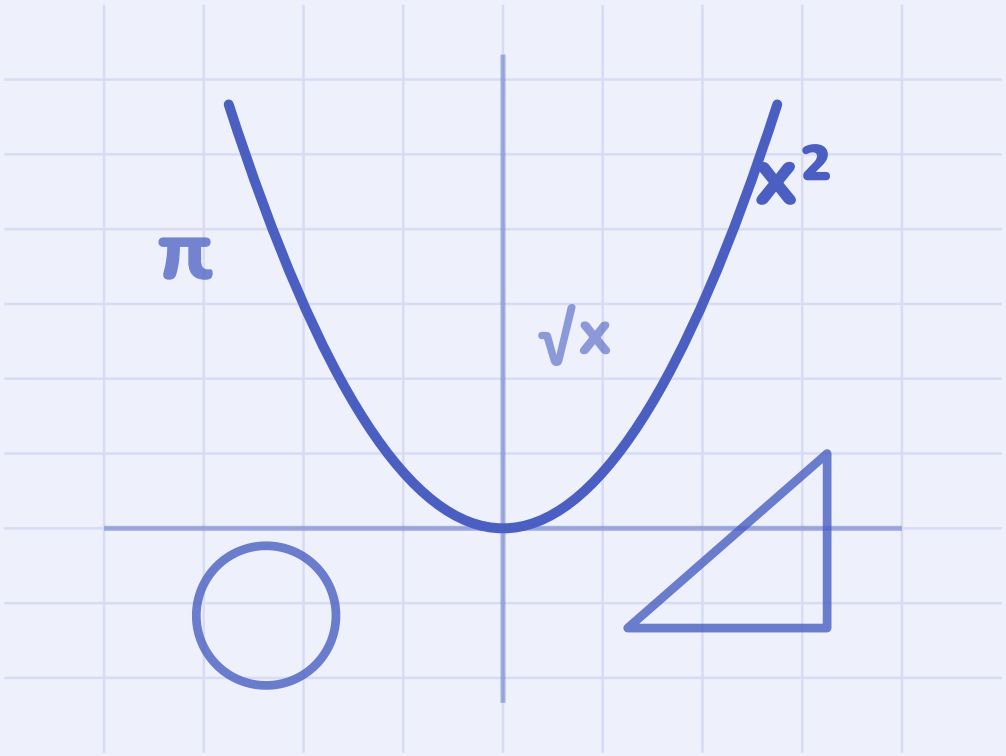
MATH PICNIC · WORKBOOK

Calculus Derivatives Workbook

8 lessons · Ages 17–18 · answer key at the back



Name _____ Date _____

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1-1

The Power Rule

WORKED EXAMPLE

$f(x) = 3x^2 - 3$

$f'(x) = 2 \cdot 3x$

$= 6x$

1. Derivatives

Find f'(x). Bring each power down in front, and take one off the power.

1 $f(x) = -3x^2 - x$

2 $f(x) = -3x^2 - 1$

3 $f(x) = 3x^2 - x - 1$

4 $f(x) = 2x^2 - 2$

5 $f(x) = -3x^2$

6 $f(x) = 2x^2 - 2x + 1$

7 $f(x) = x^2 + 2x + 1$

8 $f(x) = -2x^2 + x - 1$

9 $f(x) = -3x^2 + x - 3$

10 $f(x) = -x^2 - x - 1$

11 $f(x) = 3x^2 + x + 2$

12 $f(x) = -x^2 + x - 1$

The Product Rule

WORKED EXAMPLE

f(x) = (2x + 1)(−2x + 2)

u = 2x + 1, v = −2x + 2
u' = 2, v' = −2
f'(x) = 2(−2x + 2) + (2x + 1)(−2)
= −4x + 4 − 4x − 2
= −8x + 2

2. Derivatives

Find f'(x) with the product rule: u'v + uv'.

1 **f(x) = (x − 1)(−2x − 3)**

2 **f(x) = (−2x − 1)(−x + 1)**

3 **f(x) = (3x + 1)(2x + 3)**

4 **f(x) = (x − 3)(−2x + 1)**

5 **f(x) = (2x + 1)(3x − 1)**

6 **f(x) = (3x + 1)(−3x + 1)**

7 **f(x) = (−3x + 1)(x − 2)**

8 **f(x) = (3x − 3)(2x + 2)**

9 **f(x) = (−x + 1)(3x + 2)**

10 **f(x) = (−2x − 3)(x + 2)**

The Quotient Rule

WORKED EXAMPLE

$$f(x) = \frac{-3x - 1}{x - 3}$$

$u = -3x - 1$, $v = x - 3$

$u' = -3$, $v' = 1$

$$f'(x) = \frac{-3(x - 3) - (-3x - 1)(1)}{(x - 3)^2}$$

$$= \frac{-3x + 9 - (-3x - 1)}{(x - 3)^2}$$

$$= \frac{10}{(x - 3)^2}$$

3. Derivatives

Find f'(x) with the quotient rule: u'v − uv', all over v².

1 $f(x) = \frac{-3x}{2x - 2}$

2 $f(x) = \frac{3x - 1}{2x - 2}$

3 $f(x) = \frac{2x + 3}{x - 1}$

4 $f(x) = \frac{-2x - 2}{x - 1}$

5 $f(x) = \frac{3x + 3}{x - 2}$

6 $f(x) = \frac{-x}{x - 1}$

7 $f(x) = \frac{-3x + 2}{x - 2}$

8 $f(x) = \frac{2x}{3x - 2}$

The Chain Rule

WORKED EXAMPLE

$f(x) = (-2x - 1)^2$

$u = -2x - 1$, $u' = -2$

$f'(x) = 2(-2x - 1) \cdot (-2)$

$= -4(-2x - 1)$

4. Derivatives

Find f'(x) with the chain rule: the derivative of the outside, times u'.

1 **$f(x) = (-3x + 2)^2$**

2 **$f(x) = (-3x - 1)^3$**

3 **$f(x) = (x + 3)^2$**

4 **$f(x) = (-2x + 3)^3$**

5 **$f(x) = (2x - 3)^2$**

6 **$f(x) = (x - 1)^3$**

7 **$f(x) = (x - 2)^2$**

8 **$f(x) = (3x + 3)^3$**

9 **$f(x) = (-3x - 1)^2$**

10 **$f(x) = (x - 3)^3$**

5. Derivatives

Find f'(x). Bring each power down in front, and take one off the power.

11 **$f(x) = x^2 - 3x$**

12 **$f(x) = 2x^2 + 2x + 1$**

13 **$f(x) = -x^2 - x$**

14 **$f(x) = -2x^2 + 3x - 2$**

15 **$f(x) = -3x^2 + x$**

16 **$f(x) = -3x^2 + x + 1$**

Choosing the Rule

WORKED EXAMPLE

$f(x) = x^2 + x - 2$

$f'(x) = 2x + 1$

6. Derivatives

Find f'(x). Decide which rule the function needs before you start.

1 $f(x) = -2x^2 + 1$

2 $f(x) = (-x + 2)(-3x - 1)$

3 $f(x) = (-2x + 1)^2$

4 $f(x) = \dfrac{2x + 1}{2x - 3}$

5 $f(x) = (-2x + 1)(-x + 1)$

6 $f(x) = 2x^2 - 2x - 2$

7 $f(x) = \dfrac{x - 1}{x + 2}$

8 $f(x) = (-2x - 3)^2$

9 $f(x) = (-x - 2)(3x + 2)$

10 $f(x) = \dfrac{-x + 2}{3x + 3}$

7. Derivatives

Find f'(x) with the product rule: u'v + uv'.

11 $f(x) = (3x - 1)(x + 1)$

12 $f(x) = (2x - 2)(-2x + 2)$

13 $f(x) = (-x + 1)(x - 1)$

14 $f(x) = (3x - 2)(-3x - 3)$

15 $f(x) = (x - 3)(3x - 2)$

16 $f(x) = (-x - 1)(x - 1)$

Derivatives of sin, cos, e^x and ln x

WORKED EXAMPLE

$$f(x) = -e^x - \ln x$$

$$f'(x) = -e^x - \frac{1}{x}$$

8. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

1 $f(x) = 2 \sin x + \cos x$

2 $f(x) = 2 \sin x - e^x + 3 \ln x$

3 $f(x) = 5 \sin x - 5e^x + \ln x$

4 $f(x) = -3 \cos x + 2e^x$

5 $f(x) = -\sin x + 3e^x + \ln x$

6 $f(x) = -3 \sin x - 3 \cos x - 5 \ln x$

7 $f(x) = 4 \sin x + 2 \ln x$

8 $f(x) = -\sin x + 2 \cos x - 4 \ln x$

9 $f(x) = 4 \cos x - 2e^x - 4 \ln x$

10 $f(x) = 2 \sin x + 3 \cos x$

11 $f(x) = 3 \sin x - 2e^x - 2 \ln x$

12 $f(x) = -5 \sin x + 5e^x + 2 \ln x$

9. Derivatives

Find f'(x) with the quotient rule: u'v − uv', all over v^2.

13 $f(x) = \dfrac{2x + 2}{2x + 1}$

14 $f(x) = \dfrac{3x + 1}{2x + 2}$

15 $f(x) = \dfrac{3x + 3}{2x + 1}$

16 $f(x) = \dfrac{x - 1}{2x - 1}$

9. Derivatives (continued)

Find f'(x) with the quotient rule: u'v − uv', all over v².

17 $f(x) = \dfrac{-2x - 3}{2x + 1}$

18 $f(x) = \dfrac{3x - 3}{2x + 2}$

The Chain Rule with sin, cos, e^x and ln

WORKED EXAMPLE

$f(x) = -\ln(-x - 5)$

$u = -x - 5,\ u' = -1$

$$f'(x) = -\frac{1}{-x-5} \cdot (-1)$$

$$= \frac{1}{-x-5}$$

10. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

1 $f(x) = 5e^{-2x+1}$

2 $f(x) = e^{3x+4}$

3 $f(x) = 4\ln(3x + 4)$

4 $f(x) = 3\sin(4x + 1)$

5 $f(x) = -2e^{2x+1}$

6 $f(x) = 3\ln(-3x - 5)$

7 $f(x) = -5e^{2x-2}$

8 $f(x) = 4\sin(4x + 2)$

9 $f(x) = 3\sin(2x - 2)$

10 $f(x) = -3\cos(3x + 5)$

11. Derivatives

Find f'(x) with the chain rule: the derivative of the outside, times u'.

11 $f(x) = (-2x + 2)^2$

12 $f(x) = (-x - 2)^3$

13 $f(x) = (3x - 3)^3$

14 $f(x) = (x + 1)^3$

15 $f(x) = (x + 3)^2$

16 $f(x) = (-3x - 2)^3$

Product and Quotient Rules with Functions

WORKED EXAMPLE

$f(x) = 2e^x \cos x$

$u = 2e^x$, $v = \cos x$
$u' = 2e^x$, $v' = -\sin x$
$f'(x) = 2e^x \cos x - 2e^x \sin x$
$= 2e^x(\cos x - \sin x)$

12. Derivatives

Find f'(x). Decide whether f is a product, a quotient or a chain, then take u and v.

1 $f(x) = \dfrac{\sin x}{x}$

2 $f(x) = 4e^x \cos x$

3 $f(x) = \dfrac{3e^x}{x}$

4 $f(x) = \dfrac{\cos x}{x}$

5 $f(x) = \dfrac{5e^x}{x}$

6 $f(x) = -5e^x \cos x$

7 $f(x) = \dfrac{5 \ln x}{x}$

8 $f(x) = 2e^x \sin x$

13. Derivatives

Find f'(x). Decide which rule the function needs before you start.

9 $f(x) = (x + 2)^3$

10 $f(x) = -3x^2 + 2x$

11 $f(x) = (-2x + 3)(-3x + 3)$

12 $f(x) = (-2x - 2)^3$

13. Derivatives (continued)

Find f'(x). Decide which rule the function needs before you start.

13 $f(x) = (3x + 1)(-3x - 3)$

14 $f(x) = (-3x + 1)(2x - 1)$

Answer Key (1 of 2)

The Power Rule

Derivatives

1. $f(x) = -3x^2 - x \rightarrow f'(x) = -6x - 1$
2. $f(x) = -3x^2 - 1 \rightarrow f'(x) = -6x$
3. $f(x) = 3x^2 - x - 1 \rightarrow f'(x) = 6x - 1$
4. $f(x) = 2x^2 - 2 \rightarrow f'(x) = 4x$
5. $f(x) = -3x^2 \rightarrow f'(x) = -6x$
6. $f(x) = 2x^2 - 2x + 1 \rightarrow f'(x) = 4x - 2$
7. $f(x) = x^2 + 2x + 1 \rightarrow f'(x) = 2x + 2$
8. $f(x) = -2x^2 + x - 1 \rightarrow f'(x) = -4x + 1$
9. $f(x) = -3x^2 + x - 3 \rightarrow f'(x) = -6x + 1$
10. $f(x) = -x^2 - x - 1 \rightarrow f'(x) = -2x - 1$
11. $f(x) = 3x^2 + x + 2 \rightarrow f'(x) = 6x + 1$
12. $f(x) = -x^2 + x - 1 \rightarrow f'(x) = -2x + 1$

The Product Rule

Derivatives

1. $f(x) = (x - 1)(-2x - 3) \rightarrow f'(x) = -4x - 1$
2. $f(x) = (-2x - 1)(-x + 1) \rightarrow f'(x) = 4x - 1$
3. $f(x) = (3x + 1)(2x + 3) \rightarrow f'(x) = 12x + 11$
4. $f(x) = (x - 3)(-2x + 1) \rightarrow f'(x) = -4x + 7$
5. $f(x) = (2x + 1)(3x - 1) \rightarrow f'(x) = 12x + 1$
6. $f(x) = (3x + 1)(-3x + 1) \rightarrow f'(x) = -18x$
7. $f(x) = (-3x + 1)(x - 2) \rightarrow f'(x) = -6x + 7$
8. $f(x) = (3x - 3)(2x + 2) \rightarrow f'(x) = 12x$
9. $f(x) = (-x + 1)(3x + 2) \rightarrow f'(x) = -6x + 1$
10. $f(x) = (-2x - 3)(x + 2) \rightarrow f'(x) = -4x - 7$

The Quotient Rule

Derivatives

1. $f(x) = \frac{-3x}{2x - 2} \rightarrow f'(x) = \frac{6}{2x - 2^2}$
2. $f(x) = \frac{3x - 1}{2x - 2} \rightarrow f'(x) = -\frac{4}{2x - 2^2}$
3. $f(x) = \frac{2x + 3}{x - 1} \rightarrow f'(x) = -\frac{5}{x - 1^2}$
4. $f(x) = \frac{-2x - 2}{x - 1} \rightarrow f'(x) = \frac{4}{x - 1^2}$
5. $f(x) = \frac{3x + 3}{x - 2} \rightarrow f'(x) = -\frac{9}{x - 2^2}$
6. $f(x) = \frac{-x}{x - 1} \rightarrow f'(x) = \frac{1}{x - 1^2}$
7. $f(x) = \frac{-3x + 2}{x - 2} \rightarrow f'(x) = \frac{4}{x - 2^2}$
8. $f(x) = \frac{2x}{3x - 2} \rightarrow f'(x) = -\frac{4}{3x - 2^2}$

The Chain Rule

Derivatives

1. $f(x) = (-3x + 2)^2 \rightarrow f'(x) = -6(-3x + 2)$
2. $f(x) = (-3x - 1)^3 \rightarrow f'(x) = -9(-3x - 1)^2$
3. $f(x) = (x + 3)^2 \rightarrow f'(x) = 2(x + 3)$
4. $f(x) = (-2x + 3)^3 \rightarrow f'(x) = -6(-2x + 3)^2$
5. $f(x) = (2x - 3)^2 \rightarrow f'(x) = 4(2x - 3)$
6. $f(x) = (x - 1)^3 \rightarrow f'(x) = 3(x - 1)^2$
7. $f(x) = (x - 2)^2 \rightarrow f'(x) = 2(x - 2)$
8. $f(x) = (3x + 3)^3 \rightarrow f'(x) = 9(3x + 3)^2$

The Chain Rule (continued)

Derivatives (continued)

9. $f(x) = (-3x - 1)^2 \rightarrow f'(x) = -6(-3x - 1)$
10. $f(x) = (x - 3)^3 \rightarrow f'(x) = 3(x - 3)^2$

Derivatives

11. $f(x) = x^2 - 3x \rightarrow f'(x) = 2x - 3$
12. $f(x) = 2x^2 + 2x + 1 \rightarrow f'(x) = 4x + 2$
13. $f(x) = -x^2 - x \rightarrow f'(x) = -2x - 1$
14. $f(x) = -2x^2 + 3x - 2 \rightarrow f'(x) = -4x + 3$
15. $f(x) = -3x^2 + x \rightarrow f'(x) = -6x + 1$
16. $f(x) = -3x^2 + x + 1 \rightarrow f'(x) = -6x + 1$

Choosing the Rule

Derivatives

1. $f(x) = -2x^2 + 1 \rightarrow f'(x) = -4x$
2. $f(x) = (-x + 2)(-3x - 1) \rightarrow f'(x) = 6x - 5$
3. $f(x) = (-2x + 1)^2 \rightarrow f'(x) = -4(-2x + 1)$
4. $f(x) = \frac{2x + 1}{2x - 3} \rightarrow f'(x) = -\frac{8}{2x - 3^2}$
5. $f(x) = (-2x + 1)(-x + 1) \rightarrow f'(x) = 4x - 3$
6. $f(x) = 2x^2 - 2x - 2 \rightarrow f'(x) = 4x - 2$
7. $f(x) = \frac{x - 1}{x + 2} \rightarrow f'(x) = \frac{3}{x + 2^2}$
8. $f(x) = (-2x - 3)^2 \rightarrow f'(x) = -4(-2x - 3)$
9. $f(x) = (-x - 2)(3x + 2) \rightarrow f'(x) = -6x - 8$
10. $f(x) = \frac{-x + 2}{3x + 3} \rightarrow f'(x) = -\frac{9}{3x + 3^2}$

Derivatives

11. $f(x) = (3x - 1)(x + 1) \rightarrow f'(x) = 6x + 2$
12. $f(x) = (2x - 2)(-2x + 2) \rightarrow f'(x) = -8x + 8$
13. $f(x) = (-x + 1)(x - 1) \rightarrow f'(x) = -2x + 2$
14. $f(x) = (3x - 2)(-3x - 3) \rightarrow f'(x) = -18x - 3$
15. $f(x) = (x - 3)(3x - 2) \rightarrow f'(x) = 6x - 11$
16. $f(x) = (-x - 1)(x - 1) \rightarrow f'(x) = -2x$

Derivatives of sin, cos, e^x and ln x

Derivatives

1. $f(x) = 2 \sin x + \cos x \rightarrow f'(x) = 2 \cos x - \sin x$
2. $f(x) = 2 \sin x - e^x + 3 \ln x \rightarrow f'(x) = 2 \cos x - e^x + \frac{3}{x}$
3. $f(x) = 5 \sin x - 5e^x + \ln x \rightarrow f'(x) = 5 \cos x - 5e^x + \frac{1}{x}$
4. $f(x) = -3 \cos x + 2e^x \rightarrow f'(x) = 3 \sin x + 2e^x$
5. $f(x) = -\sin x + 3e^x + \ln x \rightarrow f'(x) = -\cos x + 3e^x + \frac{1}{x}$
6. $f(x) = -3 \sin x - 3 \cos x - 5 \ln x \rightarrow f'(x) = -3 \cos x + 3 \sin x - \frac{5}{x}$
7. $f(x) = 4 \sin x + 2 \ln x \rightarrow f'(x) = 4 \cos x + \frac{2}{x}$
8. $f(x) = -\sin x + 2 \cos x - 4 \ln x \rightarrow f'(x) = -\cos x - 2 \sin x - \frac{4}{x}$
9. $f(x) = 4 \cos x - 2e^x - 4 \ln x \rightarrow f'(x) = -4 \sin x - 2e^x - \frac{4}{x}$
10. $f(x) = 2 \sin x + 3 \cos x \rightarrow f'(x) = 2 \cos x - 3 \sin x$
11. $f(x) = 3 \sin x - 2e^x - 2 \ln x \rightarrow f'(x) = 3 \cos x - 2e^x - \frac{2}{x}$
12. $f(x) = -5 \sin x + 5e^x + 2 \ln x \rightarrow f'(x) = -5 \cos x + 5e^x + \frac{2}{x}$

Answer Key (2 of 2)

Derivatives of sin, cos, e^x and ln x (continued)

Derivatives

13. $f(x) = \frac{2x+2}{2x+1} \rightarrow f'(x) = -\frac{2}{2x+1}^2$
14. $f(x) = \frac{3x+1}{2x+2} \rightarrow f'(x) = \frac{4}{2x+2}^2$
15. $f(x) = \frac{3x+3}{2x+1} \rightarrow f'(x) = -\frac{3}{2x+1}^2$
16. $f(x) = \frac{x-1}{2x-1} \rightarrow f'(x) = \frac{1}{2x-1}^2$
17. $f(x) = \frac{-2x-3}{2x+1} \rightarrow f'(x) = \frac{4}{2x+1}^2$
18. $f(x) = \frac{3x-3}{2x+2} \rightarrow f'(x) = \frac{12}{2x+2}^2$

The Chain Rule with sin, cos, e^x and ln

Derivatives

1. $f(x) = 5e^{-2x+1} \rightarrow f'(x) = -10e^{-2x+1}$
2. $f(x) = e^{3x+4} \rightarrow f'(x) = 3e^{3x+4}$
3. $f(x) = 4\ln(3x+4) \rightarrow f'(x) = \frac{12}{3x+4}$
4. $f(x) = 3\sin(4x+1) \rightarrow f'(x) = 12\cos(4x+1)$
5. $f(x) = -2e^{2x+1} \rightarrow f'(x) = -4e^{2x+1}$
6. $f(x) = 3\ln(-3x-5) \rightarrow f'(x) = -\frac{9}{-3x-5}$
7. $f(x) = -5e^{2x-2} \rightarrow f'(x) = -10e^{2x-2}$
8. $f(x) = 4\sin(4x+2) \rightarrow f'(x) = 16\cos(4x+2)$
9. $f(x) = 3\sin(2x-2) \rightarrow f'(x) = 6\cos(2x-2)$
10. $f(x) = -3\cos(3x+5) \rightarrow f'(x) = 9\sin(3x+5)$

Derivatives

11. $f(x) = (-2x+2)^2 \rightarrow f'(x) = -4(-2x+2)$
12. $f(x) = (-x-2)^3 \rightarrow f'(x) = -3(-x-2)^2$
13. $f(x) = (3x-3)^3 \rightarrow f'(x) = 9(3x-3)^2$
14. $f(x) = (x+1)^3 \rightarrow f'(x) = 3(x+1)^2$
15. $f(x) = (x+3)^2 \rightarrow f'(x) = 2(x+3)$
16. $f(x) = (-3x-2)^3 \rightarrow f'(x) = -9(-3x-2)^2$

Product and Quotient Rules with Functions

Derivatives

1. $f(x) = \frac{\sin x}{x} \rightarrow f'(x) = \frac{x\cos x - \sin x}{x^2}$
2. $f(x) = 4e^x \cos x \rightarrow f'(x) = 4e^x(\cos x - \sin x)$
3. $f(x) = \frac{3e^x}{x} \rightarrow f'(x) = \frac{3e^x(x-1)}{x^2}$
4. $f(x) = \frac{\cos x}{x} \rightarrow f'(x) = \frac{-(x\sin x + \cos x)}{x^2}$
5. $f(x) = \frac{5e^x}{x} \rightarrow f'(x) = \frac{5e^x(x-1)}{x^2}$
6. $f(x) = -5e^x \cos x \rightarrow f'(x) = -5e^x(\cos x - \sin x)$
7. $f(x) = \frac{5\ln x}{x} \rightarrow f'(x) = \frac{5(1-\ln x)}{x^2}$
8. $f(x) = 2e^x \sin x \rightarrow f'(x) = 2e^x(\sin x + \cos x)$

Derivatives

9. $f(x) = (x+2)^3 \rightarrow f'(x) = 3(x+2)^2$
10. $f(x) = -3x^2 + 2x \rightarrow f'(x) = -6x + 2$
11. $f(x) = (-2x+3)(-3x+3) \rightarrow f'(x) = 12x - 15$
12. $f(x) = (-2x-2)^3 \rightarrow f'(x) = -6(-2x-2)^2$
13. $f(x) = (3x+1)(-3x-3) \rightarrow f'(x) = -18x - 12$
14. $f(x) = (-3x+1)(2x-1) \rightarrow f'(x) = -12x + 5$

Solutions

The Power Rule

Derivatives

1. **$f(x) = -3x^2 - x$**

$$f'(x) = -2 \cdot 3x - 1$$
$$= -6x - 1$$

A sum of powers of x: take the derivative term by term. For each term, bring the power down in front as a multiplier and take one off the power — the derivative of x^n is nx^{n-1}.

Multiply out each term. That is the derivative.

2. **$f(x) = -3x^2 - 1$**

$$f'(x) = -2 \cdot 3x$$
$$= -6x$$

A sum of powers of x: take the derivative term by term. For each term, bring the power down in front as a multiplier and take one off the power — the derivative of x^n is nx^{n-1}. The constant −1 does not change as x moves, so its derivative is 0 and it is gone.

Multiply out each term. That is the derivative.