}{(x + 1)}$

18 $\frac{(x + 7)}{(x - 9)} \div \frac{(x - 1)}{(x - 9)}$

4. Add, subtract and multiply polynomials

Work each one out.

19 $(6x^2 + 5x + 8) - (5x^2 - x + 4)$

20 $(9x^2 + 7x + 11) - (7x^2 - 9x + 9)$

21 $(2x^2 + 5x + 3) + (6x^2 + 6x + 2)$

22 $(3x + 2)(x + 4)$

23 $(5x^2 + x + 12) - (4x^2 - 5x + 9)$

24 $(9x^2 + 8x + 11) - (4x^2 - 5x + 7)$

25 $(3x^2 + 5x + 7) + (x^2 + 4x + 7)$

26 $(4x + 1)(x + 3)$

27 $(8x^2 + 7x + 12) - (5x^2 - 4x + 2)$

28 $(4x^2 + 7x + 2) + (6x^2 + 8x + 5)$

Solving Rational Equations

WORKED EXAMPLE

$$\frac{5}{(x-7)} = \frac{-5}{(x-3)}, \; x = \underline{5}$$

$x \neq 3, 7$

$5(x-3) = -5(x-7)$

$x = 5$

$x = 5$ and $x \neq 3, 7$

5. Solving rational equations

Solve each equation for x.

1 $\frac{30}{(x+7)} = 2$, $x =$ ____

2 $\frac{6}{(x-7)} = 6$, $x =$ ____

3 $\frac{-56}{(x-8)} = 7$, $x =$ ____

4 $\frac{-20}{(x-7)} = 5$, $x =$ ____

5 $\frac{56}{x} = 8$, $x =$ ____

6 $\frac{30}{(x+6)} = \frac{-5}{(x-8)}$, $x =$ ____

7 $\frac{56}{x} = \frac{63}{(x+1)}$, $x =$ ____

8 $\frac{4}{(x-9)} = \frac{-12}{(x-1)}$, $x =$ ____

9 $\frac{-45}{(x+1)} = 9$, $x =$ ____

10 $\frac{18}{(x-4)} = \frac{14}{(x-2)}$, $x =$ ____

5. Solving rational equations (continued)

Solve each equation for x.

11 $\frac{49}{(x + 2)} = 7$, x = _____

12 $\frac{49}{(x + 3)} = \frac{42}{(x + 2)}$, x = _____

13 $\frac{16}{(x + 5)} = \frac{8}{(x + 7)}$, x = _____

14 $\frac{80}{(x + 3)} = \frac{72}{(x + 2)}$, x = _____

15 $\frac{3}{(x - 6)} = \frac{-9}{(x + 2)}$, x = _____

16 $\frac{56}{(x + 9)} = 4$, x = _____

17 $\frac{18}{(x - 4)} = 6$, x = _____

18 $\frac{39}{(x + 7)} = 3$, x = _____

19 $\frac{35}{(x + 4)} = \frac{7}{x}$, x = _____

20 $\frac{-63}{(x - 1)} = 9$, x = _____

21 $\frac{72}{(x - 6)} = \frac{8}{(x + 2)}$, x = _____

22 $\frac{4}{(x + 8)} = \frac{28}{(x + 2)}$, x = _____

23 $\frac{66}{(x + 6)} = \frac{72}{(x + 7)}$, x = _____

24 $\frac{48}{(x + 3)} = \frac{54}{(x + 4)}$, x = _____

6. Dividing polynomials

Divide each polynomial. Write 0 for the remainder when it divides exactly.

25

$$
\begin{array}{rr}
 & x^2 \quad - x \quad - 4 \\
x + 4 & \overline{)\, x^3 + 3x^2 - 8x - 20} \\
 & -x^3 - 4x^2 \qquad\qquad\quad \\
 & -x^2 - 8x - 20 \\
 & x^2 + 4x \qquad\quad \\
 & -4x - 20 \\
 & 4x + 16 \\
 & -4
\end{array}
$$

26

$$
\begin{array}{rr}
 & x^2 + 5x + 6 \\
x - 1 & \overline{)\, x^3 + 4x^2 \quad + x - 6} \\
 & -x^3 \quad + x^2 \qquad\qquad \\
 & 5x^2 \quad + x - 6 \\
 & -5x^2 + 5x \qquad \\
 & 6x - 6 \\
 & -6x + 6 \\
 & 0
\end{array}
$$

27

$$
\begin{array}{rr}
 & x^2 \quad - 5x \quad - 6 \\
x + 4 & \overline{)\, x^3 \quad - x^2 - 26x - 28} \\
 & -x^3 - 4x^2 \qquad\qquad\quad \\
 & -5x^2 - 26x - 28 \\
 & 5x^2 + 20x \qquad\quad \\
 & -6x - 28 \\
 & 6x + 24 \\
 & -4
\end{array}
$$

28

$$
\begin{array}{rr}
 & x^2 \quad + 5x - 1 \\
x - 4 & \overline{)\, x^3 \quad + x^2 - 21x + 8} \\
 & -x^3 + 4x^2 \qquad\qquad\quad \\
 & 5x^2 - 21x + 8 \\
 & -5x^2 + 20x \qquad \\
 & -x + 8 \\
 & x - 4 \\
 & 4
\end{array}
$$

29

$$
\begin{array}{rr}
 & x^2 \quad - 4x + 1 \\
x + 6 & \overline{)\, x^3 + 2x^2 - 23x + 6} \\
 & -x^3 - 6x^2 \qquad\qquad\quad \\
 & -4x^2 - 23x + 6 \\
 & 4x^2 + 24x \qquad \\
 & x + 6 \\
 & -x - 6 \\
 & 0
\end{array}
$$

30

$$
\begin{array}{rr}
 & x^2 \quad + 4x \quad + 5 \\
x - 3 & \overline{)\, x^3 \quad + x^2 \quad - 7x - 16} \\
 & -x^3 + 3x^2 \qquad\qquad\quad \\
 & 4x^2 \quad - 7x - 16 \\
 & -4x^2 + 12x \qquad \\
 & 5x - 16 \\
 & -5x + 15 \\
 & -1
\end{array}
$$

31

$$
\begin{array}{rr}
 & x^2 \quad - x \quad - 4 \\
x - 6 & \overline{)\, x^3 - 7x^2 + 2x + 20} \\
 & -x^3 + 6x^2 \qquad\qquad\quad \\
 & -x^2 + 2x + 20 \\
 & x^2 - 6x \qquad\quad \\
 & -4x + 20 \\
 & 4x - 24 \\
 & -4
\end{array}
$$

32

$$
\begin{array}{rr}
 & x^2 \quad + 0 - 3 \\
x - 3 & \overline{)\, x^3 - 3x^2 - 3x} \\
 & -x^3 + 3x^2 \qquad\quad \\
 & -3x \\
 & 3x - 9 \\
 & -9
\end{array}
$$

Answer Key (1 of 2)

Simplifying Rational Expressions

Simplifying rational expressions

1. $(x^2 + 5x - 66) / (x^2 - 10x + 24) = (x + 11) / (x - 4)$
2. $(x^2 + 3x + 2) / (x^2 + 22x + 40) = (x + 1) / (x + 20)$
3. $(x^2 - 22x + 112) / (x^2 - 14x) = (x - 8) / x$
4. $(x^2 - 10x + 16) / (x^2 - 8x) = (x - 2) / x$
5. $(x^2 - 18x + 72) / (x^2 - 4x - 12) = (x - 12) / (x + 2)$
6. $(x^2 - 19x + 88) / (x^2 - x - 110) = (x - 8) / (x + 10)$
7. $(x^2 + 8x - 33) / (x^2 + 3x - 18) = (x + 11) / (x + 6)$
8. $(x^2 - 20x + 51) / (x^2 - 10x + 21) = (x - 17) / (x - 7)$
9. $(x^2 - 7x - 228) / (x^2 - 36x + 323) = (x + 12) / (x - 17)$
10. $(x^2 - 2x - 8) / (x^2 - 5x + 4) = (x + 2) / (x - 1)$
11. $(x^2 + x - 30) / (x^2 - 12x - 108) = (x - 5) / (x - 18)$
12. $(x^2 - 14x - 51) / (x^2 + 18x + 45) = (x - 17) / (x + 15)$
13. $(x^2 + 18x - 19) / (x^2 - 16x + 15) = (x + 19) / (x - 15)$
14. $(x^2 - 6x - 112) / (x^2 - 7x - 120) = (x - 14) / (x - 15)$
15. $(x^2 + 15x + 36) / (x^2 - 10x - 39) = (x + 12) / (x - 13)$
16. $(x^2 + 7x + 12) / (x^2 + 24x + 80) = (x + 3) / (x + 20)$
17. $(x^2 - 13x - 114) / (x^2 - 15x - 76) = (x + 6) / (x + 4)$
18. $(x^2 + 18x + 72) / (x^2 + 12x) = (x + 6) / x$
19. $(x^2 - 9x + 18) / (x^2 - 13x + 30) = (x - 6) / (x - 10)$
20. $(x^2 - 8x - 180) / (x^2 + 16x + 60) = (x - 18) / (x + 6)$
21. $(x^2 + 9x - 52) / (x^2 - 9x + 20) = (x + 13) / (x - 5)$

Simplifying radicals

22. $\sqrt[3]{72} = 2\sqrt[3]{9}$
23. $\sqrt[3]{32} = 2\sqrt[3]{4}$
24. $\sqrt[3]{324} = 3\sqrt[3]{12}$
25. $\sqrt[3]{56} = 2\sqrt[3]{7}$
26. $\sqrt[3]{135} = 3\sqrt[3]{5}$
27. $\sqrt[3]{320} = 4\sqrt[3]{5}$
28. $\sqrt[3]{81} = 3\sqrt[3]{3}$
29. $\sqrt[3]{108} = 3\sqrt[3]{4}$
30. $\sqrt[3]{500} = 5\sqrt[3]{4}$
31. $\sqrt[3]{189} = 3\sqrt[3]{7}$

Multiplying and Dividing Rational Expressions

Multiplying and dividing rational expressions

1. $(x - 1)/(x - 6) \div (x - 5)/(x - 6) = (x - 1) / (x - 5)$ $(x \neq 5, 6)$
2. $(x + 8)/(x - 3) \div (x + 4)/(x - 3) = (x + 8) / (x + 4)$ $(x \neq -4, 3)$
3. $(x + 3)/(x + 4) \times (x + 4)/(x + 7) = (x + 3) / (x + 7)$ $(x \neq -7, -4)$
4. $(x - 9)/(x - 8) \div (x - 6)/(x - 8) = (x - 9) / (x - 6)$ $(x \neq 6, 8)$
5. $(x - 5)/(x + 6) \times (x + 6)/(x - 1) = (x - 5) / (x - 1)$ $(x \neq -6, 1)$
6. $(x - 6)/(x + 8) \times (x + 8)/(x + 2) = (x - 6) / (x + 2)$ $(x \neq -8, -2)$
7. $(x - 7)/(x - 4) \times (x - 4)/(x - 3) = (x - 7) / (x - 3)$ $(x \neq 3, 4)$
8. $(x - 6)/(x + 9) \times (x + 9)/(x + 6) = (x - 6) / (x + 6)$ $(x \neq -9, -6)$
9. $(x - 4)/(x + 3) \times (x + 3)/(x - 8) = (x - 4) / (x - 8)$ $(x \neq -3, 8)$
10. $(x + 2)/(x - 7) \times (x - 7)/(x + 5) = (x + 2) / (x + 5)$ $(x \neq -5, 7)$
11. $(x - 8)/(x - 9) \times (x - 9)/(x - 5) = (x - 8) / (x - 5)$ $(x \neq 5, 9)$
12. $(x - 1)/(x - 3) \div (x + 6)/(x - 3) = (x - 1) / (x + 6)$ $(x \neq -6, 3)$
13. $(x + 8)/(x + 7) \div (x - 3)/(x + 7) = (x + 8) / (x - 3)$ $(x \neq -7, 3)$

Multiplying and Dividing Rational Expressions (continued)

Multiplying and dividing rational expressions (continued)

14. $(x - 5)/(x + 4) \div (x - 9)/(x + 4) = (x - 5) / (x - 9)$ $(x \neq -4, 9)$
15. $(x + 3)/(x + 1) \times (x + 1)/(x - 5) = (x + 3) / (x - 5)$ $(x \neq -1, 5)$
16. $(x + 4)/(x - 7) \div (x - 8)/(x - 7) = (x + 4) / (x - 8)$ $(x \neq 7, 8)$
17. $(x + 4)/(x + 7) \times (x + 7)/(x + 1) = (x + 4) / (x + 1)$ $(x \neq -7, -1)$
18. $(x + 7)/(x - 9) \div (x - 1)/(x - 9) = (x + 7) / (x - 1)$ $(x \neq 1, 9)$

Add, subtract and multiply polynomials

19. $(6x^2 + 5x + 8) - (5x^2 - x + 4) \rightarrow 1x^2 + 6x + 4$
20. $(9x^2 + 7x + 11) - (7x^2 - 9x + 9) \rightarrow 2x^2 + 16x + 2$
21. $(2x^2 + 5x + 3) + (6x^2 + 6x + 2) \rightarrow 8x^2 + 11x + 5$
22. $(3x + 2)(x + 4) \rightarrow 3x^2 + 14x + 8$
23. $(5x^2 + x + 12) - (4x^2 - 5x + 9) \rightarrow 1x^2 + 6x + 3$
24. $(9x^2 + 8x + 11) - (4x^2 - 5x + 7) \rightarrow 5x^2 + 13x + 4$
25. $(3x^2 + 5x + 7) + (x^2 + 4x + 7) \rightarrow 4x^2 + 9x + 14$
26. $(4x + 1)(x + 3) \rightarrow 4x^2 + 13x + 3$
27. $(8x^2 + 7x + 12) - (5x^2 - 4x + 2) \rightarrow 3x^2 + 11x + 10$
28. $(4x^2 + 7x + 2) + (6x^2 + 8x + 5) \rightarrow 10x^2 + 15x + 7$

Solving Rational Equations

Solving rational equations

1. $30/(x + 7) = 2 \rightarrow x = 8$ $(x \neq -7)$
2. $6/(x - 7) = 6 \rightarrow x = 8$ $(x \neq 7)$
3. $-56/(x - 8) = 7 \rightarrow x = 0$ $(x \neq 8)$
4. $-20/(x - 7) = 5 \rightarrow x = 3$ $(x \neq 7)$
5. $56/x = 8 \rightarrow x = 7$ $(x \neq 0)$
6. $30/(x + 6) = -5/(x - 8) \rightarrow x = 6$ $(x \neq -6, 8)$
7. $56/x = 63/(x + 1) \rightarrow x = 8$ $(x \neq -1, 0)$
8. $4/(x - 9) = -12/(x - 1) \rightarrow x = 7$ $(x \neq 1, 9)$
9. $-45/(x + 1) = 9 \rightarrow x = -6$ $(x \neq -1)$
10. $18/(x - 4) = 14/(x - 2) \rightarrow x = -5$ $(x \neq 2, 4)$
11. $49/(x + 2) = 7 \rightarrow x = 5$ $(x \neq -2)$
12. $49/(x + 3) = 42/(x + 2) \rightarrow x = 4$ $(x \neq -3, -2)$
13. $16/(x + 5) = 8/(x + 7) \rightarrow x = -9$ $(x \neq -7, -5)$
14. $80/(x + 3) = 72/(x + 2) \rightarrow x = 7$ $(x \neq -3, -2)$
15. $3/(x - 6) = -9/(x + 2) \rightarrow x = 4$ $(x \neq -2, 6)$
16. $56/(x + 9) = 4 \rightarrow x = 5$ $(x \neq -9)$
17. $18/(x - 4) = 6 \rightarrow x = 7$ $(x \neq 4)$
18. $39/(x + 7) = 3 \rightarrow x = 6$ $(x \neq -7)$
19. $35/(x + 4) = 7/x \rightarrow x = 1$ $(x \neq -4, 0)$
20. $-63/(x - 1) = 9 \rightarrow x = -6$ $(x \neq 1)$
21. $72/(x - 6) = 8/(x + 2) \rightarrow x = -3$ $(x \neq -2, 6)$
22. $4/(x + 8) = 28/(x + 2) \rightarrow x = -9$ $(x \neq -8, -2)$
23. $66/(x + 6) = 72/(x + 7) \rightarrow x = 5$ $(x \neq -7, -6)$
24. $48/(x + 3) = 54/(x + 4) \rightarrow x = 5$ $(x \neq -4, -3)$

Dividing polynomials

25. $(x^3 + 3x^2 - 8x - 20) \div (x + 4) = x^2 - x - 4$ remainder -4
26. $(x^3 + 4x^2 + x - 6) \div (x - 1) = x^2 + 5x + 6$
27. $(x^3 - x^2 - 26x - 28) \div (x + 4) = x^2 - 5x - 6$ remainder -4
28. $(x^3 + x^2 - 21x + 8) \div (x - 4) = x^2 + 5x - 1$ remainder 4

Answer Key (2 of 2)

Solving Rational Equations (continued)

Dividing polynomials (continued)

29. $(x^3 + 2x^2 - 23x + 6) \div (x + 6) = x^2 - 4x + 1$
30. $(x^3 + x^2 - 7x - 16) \div (x - 3) = x^2 + 4x + 5$ remainder -1
31. $(x^3 - 7x^2 + 2x + 20) \div (x - 6) = x^2 - x - 4$ remainder -4
32. $(x^3 - 3x^2 - 3x) \div (x - 3) = x^2 - 3$ remainder -9

Simplifying Rational Expressions

Simplifying rational expressions

1. $\dfrac{x^2 + 5x - 66}{x^2 - 10x + 24}$

$$①/② \quad \frac{(x-6)(x+11)}{(x-6)(x-4)}$$
$$= \frac{(x+11)}{(x-4)}$$

Nothing can cancel while these are sums. Factor the top first — two numbers that multiply to −66 and add to 5. Call it ①.

Now the bottom, the same way — call it ②. (x − 6) turns up again, and that is not a coincidence: it is the whole reason this expression simplifies.

The question is ① over ②. (x − 6) is a FACTOR of both, so it divides out and leaves 1. Only a factor can go this way — striking the x^2 off the top and the bottom of the sums above is the mistake this page exists to catch. And the cancelling changed something quietly: x may not be 4 or 6, because the original has no value there.

2. $\dfrac{x^2 + 3x + 2}{x^2 + 22x + 40}$

$$①/② \quad \frac{(x+2)(x+1)}{(x+2)(x+20)}$$
$$= \frac{(x+1)}{(x+20)}$$

Nothing can cancel while these are sums. Factor the top first — two numbers that multiply to 2 and add to 3. Call it ①.

Now the bottom, the same way — call it ②. (x + 2) turns up again, and that is not a coincidence: it is the whole reason this expression simplifies.

The question is ① over ②. (x + 2) is a FACTOR of both, so it divides out and leaves 1. Only a factor can go this way — striking the x^2 off the top and the bottom of the sums above is the mistake this page exists to catch. And the cancelling changed something quietly: x may not be −20 or −2, because the original has no value there.

Simplifying Rational Expressions (continued)

Simplifying radicals

22. $\sqrt[3]{72}$

$$\sqrt[3]{72} = 2\sqrt[3]{9}$$

Split 72 into a perfect cube times whatever is left: 72 = 8 × 9. 8 is 2 cubed, and 9 has no cube factor of its own — that last part is what makes this the LARGEST perfect cube inside, and it is where the answer stops.

A root splits across a product, so the one root becomes two.

$\sqrt[3]{8}$ is 2, so it comes out in front. $\sqrt[3]{9}$ stays under the sign, because 9 has nothing more to give.

23. $\sqrt[3]{32}$

$$\sqrt[3]{32} = 2\sqrt[3]{4}$$

Split 32 into a perfect cube times whatever is left: 32 = 8 × 4. 8 is 2 cubed, and 4 has no cube factor of its own — that last part is what makes this the LARGEST perfect cube inside, and it is where the answer stops.

A root splits across a product, so the one root becomes two.

$\sqrt[3]{8}$ is 2, so it comes out in front. $\sqrt[3]{4}$ stays under the sign, because 4 has nothing more to give.

Solutions (3 of 6)

Multiplying and Dividing Rational Expressions

Multiplying and dividing rational expressions

1. $\dfrac{(x-1)}{(x-6)} \div \dfrac{(x-5)}{(x-6)}$

$(x - 1)/(x - 6) \div (x - 5)/(x - 6)$

$(x - 1)/(x - 6) \times (x - 6)/(x - 5)$

$(x - 1)/(x - 6) \times (x - 6)/(x - 5)$

$(x - 6)$ cancels

$(x - 1) / (x - 5)$

$x \neq 5, 6$

Dividing by a fraction is multiplying by it upside down, and that has to happen FIRST. Turn (x – 5)/(x – 6) over and change the ÷ to a ×. Cancelling before the flip is cancelling across a division sign, which is not a move.

Now it is one product, so anything on top may cancel with anything underneath — the two fractions are no longer separate. (x – 6) appears in both, so it goes.

What is left is (x – 1) / (x – 5).

One more thing, and it is the half that gets dropped: x may not be 5 or 6. Those are the numbers that made a denominator zero in the ORIGINAL — including (x – 6), which cancelled. A bracket does not stop mattering because it left the page.

2. $\dfrac{(x+8)}{(x-3)} \div \dfrac{(x+4)}{(x-3)}$

$(x + 8)/(x - 3) \div (x + 4)/(x - 3)$

$(x + 8)/(x - 3) \times (x - 3)/(x + 4)$

$(x + 8)/(x - 3) \times (x - 3)/(x + 4)$

$(x - 3)$ cancels

$(x + 8) / (x + 4)$

$x \neq -4, 3$

Dividing by a fraction is multiplying by it upside down, and that has to happen FIRST. Turn (x + 4)/(x – 3) over and change the ÷ to a ×. Cancelling before the flip is cancelling across a division sign, which is not a move.

Now it is one product, so anything on top may cancel with anything underneath — the two fractions are no longer separate. (x – 3) appears in both, so it goes.

What is left is (x + 8) / (x + 4).

One more thing, and it is the half that gets dropped: x may not be −4 or 3. Those are the numbers that made a denominator zero in the ORIGINAL — including (x – 3), which cancelled. A bracket does not stop mattering because it left the page.

Multiplying and Dividing Rational Expressions (continued)

Add, subtract and multiply polynomials

19. $(6x^2 + 5x + 8) - (5x^2 - x + 4)$

$x^2 + 6x + 4$

Only matching terms combine: x^2 with x^2, x with x, numbers with numbers. Line the two brackets up by power before doing anything, and nothing can drift into the wrong column.

Subtracting a bracket flips the sign of EVERY term inside it, not just the first: taking away $5x^2 - x + 4$ is the same as adding $-5x^2 + x - 4$. Rewrite it that way before adding — it is slower and it is the step that gets marks.

Now take the columns one at a time, adding the rewritten signs. x^2: $6 + (-5) = 1$. x: $5 + 1 = 6$. Numbers: $8 + (-4) = 4$.

So the answer is $x^2 + 6x + 4$. Read it back and count the terms — three went in on each side, and three come out.

20. $(9x^2 + 7x + 11) - (7x^2 - 9x + 9)$

$2x^2 + 16x + 2$

Only matching terms combine: x^2 with x^2, x with x, numbers with numbers. Line the two brackets up by power before doing anything, and nothing can drift into the wrong column.

Subtracting a bracket flips the sign of EVERY term inside it, not just the first: taking away $7x^2 - 9x + 9$ is the same as adding $-7x^2 + 9x - 9$. Rewrite it that way before adding — it is slower and it is the step that gets marks.

Now take the columns one at a time, adding the rewritten signs. x^2: $9 + (-7) = 2$. x: $7 + 9 = 16$. Numbers: $11 + (-9) = 2$.

So the answer is $2x^2 + 16x + 2$. Read it back and count the terms — three went in on each side, and three come out.

Solving Rational Equations

Solving rational equations

1. $\dfrac{30}{(x + 7)} = 2$, x = ____

$$x \neq -7$$

$$30/(x + 7) = 2$$

$$30 = 2(x + 7)$$

$$x = 8$$

$$x = 8 \quad \text{and} \quad x \neq -7$$

Before anything else, note what x cannot be. (x + 7) is a denominator, so x may not be −7. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Now clear the fraction: multiply both sides by (x + 7). The left side is left with 30 and the right becomes 2((x + 7)).

Multiply out, gather the x terms on one side and the numbers on the other, and x = 8.

Last, check it against the first line: x may not be −7, and 8 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

2. $\dfrac{6}{(x - 7)} = 6$, x = ____

$$x \neq 7$$

$$6/(x - 7) = 6$$

$$6 = 6(x - 7)$$

$$x = 8$$

$$x = 8 \quad \text{and} \quad x \neq 7$$

Before anything else, note what x cannot be. (x − 7) is a denominator, so x may not be 7. Writing that down FIRST is the habit — after the fractions are cleared there is nothing left on the page to remind you.

Now clear the fraction: multiply both sides by (x − 7). The left side is left with 6 and the right becomes 6((x − 7)).

Multiply out, gather the x terms on one side and the numbers on the other, and x = 8.

Last, check it against the first line: x may not be 7, and 8 is not one of those — so it is a real solution. On another question it would not have been, which is why this step is not optional.

Solutions (6 of 6)

Solving Rational Equations (continued)

Dividing polynomials

25.

$$\begin{array}{rl}
 & \quad x^2 \quad - x \quad - 4 \\
x + 4 & \big)\overline{\,x^3 + 3x^2 - 8x - 20} \\
 & \underline{-x^3 - 4x^2} \\
 & \qquad -x^2 - 8x - 20 \\
 & \qquad \underline{\;\; x^2 + 4x} \\
 & \qquad\qquad -4x - 20 \\
 & \qquad\qquad \underline{\;\; 4x + 16} \\
 & \qquad\qquad\qquad -4
\end{array}$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x + 4)$ is $x^3 + 4x^2$ — and take that away, leaving $-x^2 - 8x - 20$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$-x^2 \div x$ is $-x$, so $-x$ goes in the x place. Multiply the WHOLE divisor by it — $-x(x + 4)$ is $-x^2 - 4x$ — and take that away, leaving $-4x - 20$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$-4x \div x$ is -4, so -4 goes in the constant place. Multiply the WHOLE divisor by it — $-4(x + 4)$ is $-4x - 16$ — and take that away, leaving -4. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

-4 is left and there is no x in it to divide, so that is the remainder. $(x^3 + 3x^2 - 8x - 20) \div (x + 4) = x^2 - x - 4$ remainder -4.

26.

$$\begin{array}{rl}
 & \quad x^2 + 5x + 6 \\
x - 1 & \big)\overline{\,x^3 + 4x^2 \quad + x - 6} \\
 & \underline{-x^3 \quad + x^2} \\
 & \qquad 5x^2 \quad + x - 6 \\
 & \qquad \underline{-5x^2 + 5x} \\
 & \qquad\qquad 6x - 6 \\
 & \qquad\qquad \underline{-6x + 6} \\
 & \qquad\qquad\qquad 0
\end{array}$$

A polynomial goes into the house the same way a number does, and long division starts at the LEFT. Every term sits in its own column, and that is what makes the subtractions line up.

$x^3 \div x$ is x^2, so x^2 goes in the x^2 place. Multiply the WHOLE divisor by it — $x^2(x - 1)$ is $x^3 - x^2$ — and take that away, leaving $5x^2 + x - 6$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$5x^2 \div x$ is $5x$, so $5x$ goes in the x place. Multiply the WHOLE divisor by it — $5x(x - 1)$ is $5x^2 - 5x$ — and take that away, leaving $6x - 6$. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

$6x \div x$ is 6, so 6 goes in the constant place. Multiply the WHOLE divisor by it — $6(x - 1)$ is $6x - 6$ — and take that away, leaving 0. That subtraction is the step that gets skipped, and skipping it is invisible until the last line comes out wrong.

Nothing is left, so $x - 1$ divides it exactly: $(x^3 + 4x^2 + x - 6) \div (x - 1) = x^2 + 5x + 6$.