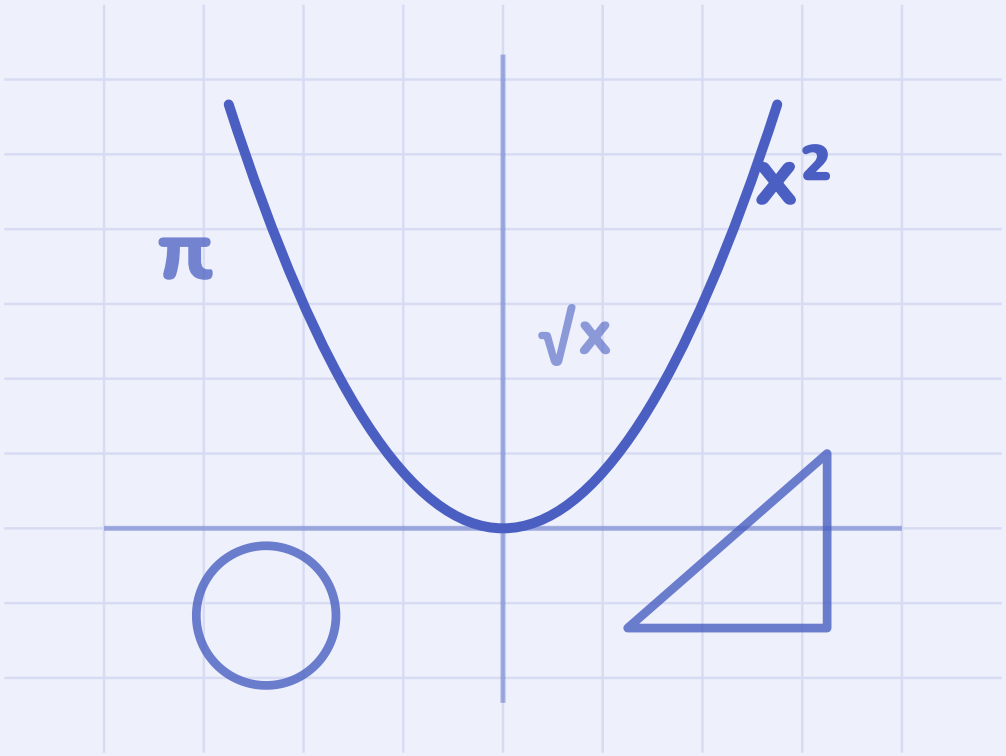
MATH PICNIC · WORKBOOK

# Calculus Advanced Workbook

22 lessons · Ages 17–18 · answer key at the back




Name _____ Date _____

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# The Power Rule

WORKED EXAMPLE

$f(x) = 3x^2 - 3$

$f'(x) = 2 \cdot 3x$

$= 6x$

## 1. Derivatives

Find f'(x). Bring each power down in front, and take one off the power.

1 $f(x) = 3x^2 - x - 1$

2 $f(x) = -3x^2 + x - 3$

3 $f(x) = 2x^2 - 2x + 1$

4 $f(x) = 3x^2 - 3x + 3$

5 $f(x) = 3x^2 + x + 2$

6 $f(x) = -3x^2 + 3x + 1$

7 $f(x) = 3x^2 - 2x + 2$

8 $f(x) = 2x^2 - 2x - 2$

9 $f(x) = 3x^2 + 2x - 3$

10 $f(x) = 2x^2 + 3x - 3$

11 $f(x) = x^2 + 3x + 1$

12 $f(x) = -3x^2 + 2x + 2$

# The Product Rule

WORKED EXAMPLE

**f(x) = (2x + 1)(−2x + 2)**

u = 2x + 1, v = −2x + 2
u' = 2, v' = −2
f'(x) = 2(−2x + 2) + (2x + 1)(−2)
= −4x + 4 − 4x − 2
= −8x + 2

## 2. Derivatives

Find f'(x) with the product rule: u'v + uv'.

1 **f(x) = (3x + 1)(2x + 3)**

2 **f(x) = (3x − 3)(2x + 2)**

3 **f(x) = (3x + 1)(−3x + 1)**

4 **f(x) = (3x + 3)(x − 3)**

5 **f(x) = (−2x − 3)(x + 2)**

6 **f(x) = (−3x − 2)(2x + 3)**

7 **f(x) = (−2x + 3)(3x − 3)**

8 **f(x) = (−3x − 2)(2x − 1)**

9 **f(x) = (−3x − 2)(−2x + 3)**

10 **f(x) = (−2x + 3)(−3x + 3)**

# The Quotient Rule

WORKED EXAMPLE

$$f(x) = \frac{-3x}{2x - 2}$$

$u = -3x$, $v = 2x - 2$

$u' = -3$, $v' = 2$

$$f'(x) = \frac{-3(2x - 2) - (-3x)(2)}{(2x - 2)^2}$$

$$= \frac{-6x + 6 - (-6x)}{(2x - 2)^2}$$

$$= \frac{6}{(2x - 2)^2}$$

## 3. Derivatives

Find $f'(x)$ with the quotient rule: $u'v - uv'$, all over $v^2$.

1 $f(x) = \dfrac{3x - 1}{2x - 2}$

2 $f(x) = \dfrac{3x + 3}{x - 2}$

3 $f(x) = \dfrac{-3x + 2}{x - 2}$

4 $f(x) = \dfrac{3x - 2}{3x + 2}$

5 $f(x) = \dfrac{-2x - 3}{2x + 1}$

6 $f(x) = \dfrac{-2x + 2}{2x - 3}$

## 3. Derivatives (continued)

Find f'(x) with the quotient rule: u'v − uv', all over v².

7 $f(x) = \dfrac{-2x - 2}{2x + 3}$

8 $f(x) = \dfrac{-3x - 1}{3x - 1}$

# The Chain Rule

WORKED EXAMPLE

**$f(x) = (-2x - 1)^2$**

$u = -2x - 1$, $u' = -2$

$f'(x) = 2(-2x - 1) \cdot (-2)$

$= -4(-2x - 1)$

## 4. Derivatives

Find f'(x) with the chain rule: the derivative of the outside, times u'.

1 $f(x) = (-3x - 1)^3$

2 $f(x) = (-2x + 3)^3$

3 $f(x) = (x - 1)^3$

4 $f(x) = (3x + 3)^3$

5 $f(x) = (x - 3)^3$

6 $f(x) = (3x - 3)^3$

7 $f(x) = (-3x - 2)^3$

8 $f(x) = (-x - 1)^3$

9 $f(x) = (2x - 3)^3$

10 $f(x) = (2x + 3)^3$

## 5. Derivatives

Find f'(x). Bring each power down in front, and take one off the power.

11 $f(x) = 2x^2 + 2x + 1$

12 $f(x) = -2x^2 + 3x - 2$

13 $f(x) = -3x^2 + x + 1$

14 $f(x) = -x^2 + 3x + 3$

15 $f(x) = -x^2 - 3x - 1$

16 $f(x) = 3x^2 - x + 3$

# Choosing the Rule

WORKED EXAMPLE

$f(x) = x^2 + x - 2$

$f'(x) = 2x + 1$

## 6. Derivatives

Find f'(x). Decide which rule the function needs before you start.

1 $f(x) = (-x + 2)(-3x - 1)$

2 $f(x) = \dfrac{2x + 1}{2x - 3}$

3 $f(x) = (-2x + 1)(-x + 1)$

4 $f(x) = \dfrac{x - 1}{x + 2}$

5 $f(x) = (-x - 2)(3x + 2)$

6 $f(x) = \dfrac{-x + 2}{3x + 3}$

7 $f(x) = \dfrac{x + 2}{3x - 2}$

8 $f(x) = (x + 3)(-x + 3)$

9 $f(x) = \dfrac{-x + 3}{3x + 2}$

10 $f(x) = \dfrac{2x - 2}{3x - 2}$

## 7. Derivatives

Find f'(x) with the product rule: u'v + uv'.

11 $f(x) = (2x - 2)(-2x + 2)$

12 $f(x) = (3x - 2)(-3x - 3)$

13 $f(x) = (x - 3)(3x - 2)$

14 $f(x) = (-3x - 2)(2x + 3)$

15 $f(x) = (-2x - 3)(-x - 3)$

16 $f(x) = (3x + 2)(2x + 3)$

# Derivatives of sin, cos, $e^x$ and ln x

WORKED EXAMPLE

$$f(x) = -e^x - \ln x$$

$$f'(x) = -e^x - \frac{1}{x}$$

## 8. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

1 $f(x) = 2\sin x - e^x + 3\ln x$

2 $f(x) = 5\sin x - 5e^x + \ln x$

3 $f(x) = -\sin x + 3e^x + \ln x$

4 $f(x) = -3\sin x - 3\cos x - 5\ln x$

5 $f(x) = -\sin x + 2\cos x - 4\ln x$

6 $f(x) = 4\cos x - 2e^x - 4\ln x$

7 $f(x) = -5\sin x + 5e^x + 2\ln x$

8 $f(x) = 3\sin x - 2e^x - 2\ln x$

9 $f(x) = -5\cos x + 5e^x - 4\ln x$

10 $f(x) = 4\cos x - 4e^x - \ln x$

11 $f(x) = 4\cos x - 4\ln x$

12 $f(x) = 5\cos x - 4e^x + 3\ln x$

## 9. Derivatives

Find f'(x) with the quotient rule: u'v − uv', all over $v^2$.

13 $f(x) = \dfrac{3x + 1}{2x + 2}$

14 $f(x) = \dfrac{3x + 3}{2x + 1}$

15 $f(x) = \dfrac{-2x - 3}{2x + 1}$

16 $f(x) = \dfrac{3x - 3}{2x + 2}$

## 9. Derivatives (continued)

Find f'(x) with the quotient rule: u'v − uv', all over v².

17 $f(x) = \dfrac{3x + 2}{2x - 1}$

18 $f(x) = \dfrac{3x + 3}{x - 3}$

# The Chain Rule with sin, cos, $e^x$ and ln

WORKED EXAMPLE

**$f(x) = -\ln(-x - 5)$**

$u = -x - 5,\ u' = -1$

$$f'(x) = -\frac{1}{-x-5} \cdot (-1)$$

$$= \frac{1}{-x-5}$$

## 10. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

1 **$f(x) = e^{3x+4}$**

2 **$f(x) = 4\ln(3x + 4)$**

3 **$f(x) = 3\sin(4x + 1)$**

4 **$f(x) = 3\ln(-3x - 5)$**

5 **$f(x) = -5e^{2x-2}$**

6 **$f(x) = 4\sin(4x + 2)$**

7 **$f(x) = -3\cos(3x + 5)$**

8 **$f(x) = 5\ln(2x + 3)$**

9 **$f(x) = -4\cos(3x + 4)$**

10 **$f(x) = -3\sin(3x - 5)$**

## 11. Derivatives

Find f'(x) with the chain rule: the derivative of the outside, times u'.

11 **$f(x) = (-x - 2)^3$**

12 **$f(x) = (3x - 3)^3$**

13 **$f(x) = (x + 1)^3$**

14 **$f(x) = (-3x - 2)^3$**

15 **$f(x) = (3x + 1)^3$**

16 **$f(x) = (3x + 2)^3$**

# Product and Quotient Rules with Functions

WORKED EXAMPLE

**$f(x) = 2e^x \cos x$**

$u = 2e^x$, $v = \cos x$
$u' = 2e^x$, $v' = -\sin x$
$f'(x) = 2e^x \cos x - 2e^x \sin x$
$= 2e^x(\cos x - \sin x)$

## 12. Derivatives

Find f'(x). Decide whether f is a product, a quotient or a chain, then take u and v.

1 $f(x) = \dfrac{\sin x}{x}$

2 $f(x) = \dfrac{3e^x}{x}$

3 $f(x) = \dfrac{\cos x}{x}$

4 $f(x) = \dfrac{5e^x}{x}$

5 $f(x) = -5e^x \cos x$

6 $f(x) = \dfrac{5 \ln x}{x}$

7 $f(x) = \dfrac{4 \sin x}{x}$

8 $f(x) = 5e^x \cos x$

## 13. Derivatives

Find f'(x). Decide which rule the function needs before you start.

9 $f(x) = (x + 2)^3$

10 $f(x) = (-2x + 3)(-3x + 3)$

## 13. Derivatives (continued)

Find f'(x). Decide which rule the function needs before you start.

11 $f(x) = (-2x - 2)^3$

12 $f(x) = (3x + 1)(-3x - 3)$

13 $f(x) = (-3x + 1)(2x - 1)$

14 $f(x) = \dfrac{x}{x - 3}$

# The Slope of a Curve

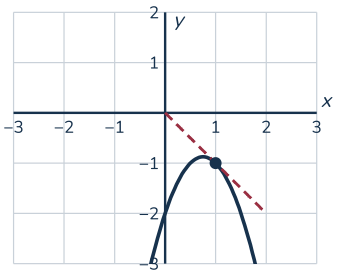
WORKED EXAMPLE

**$f(x) = -2x^2 + 3x - 2,\ x = 1$**

$f'(x) = -4x + 3$

$f'(1) = -4(1) + 3$

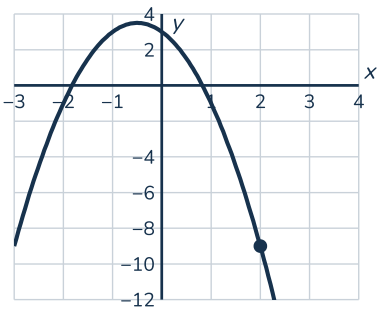
$= -1$



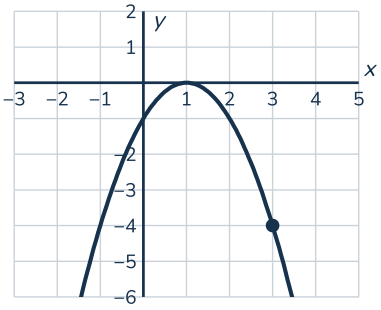

## 14. Using derivatives

Find the slope of each curve at the point: find f'(x), then put x in.

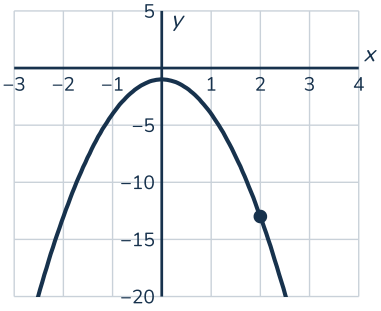
1 **$f(x) = -2x^2 - 2x + 3,\ x = 2$**




2 **$f(x) = -x^2 + 2x - 1,\ x = 3$**



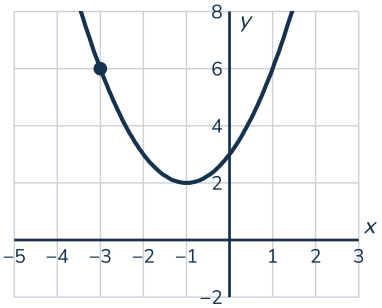

3 **$f(x) = -3x^2 - 1,\ x = 2$**



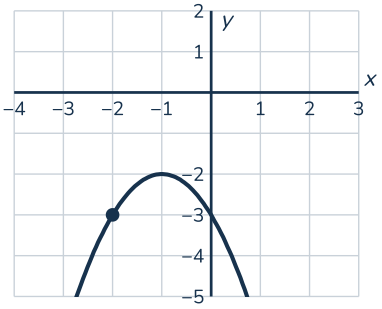
## 14. Using derivatives (continued)

Find the slope of each curve at the point: find f'(x), then put x in.

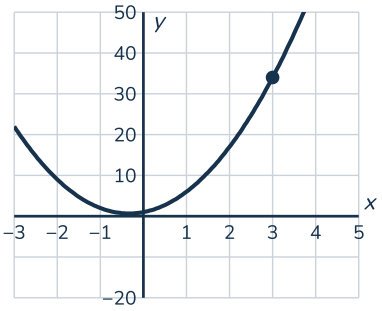
4 **$f(x) = x^2 + 2x + 3,\ x = -3$**




5 **$f(x) = -x^2 - 2x - 3,\ x = -2$**




6 **$f(x) = 3x^2 + 2x + 1,\ x = 3$**




## 15. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

7 **$f(x) = -\sin x + 2\cos x - 4\ln x$**

8 **$f(x) = 4\cos x - 2e^x - 4\ln x$**

9 **$f(x) = 5\cos x - 4e^x$**

10 **$f(x) = 5\cos x + 2e^x + 5\ln x$**

11 **$f(x) = -2\cos x + 5\ln x$**

12 **$f(x) = 5\cos x + 4e^x + 2\ln x$**

# Tangent Lines

WORKED EXAMPLE

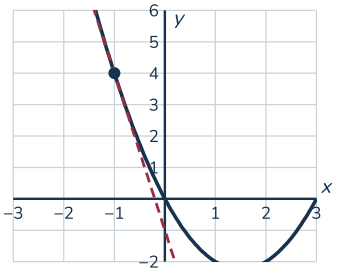
**$f(x) = x^2 - 3x,\ x = -1$**

$f'(x) = 2x - 3$

$m = f'(-1) = -5,\ f(-1) = 4$

$y - 4 = -5(x + 1)$

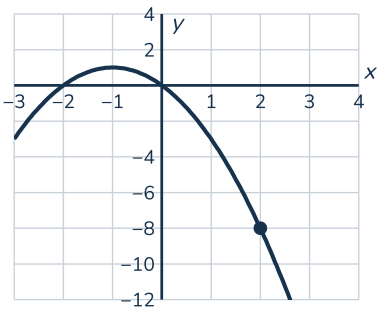
$y = -5x - 1$



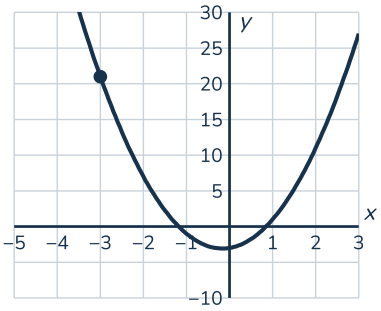

## 16. Using derivatives

Find the equation of the tangent line at the point given.

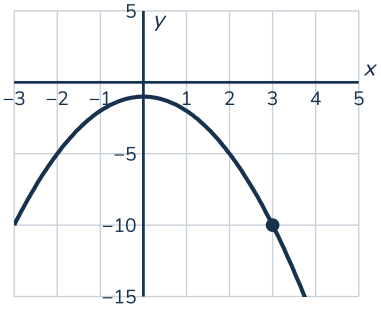
1 **$f(x) = -x^2 - 2x,\ x = 2$**




2 **$f(x) = 3x^2 + x - 3,\ x = -3$**



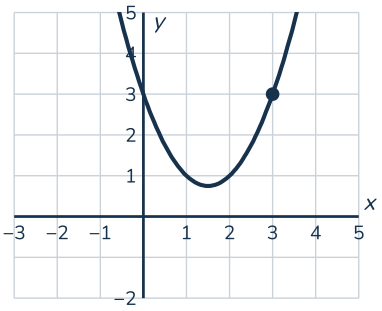

3 **$f(x) = -x^2 - 1,\ x = 3$**



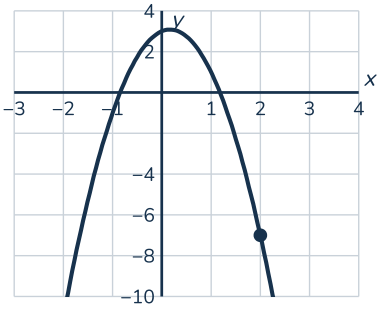
## 16. Using derivatives (continued)

Find the equation of the tangent line at the point given.

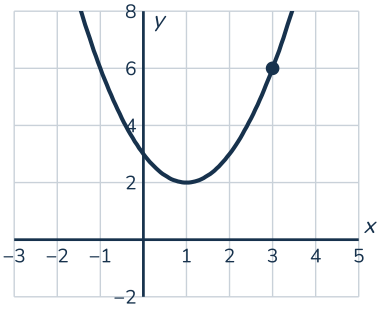
4 $f(x) = x^2 - 3x + 3,\ x = 3$




5 $f(x) = -3x^2 + x + 3,\ x = 2$




6 $f(x) = x^2 - 2x + 3,\ x = 3$




## 17. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

7 $f(x) = -5e^{3x-1}$

8 $f(x) = 4e^{3x-5}$

9 $f(x) = \ln(4x + 3)$

10 $f(x) = -3\sin(4x - 3)$

11 $f(x) = 2\sin(4x + 1)$

12 $f(x) = -4\ln(4x - 1)$

# Stationary Points

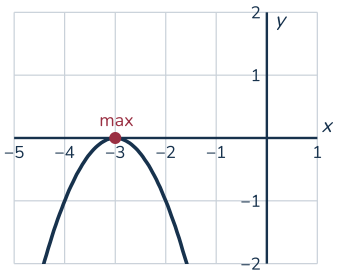
WORKED EXAMPLE

**f(x) = −x² − 6x − 9**

f'(x) = −2x − 6 = 0

x = −3, f(−3) = 0

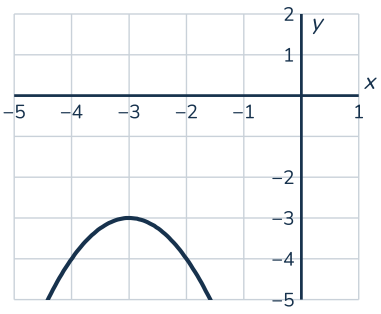
f''(x) = −2 < 0, so a maximum



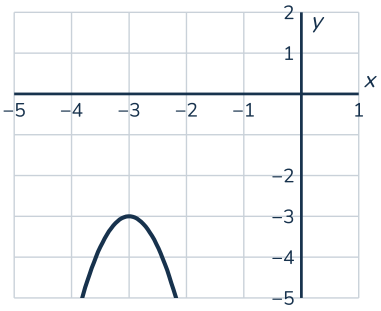

## 18. Using derivatives

Find where f'(x) = 0 and say if each point is a maximum or a minimum.

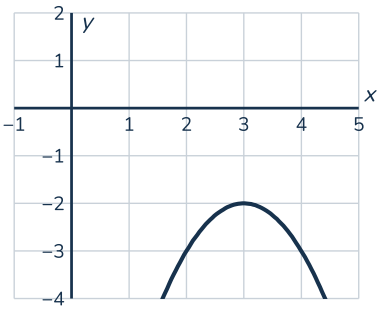
1 **f(x) = −x² − 6x − 12**




2 **f(x) = −3x² − 18x − 30**



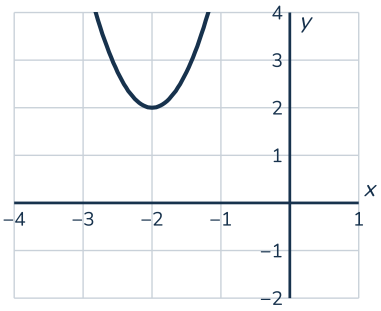

3 **f(x) = −x² + 6x − 11**



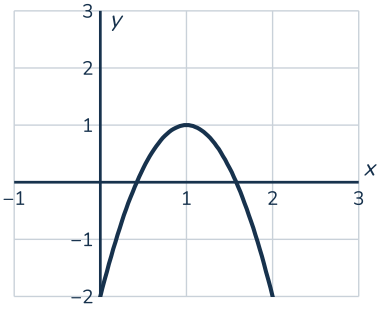
## 18. Using derivatives (continued)

Find where f'(x) = 0 and say if each point is a maximum or a minimum.

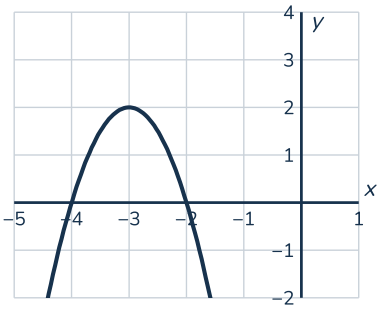
4 $f(x) = 3x^2 + 12x + 14$




5 $f(x) = -3x^2 + 6x - 2$




6 $f(x) = -2x^2 - 12x - 16$




## 19. Derivatives

Find f'(x). Decide whether f is a product, a quotient or a chain, then take u and v.

7 $f(x) = \dfrac{2 \ln x}{x}$

8 $f(x) = \dfrac{3 \cos x}{x}$

9 $f(x) = \dfrac{e^x}{x}$

10 $f(x) = \dfrac{3 \sin x}{x}$

# Indefinite Integrals

WORKED EXAMPLE

$$\int (4x)\,dx = \frac{4x^2}{2} + C = 2x^2 + C$$

## 20. Integrals

Raise each power by one and divide by the new power. Don't forget + C.

1 $\int (-4x + 2)\,dx$

2 $\int (-6x - 2)\,dx$

3 $\int (2x + 3)\,dx$

4 $\int (6x - 3)\,dx$

5 $\int (-4x - 2)\,dx$

6 $\int (-4x - 3)\,dx$

7 $\int (-6x + 3)\,dx$

8 $\int (-2x - 3)\,dx$

9 $\int (4x + 3)\,dx$

10 $\int (-6x - 3)\,dx$

11 $\int (2x - 3)\,dx$

12 $\int (-4x + 3)\,dx$

## 21. Derivatives

Find f'(x). Bring each power down in front, and take one off the power.

13 $f(x) = -3x^2 + x - 2$

14 $f(x) = 2x^2 - 3x - 2$

15 $f(x) = -3x^2 + x + 2$

16 $f(x) = 3x^2 - 3x + 1$

17 $f(x) = -2x^2 + x - 3$

18 $f(x) = -3x^2 - 2x + 3$

# Definite Integrals

WORKED EXAMPLE

$$\int_0^1 (-4x)\,dx$$
$$= [-2x^2]_0^1$$
$$= (-2(1)^2) - 0$$
$$= -2 - 0$$
$$= -2$$

## 22. Integrals

Find the antiderivative, then take its value at the bottom from the top.

1 $\int_{-1}^{0} (2x - 2)\,dx$

2 $\int_{-2}^{2} (4x - 2)\,dx$

3 $\int_{-1}^{1} (-4x + 2)\,dx$

4 $\int_{-1}^{2} (4x - 1)\,dx$

5 $\int_{-1}^{0} (4x + 2)\,dx$

6 $\int_{-2}^{-1} (4x - 1)\,dx$

7 $\int_{-2}^{1} (4x - 2)\,dx$

8 $\int_{-2}^{1} (-2x - 1)\,dx$

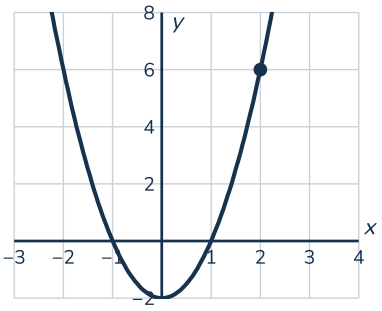
9 $\int_{-1}^{2} (2x - 2)\,dx$

10 $\int_{-1}^{2} (-4x - 1)\,dx$

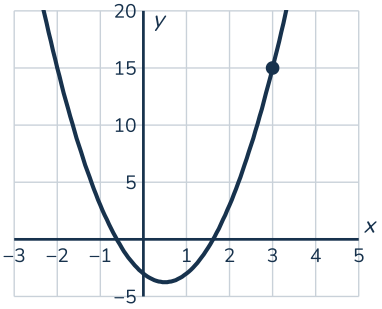
## 23. Using derivatives

Find the slope of each curve at the point: find f'(x), then put x in.

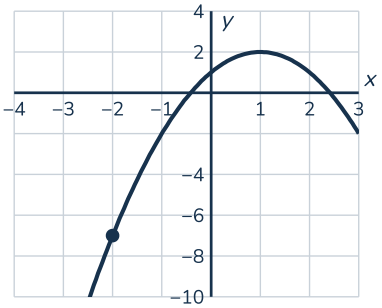
11 **$f(x) = 2x^2 - 2,\ x = 2$**



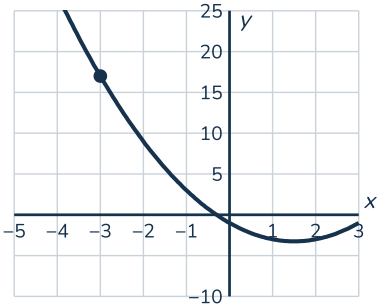

12 **$f(x) = 3x^2 - 3x - 3,\ x = 3$**




13 **$f(x) = -x^2 + 2x + 1,\ x = -2$**




14 **$f(x) = x^2 - 3x - 1,\ x = -3$**



# Integration by Substitution

WORKED EXAMPLE

$\int -3(-x + 3)^2\,dx$

$u = -x + 3,\ du = -\,dx$

$= \int 3u^2\,du$

$= u^3 + C$

$= (-x + 3)^3 + C$

## 24. Integrals

Find each integral by substitution: let u be the bracket.

1 $\int -9(-3x + 3)^2\,dx$

2 $\int -12(-3x - 3)^3\,dx$

3 $\int 8(2x + 1)^3\,dx$

4 $\int 12(3x + 3)^3\,dx$

5 $\int -4(-x + 2)^3\,dx$

6 $\int -12(-3x - 2)^3\,dx$

7 $\int 8(2x + 3)^3\,dx$

8 $\int -4(-x - 1)^3\,dx$

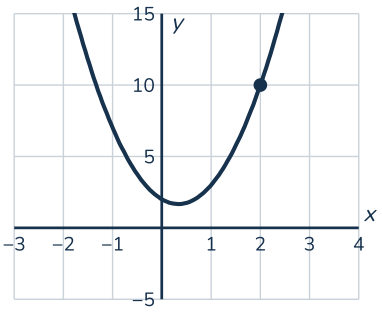
9 $\int 12(3x + 2)^3\,dx$

10 $\int -8(-2x - 3)^3\,dx$

## 25. Using derivatives

Find the equation of the tangent line at the point given.

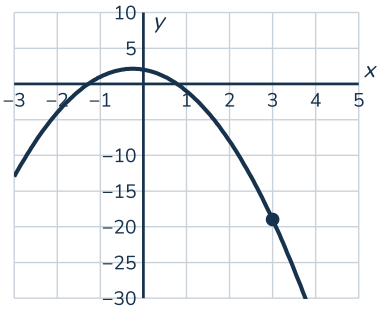
11 $f(x) = 3x^2 - 2x + 2,\ x = 2$



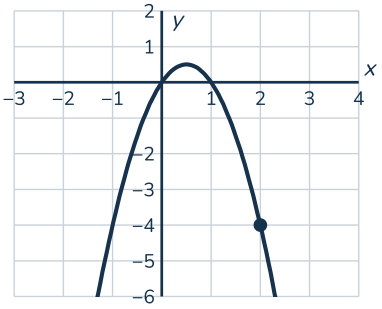
## 25. Using derivatives (continued)

Find the equation of the tangent line at the point given.

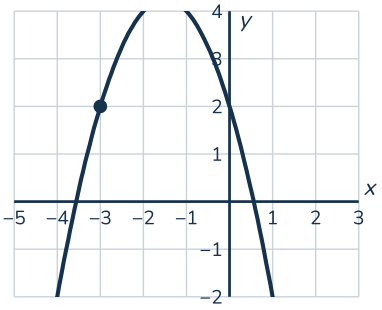
12 $f(x) = -2x^2 - x + 2,\ x = 3$




13 $f(x) = -2x^2 + 2x,\ x = 2$




14 $f(x) = -x^2 - 3x + 2,\ x = -3$



# Integrals of sin, cos, $e^x$ and 1/x

WORKED EXAMPLE

$$\int \left(-5 \sin x + \frac{1}{x}\right) dx$$
$$= 5 \cos x + \ln|x| + C$$

## 26. Integrals

Find each integral. Use the integral of each function, and divide by what the inside multiplies x by.

1 $\int \left(-\cos x - \sin x - \frac{1}{x}\right) dx$

2 $\int \left(\cos x - 2 \sin x - \frac{5}{x}\right) dx$

3 $\int (-3 \sin x + 4e^x) \, dx$

4 $\int \left(-4 \sin x + 2e^x - \frac{5}{x}\right) dx$

5 $\int (\cos x + 2 \sin x + 2e^x) \, dx$

6 $\int \left(-2 \sin x - e^x + \frac{5}{x}\right) dx$

7 $\int (-3 \cos x - 3 \sin x - 3e^x) \, dx$

8 $\int (2 \sin x + 5e^x) \, dx$

9 $\int \left(-4 \cos x - 4 \sin x + \frac{3}{x}\right) dx$

10 $\int \left(-\sin x + 4e^x + \frac{3}{x}\right) dx$

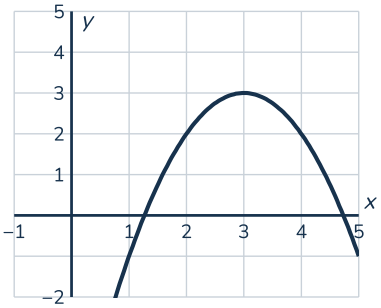
11 $\int \left(-4 \cos x + 2e^x + \frac{1}{x}\right) dx$

12 $\int (4 \cos x - 5 \sin x + 3e^x) \, dx$

## 27. Using derivatives

Find where f'(x) = 0 and say if each point is a maximum or a minimum.

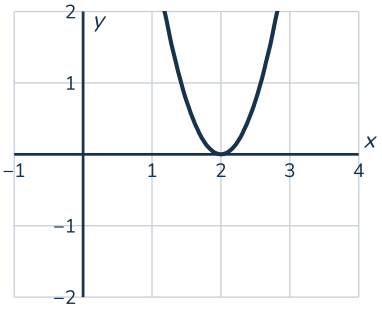
13 $f(x) = -x^2 + 6x - 6$



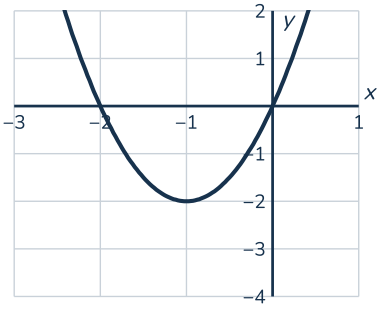
## 27. Using derivatives (continued)

Find where f'(x) = 0 and say if each point is a maximum or a minimum.

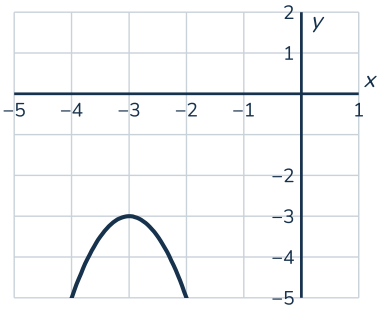
14 **$f(x) = 3x^2 - 12x + 12$**




15 **$f(x) = 2x^2 + 4x$**




16 **$f(x) = -2x^2 - 12x - 21$**



# Integration by Parts

WORKED EXAMPLE

**$\int -2x \sin x \, dx$**

$u = -2x$, $dv = \sin x \, dx$
$du = -2 \, dx$, $v = -\cos x$
$= 2x \cos x - \int 2 \cos x \, dx$
$= 2x \cos x - 2 \sin x + C$

## 28. Integrals

Find each integral by parts: $\int u \, dv = uv - \int v \, du$.

1 $\int -4x \cos x \, dx$

2 $\int 5x \sin x \, dx$

3 $\int -4xe^x \, dx$

4 $\int -5xe^x \, dx$

5 $\int 3x \cos x \, dx$

6 $\int -5x \cos x \, dx$

7 $\int 5xe^x \, dx$

8 $\int 4x \sin x \, dx$

## 29. Derivatives

Find f'(x) with the product rule: u'v + uv'.

9 $f(x) = (-2x + 3)(x + 3)$

10 $f(x) = (3x - 3)(-2x - 2)$

11 $f(x) = (-2x - 2)(-x - 3)$

12 $f(x) = (-2x - 2)(3x - 3)$

13 $f(x) = (2x + 2)(3x - 1)$

14 $f(x) = (2x - 3)(3x + 3)$

# The nth-Term Test

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} \frac{2}{n^2 - n + 3}$$

$$\lim_{n\to\infty} \frac{2}{n^2 - n + 3} = \lim_{n\to\infty} \frac{2/n^2}{1 - 1/n + 3/n^2}$$

$$= \frac{0}{1 + 0 + 0} = 0$$

$\lim a_n = 0$: the test says nothing

## 30. Series: converge or diverge?

Find $\lim a_n$. Not 0: the series diverges. 0: the test says nothing.

1 $$\sum_{n=1}^{\infty} \frac{-n^2 - 3}{2n^2 + 1}$$

2 $$\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 3}$$

3 $$\sum_{n=1}^{\infty} \frac{n^2 - 3}{2n^2 + 1}$$

4 $$\sum_{n=1}^{\infty} \frac{-n^2 - 2}{2n^2 - 1}$$

5 $$\sum_{n=1}^{\infty} \frac{-n^2}{2n^2 + 3}$$

6 $$\sum_{n=1}^{\infty} \frac{-2n - 2}{2n^2 + 3}$$

7 $$\sum_{n=1}^{\infty} \frac{n^2 - 1}{2n^2 + 2}$$

8 $$\sum_{n=1}^{\infty} \frac{-n^2 + 3}{2n^2 - 1}$$

## 31. Integrals

Find the antiderivative, then take its value at the bottom from the top.

9 $\int_{-1}^{1} (2x)\,dx$

10 $\int_{-2}^{2} (2x)\,dx$

11 $\int_{-2}^{1} (-2x + 1)\,dx$

12 $\int_{-2}^{-1} (4x - 1)\,dx$

# Geometric and Telescoping Series

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} 3(-4)^{n-1}$$

$a = 3$ and $r = -4$

$|r| = 4 > 1$, so the series diverges

## 32. Series: converge or diverge?

Does each series converge? If it does, find its sum.

1 $$\sum_{n=1}^{\infty} -2\left(-\frac{1}{4}\right)^{n-1}$$

2 $$\sum_{n=1}^{\infty} \frac{2}{(n+2)(n+3)}$$

3 $$\sum_{n=1}^{\infty} 3(-2)^{n}$$

4 $$\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+3)}$$

5 $$\sum_{n=1}^{\infty} 3\left(-\frac{1}{4}\right)^{n-1}$$

6 $$\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$$

7 $$\sum_{n=1}^{\infty} \frac{4}{(n+1)(n+3)}$$

8 $$\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$$

## 33. Integrals

Find each integral by substitution: let u be the bracket.

9 $\int -8(-2x + 1)^3 \, dx$

10 $\int 12(3x - 2)^3 \, dx$

11 $\int -8(-2x + 2)^3 \, dx$

12 $\int 8(2x + 3)^3 \, dx$

# p-Series and the Comparison Test

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}$$

A number times a p-series: $\frac{2}{\sqrt{n}} = 2 \cdot \frac{1}{\sqrt{n}}$

$p = \frac{1}{2} \le 1$, so the series diverges

## 34. Series: converge or diverge?

Does each series converge? Compare it with a p-series, $\sum 1/n^p$.

1 $$\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$$

2 $$\sum_{n=1}^{\infty} \frac{n + 1}{2n^3 + 1}$$

3 $$\sum_{n=1}^{\infty} \frac{1}{n^2\sqrt{n}}$$

4 $$\sum_{n=1}^{\infty} \frac{n + 2}{n^3 + 2}$$

5 $$\sum_{n=1}^{\infty} \frac{n + 2}{n^2 + 2}$$

6 $$\sum_{n=1}^{\infty} \frac{n + 3}{2n^3 + 3}$$

7 $$\sum_{n=1}^{\infty} \frac{2}{n + 2}$$

8 $$\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}$$

## 35. Integrals

Find each integral. Use the integral of each function, and divide by what the inside multiplies x by.

9 $\int (5 \sin x + \frac{2}{x})\, dx$

10 $\int (3 \sin x - 3e^x + \frac{4}{x})\, dx$

11 $\int (-5 \cos x + \frac{2}{x})\, dx$

12 $\int (3 \cos x + 4 \sin x + \frac{4}{x})\, dx$

# The Ratio Test

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} \frac{n}{3^n}$$

$$\frac{a_{n+1}}{a_n} = \frac{n+1}{3^{n+1}} \cdot \frac{3^n}{n}$$

$$= \frac{n+1}{n} \cdot \frac{1}{3}$$

$$L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1 \cdot \frac{1}{3} = \frac{1}{3}$$

$L < 1$, so the series converges

## 36. Series: converge or diverge?

Does each series converge? Use the ratio test: $L = \lim |a_{n+1} \div a_n|$.

1 $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2}$

2 $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n!}$

3 $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$

4 $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{4^n}$

5 $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{3^n}$

6 $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n^2}$

7 $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{4^n}$

8 $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n}$

## 37. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

9 $f(x) = 5 \sin x - 5e^x$

10 $f(x) = -5 \cos x + 3e^x - \ln x$

11 $f(x) = -5 \sin x - 5 \cos x$

12 $f(x) = -3 \sin x - 5 \cos x - 2 \ln x$

13 $f(x) = 2 \sin x - e^x + 3 \ln x$

14 $f(x) = -3 \sin x + 4 \cos x + 5 \ln x$

# Alternating Series

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n\sqrt{n}}$$

$\sum |a_n| = \sum \frac{1}{n\sqrt{n}}$ : $p = \frac{3}{2} > 1$, converges

The series converges absolutely

## 38. Series: converge or diverge?

Does each series converge absolutely, converge conditionally, or diverge?

1 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$$

2 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n^2\sqrt{n}}$$

3 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2\sqrt{n}}$$

4 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n}$$

5 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{\sqrt{n}}$$

6 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n^2\sqrt{n}}$$

7 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n}$$

8 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n+3}$$

## 39. Integrals

Raise each power by one and divide by the new power. Don't forget + C.

9 $\int (-6x + 1)\,dx$

10 $\int (4x - 3)\,dx$

11 $\int (6x - 1)\,dx$

12 $\int (-6x + 2)\,dx$

13 $\int (2x - 3)\,dx$

14 $\int (-6x + 3)\,dx$

# Interval of Convergence

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{3^n}$$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{|x+2|}{3}$$

$L = \frac{|x+2|}{3} < 1$: $R = 3$, $-5 < x < 1$

$x = 1$: $\sum 1$ diverges ($a_n \not\to 0$)

$x = -5$: $\sum (-1)^n$ diverges ($a_n \not\to 0$)

Interval: $(-5, 1)$

## 40. Series: converge or diverge?

Find the interval of convergence. Check both endpoints.

1 $$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n \cdot 2^n}$$

2 $$\sum_{n=1}^{\infty} \frac{(-1)^n(x+1)^n}{n \cdot 3^n}$$

3 $$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n^2}$$

4 $$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n^2 \cdot 3^n}$$

## 41. Derivatives

Find f'(x) with the product rule: u'v + uv'.

5 $f(x) = (3x + 2)(-3x + 1)$

6 $f(x) = (3x + 3)(x - 3)$

7 $f(x) = (x - 2)(3x - 2)$

8 $f(x) = (-3x - 3)(2x + 3)$

# Answer Key (1 of 5)

## The Power Rule

Derivatives

1. $f(x) = 3x^2 - x - 1 \rightarrow f'(x) = 6x - 1$
2. $f(x) = -3x^2 + x - 3 \rightarrow f'(x) = -6x + 1$
3. $f(x) = 2x^2 - 2x + 1 \rightarrow f'(x) = 4x - 2$
4. $f(x) = 3x^2 - 3x + 3 \rightarrow f'(x) = 6x - 3$
5. $f(x) = 3x^2 + x + 2 \rightarrow f'(x) = 6x + 1$
6. $f(x) = -3x^2 + 3x + 1 \rightarrow f'(x) = -6x + 3$
7. $f(x) = 3x^2 - 2x + 2 \rightarrow f'(x) = 6x - 2$
8. $f(x) = 2x^2 - 2x - 2 \rightarrow f'(x) = 4x - 2$
9. $f(x) = 3x^2 + 2x - 3 \rightarrow f'(x) = 6x + 2$
10. $f(x) = 2x^2 + 3x - 3 \rightarrow f'(x) = 4x + 3$
11. $f(x) = x^2 + 3x + 1 \rightarrow f'(x) = 2x + 3$
12. $f(x) = -3x^2 + 2x + 2 \rightarrow f'(x) = -6x + 2$

## The Product Rule

Derivatives

1. $f(x) = (3x + 1)(2x + 3) \rightarrow f'(x) = 12x + 11$
2. $f(x) = (3x - 3)(2x + 2) \rightarrow f'(x) = 12x$
3. $f(x) = (3x + 1)(-3x + 1) \rightarrow f'(x) = -18x$
4. $f(x) = (3x + 3)(x - 3) \rightarrow f'(x) = 6x - 6$
5. $f(x) = (-2x - 3)(x + 2) \rightarrow f'(x) = -4x - 7$
6. $f(x) = (-3x - 2)(2x + 3) \rightarrow f'(x) = -12x - 13$
7. $f(x) = (-2x + 3)(3x - 3) \rightarrow f'(x) = -12x + 15$
8. $f(x) = (-3x - 2)(2x - 1) \rightarrow f'(x) = -12x - 1$
9. $f(x) = (-3x - 2)(-2x + 3) \rightarrow f'(x) = 12x - 5$
10. $f(x) = (-2x + 3)(-3x + 3) \rightarrow f'(x) = 12x - 15$

## The Quotient Rule

Derivatives

1. $f(x) = \frac{3x - 1}{2x - 2} \rightarrow f'(x) = -\frac{4}{2x - 2}^2$
2. $f(x) = \frac{3x + 3}{x - 2} \rightarrow f'(x) = -\frac{9}{x - 2}^2$
3. $f(x) = \frac{-3x + 2}{x - 2} \rightarrow f'(x) = \frac{4}{x - 2}^2$
4. $f(x) = \frac{3x - 2}{3x + 2} \rightarrow f'(x) = \frac{12}{3x + 2}^2$
5. $f(x) = \frac{-2x - 3}{2x + 1} \rightarrow f'(x) = \frac{4}{2x + 1}^2$
6. $f(x) = \frac{-2x + 2}{2x - 3} \rightarrow f'(x) = \frac{2}{2x - 3}^2$
7. $f(x) = \frac{-2x - 2}{2x + 3} \rightarrow f'(x) = -\frac{2}{2x + 3}^2$
8. $f(x) = \frac{-3x - 1}{3x - 1} \rightarrow f'(x) = \frac{6}{3x - 1}^2$

## The Chain Rule

Derivatives

1. $f(x) = (-3x - 1)^3 \rightarrow f'(x) = -9(-3x - 1)^2$
2. $f(x) = (-2x + 3)^3 \rightarrow f'(x) = -6(-2x + 3)^2$
3. $f(x) = (x - 1)^3 \rightarrow f'(x) = 3(x - 1)^2$
4. $f(x) = (3x + 3)^3 \rightarrow f'(x) = 9(3x + 3)^2$
5. $f(x) = (x - 3)^3 \rightarrow f'(x) = 3(x - 3)^2$
6. $f(x) = (3x - 3)^3 \rightarrow f'(x) = 9(3x - 3)^2$
7. $f(x) = (-3x - 2)^3 \rightarrow f'(x) = -9(-3x - 2)^2$
8. $f(x) = (-x - 1)^3 \rightarrow f'(x) = -3(-x - 1)^2$

## The Chain Rule (continued)

Derivatives (continued)

9. $f(x) = (2x - 3)^3 \rightarrow f'(x) = 6(2x - 3)^2$
10. $f(x) = (2x + 3)^3 \rightarrow f'(x) = 6(2x + 3)^2$

Derivatives

11. $f(x) = 2x^2 + 2x + 1 \rightarrow f'(x) = 4x + 2$
12. $f(x) = -2x^2 + 3x - 2 \rightarrow f'(x) = -4x + 3$
13. $f(x) = -3x^2 + x + 1 \rightarrow f'(x) = -6x + 1$
14. $f(x) = -x^2 + 3x + 3 \rightarrow f'(x) = -2x + 3$
15. $f(x) = -x^2 - 3x - 1 \rightarrow f'(x) = -2x - 3$
16. $f(x) = 3x^2 - x + 3 \rightarrow f'(x) = 6x - 1$

## Choosing the Rule

Derivatives

1. $f(x) = (-x + 2)(-3x - 1) \rightarrow f'(x) = 6x - 5$
2. $f(x) = \frac{2x + 1}{2x - 3} \rightarrow f'(x) = -\frac{8}{2x - 3}^2$
3. $f(x) = (-2x + 1)(-x + 1) \rightarrow f'(x) = 4x - 3$
4. $f(x) = \frac{x - 1}{x + 2} \rightarrow f'(x) = \frac{3}{x + 2}^2$
5. $f(x) = (-x - 2)(3x + 2) \rightarrow f'(x) = -6x - 8$
6. $f(x) = \frac{-x + 2}{3x + 3} \rightarrow f'(x) = -\frac{9}{3x + 3}^2$
7. $f(x) = \frac{x + 2}{3x - 2} \rightarrow f'(x) = -\frac{8}{3x - 2}^2$
8. $f(x) = (x + 3)(-x + 3) \rightarrow f'(x) = -2x$
9. $f(x) = \frac{-x + 3}{3x + 2} \rightarrow f'(x) = -\frac{11}{3x + 2}^2$
10. $f(x) = \frac{2x - 2}{3x - 2} \rightarrow f'(x) = \frac{2}{3x - 2}^2$

Derivatives

11. $f(x) = (2x - 2)(-2x + 2) \rightarrow f'(x) = -8x + 8$
12. $f(x) = (3x - 2)(-3x - 3) \rightarrow f'(x) = -18x - 3$
13. $f(x) = (x - 3)(3x - 2) \rightarrow f'(x) = 6x - 11$
14. $f(x) = (-3x - 2)(2x + 3) \rightarrow f'(x) = -12x - 13$
15. $f(x) = (-2x - 3)(-x - 3) \rightarrow f'(x) = 4x + 9$
16. $f(x) = (3x + 2)(2x + 3) \rightarrow f'(x) = 12x + 13$

## Derivatives of sin, cos, $e^x$ and ln x

Derivatives

1. $f(x) = 2 \sin x - e^x + 3 \ln x \rightarrow f'(x) = 2 \cos x - e^x + \frac{3}{x}$
2. $f(x) = 5 \sin x - 5e^x + \ln x \rightarrow f'(x) = 5 \cos x - 5e^x + \frac{1}{x}$
3. $f(x) = -\sin x + 3e^x + \ln x \rightarrow f'(x) = -\cos x + 3e^x + \frac{1}{x}$
4. $f(x) = -3 \sin x - 3 \cos x - 5 \ln x \rightarrow f'(x) = -3 \cos x + 3 \sin x - \frac{5}{x}$
5. $f(x) = -\sin x + 2 \cos x - 4 \ln x \rightarrow f'(x) = -\cos x - 2 \sin x - \frac{4}{x}$
6. $f(x) = 4 \cos x - 2e^x - 4 \ln x \rightarrow f'(x) = -4 \sin x - 2e^x - \frac{4}{x}$
7. $f(x) = -5 \sin x + 5e^x + 2 \ln x \rightarrow f'(x) = -5 \cos x + 5e^x + \frac{2}{x}$
8. $f(x) = 3 \sin x - 2e^x - 2 \ln x \rightarrow f'(x) = 3 \cos x - 2e^x - \frac{2}{x}$
9. $f(x) = -5 \cos x + 5e^x - 4 \ln x \rightarrow f'(x) = 5 \sin x + 5e^x - \frac{4}{x}$
10. $f(x) = 4 \cos x - 4e^x - \ln x \rightarrow f'(x) = -4 \sin x - 4e^x - \frac{1}{x}$
11. $f(x) = 4 \cos x - 4 \ln x \rightarrow f'(x) = -4 \sin x - \frac{4}{x}$
12. $f(x) = 5 \cos x - 4e^x + 3 \ln x \rightarrow f'(x) = -5 \sin x - 4e^x + \frac{3}{x}$

# Answer Key (2 of 5)

## Derivatives of sin, cos, $e^x$ and ln x (continued)

Derivatives

13. $f(x) = \frac{3x+1}{2x+2} \rightarrow f'(x) = \frac{4}{2x+2}^2$
14. $f(x) = \frac{3x+3}{2x+1} \rightarrow f'(x) = -\frac{3}{2x+1}^2$
15. $f(x) = \frac{-2x-3}{2x+1} \rightarrow f'(x) = \frac{4}{2x+1}^2$
16. $f(x) = \frac{3x-3}{2x+2} \rightarrow f'(x) = \frac{12}{2x+2}^2$
17. $f(x) = \frac{3x+2}{2x-1} \rightarrow f'(x) = -\frac{7}{2x-1}^2$
18. $f(x) = \frac{3x+3}{x-3} \rightarrow f'(x) = -\frac{12}{x-3}^2$

## The Chain Rule with sin, cos, $e^x$ and ln

Derivatives

1. $f(x) = e^{3x+4} \rightarrow f'(x) = 3e^{3x+4}$
2. $f(x) = 4\ln(3x+4) \rightarrow f'(x) = \frac{12}{3x+4}$
3. $f(x) = 3\sin(4x+1) \rightarrow f'(x) = 12\cos(4x+1)$
4. $f(x) = 3\ln(-3x-5) \rightarrow f'(x) = -\frac{9}{-3x-5}$
5. $f(x) = -5e^{2x-2} \rightarrow f'(x) = -10e^{2x-2}$
6. $f(x) = 4\sin(4x+2) \rightarrow f'(x) = 16\cos(4x+2)$
7. $f(x) = -3\cos(3x+5) \rightarrow f'(x) = 9\sin(3x+5)$
8. $f(x) = 5\ln(2x+3) \rightarrow f'(x) = \frac{10}{2x+3}$
9. $f(x) = -4\cos(3x+4) \rightarrow f'(x) = 12\sin(3x+4)$
10. $f(x) = -3\sin(3x-5) \rightarrow f'(x) = -9\cos(3x-5)$

Derivatives

11. $f(x) = (-x-2)^3 \rightarrow f'(x) = -3(-x-2)^2$
12. $f(x) = (3x-3)^3 \rightarrow f'(x) = 9(3x-3)^2$
13. $f(x) = (x+1)^3 \rightarrow f'(x) = 3(x+1)^2$
14. $f(x) = (-3x-2)^3 \rightarrow f'(x) = -9(-3x-2)^2$
15. $f(x) = (3x+1)^3 \rightarrow f'(x) = 9(3x+1)^2$
16. $f(x) = (3x+2)^3 \rightarrow f'(x) = 9(3x+2)^2$

## Product and Quotient Rules with Functions

Derivatives

1. $f(x) = \frac{\sin x}{x} \rightarrow f'(x) = \frac{x\cos x - \sin x}{x^2}$
2. $f(x) = \frac{3e^x}{x} \rightarrow f'(x) = \frac{3e^x(x-1)}{x^2}$
3. $f(x) = \frac{\cos x}{x} \rightarrow f'(x) = \frac{-(x\sin x + \cos x)}{x^2}$
4. $f(x) = \frac{5e^x}{x} \rightarrow f'(x) = \frac{5e^x(x-1)}{x^2}$
5. $f(x) = -5e^x\cos x \rightarrow f'(x) = -5e^x(\cos x - \sin x)$
6. $f(x) = \frac{5\ln x}{x} \rightarrow f'(x) = \frac{5(1-\ln x)}{x^2}$
7. $f(x) = \frac{4\sin x}{x} \rightarrow f'(x) = \frac{4(x\cos x - \sin x)}{x^2}$
8. $f(x) = 5e^x\cos x \rightarrow f'(x) = 5e^x(\cos x - \sin x)$

Derivatives

9. $f(x) = (x+2)^3 \rightarrow f'(x) = 3(x+2)^2$
10. $f(x) = (-2x+3)(-3x+3) \rightarrow f'(x) = 12x - 15$
11. $f(x) = (-2x-2)^3 \rightarrow f'(x) = -6(-2x-2)^2$
12. $f(x) = (3x+1)(-3x-3) \rightarrow f'(x) = -18x - 12$
13. $f(x) = (-3x+1)(2x-1) \rightarrow f'(x) = -12x + 5$
14. $f(x) = x/(x-3) \rightarrow f'(x) = -\frac{3}{x-3}^2$

## The Slope of a Curve

Using derivatives

1. $f(x) = -2x^2 - 2x + 3$; slope at $x = 2 \rightarrow f'(2) = -10$
2. $f(x) = -x^2 + 2x - 1$; slope at $x = 3 \rightarrow f'(3) = -4$
3. $f(x) = -3x^2 - 1$; slope at $x = 2 \rightarrow f'(2) = -12$
4. $f(x) = x^2 + 2x + 3$; slope at $x = -3 \rightarrow f'(-3) = -4$
5. $f(x) = -x^2 - 2x - 3$; slope at $x = -2 \rightarrow f'(-2) = 2$
6. $f(x) = 3x^2 + 2x + 1$; slope at $x = 3 \rightarrow f'(3) = 20$

Derivatives

7. $f(x) = -\sin x + 2\cos x - 4\ln x \rightarrow f'(x) = -\cos x - 2\sin x - \frac{4}{x}$
8. $f(x) = 4\cos x - 2e^x - 4\ln x \rightarrow f'(x) = -4\sin x - 2e^x - \frac{4}{x}$
9. $f(x) = 5\cos x - 4e^x \rightarrow f'(x) = -5\sin x - 4e^x$
10. $f(x) = 5\cos x + 2e^x + 5\ln x \rightarrow f'(x) = -5\sin x + 2e^x + \frac{5}{x}$
11. $f(x) = -2\cos x + 5\ln x \rightarrow f'(x) = 2\sin x + \frac{5}{x}$
12. $f(x) = 5\cos x + 4e^x + 2\ln x \rightarrow f'(x) = -5\sin x + 4e^x + \frac{2}{x}$

## Tangent Lines

Using derivatives

1. $f(x) = -x^2 - 2x$; tangent at $x = 2 \rightarrow y = -6x + 4$
2. $f(x) = 3x^2 + x - 3$; tangent at $x = -3 \rightarrow y = -17x - 30$
3. $f(x) = -x^2 - 1$; tangent at $x = 3 \rightarrow y = -6x + 8$
4. $f(x) = x^2 - 3x + 3$; tangent at $x = 3 \rightarrow y = 3x - 6$
5. $f(x) = -3x^2 + x + 3$; tangent at $x = 2 \rightarrow y = -11x + 15$
6. $f(x) = x^2 - 2x + 3$; tangent at $x = 3 \rightarrow y = 4x - 6$

Derivatives

7. $f(x) = -5e^{3x-1} \rightarrow f'(x) = -15e^{3x-1}$
8. $f(x) = 4e^{3x-5} \rightarrow f'(x) = 12e^{3x-5}$
9. $f(x) = \ln(4x+3) \rightarrow f'(x) = \frac{4}{4x+3}$
10. $f(x) = -3\sin(4x-3) \rightarrow f'(x) = -12\cos(4x-3)$
11. $f(x) = 2\sin(4x+1) \rightarrow f'(x) = 8\cos(4x+1)$
12. $f(x) = -4\ln(4x-1) \rightarrow f'(x) = -\frac{16}{4x-1}$

## Stationary Points

Using derivatives

1. $f(x) = -x^2 - 6x - 12$; stationary points → maximum $(-3, -3)$
2. $f(x) = -3x^2 - 18x - 30$; stationary points → maximum $(-3, -3)$
3. $f(x) = -x^2 + 6x - 11$; stationary points → maximum $(3, -2)$
4. $f(x) = 3x^2 + 12x + 14$; stationary points → minimum $(-2, 2)$
5. $f(x) = -3x^2 + 6x - 2$; stationary points → maximum $(1, 1)$
6. $f(x) = -2x^2 - 12x - 16$; stationary points → maximum $(-3, 2)$

Derivatives

7. $f(x) = \frac{2\ln x}{x} \rightarrow f'(x) = \frac{2(1-\ln x)}{x^2}$
8. $f(x) = \frac{3\cos x}{x} \rightarrow f'(x) = \frac{-3(x\sin x + \cos x)}{x^2}$
9. $f(x) = \frac{e^x}{x} \rightarrow f'(x) = \frac{e^x(x-1)}{x^2}$
10. $f(x) = \frac{3\sin x}{x} \rightarrow f'(x) = \frac{3(x\cos x - \sin x)}{x^2}$

# Answer Key (3 of 5)

## Indefinite Integrals

Integrals

1. $\int(-4x + 2)\,dx = -2x^2 + 2x + C$
2. $\int(-6x - 2)\,dx = -3x^2 - 2x + C$
3. $\int(2x + 3)\,dx = x^2 + 3x + C$
4. $\int(6x - 3)\,dx = 3x^2 - 3x + C$
5. $\int(-4x - 2)\,dx = -2x^2 - 2x + C$
6. $\int(-4x - 3)\,dx = -2x^2 - 3x + C$
7. $\int(-6x + 3)\,dx = -3x^2 + 3x + C$
8. $\int(-2x - 3)\,dx = -x^2 - 3x + C$
9. $\int(4x + 3)\,dx = 2x^2 + 3x + C$
10. $\int(-6x - 3)\,dx = -3x^2 - 3x + C$
11. $\int(2x - 3)\,dx = x^2 - 3x + C$
12. $\int(-4x + 3)\,dx = -2x^2 + 3x + C$

Derivatives

13. $f(x) = -3x^2 + x - 2 \rightarrow f'(x) = -6x + 1$
14. $f(x) = 2x^2 - 3x - 2 \rightarrow f'(x) = 4x - 3$
15. $f(x) = -3x^2 + x + 2 \rightarrow f'(x) = -6x + 1$
16. $f(x) = 3x^2 - 3x + 1 \rightarrow f'(x) = 6x - 3$
17. $f(x) = -2x^2 + x - 3 \rightarrow f'(x) = -4x + 1$
18. $f(x) = -3x^2 - 2x + 3 \rightarrow f'(x) = -6x - 2$

## Definite Integrals

Integrals

1. $\int$ from $-1$ to $0$ of $(2x - 2)\,dx = -3$
2. $\int$ from $-2$ to $2$ of $(4x - 2)\,dx = -8$
3. $\int$ from $-1$ to $1$ of $(-4x + 2)\,dx = 4$
4. $\int$ from $-1$ to $2$ of $(4x - 1)\,dx = 3$
5. $\int$ from $-1$ to $0$ of $(4x + 2)\,dx = 0$
6. $\int$ from $-2$ to $-1$ of $(4x - 1)\,dx = -7$
7. $\int$ from $-2$ to $1$ of $(4x - 2)\,dx = -12$
8. $\int$ from $-2$ to $1$ of $(-2x - 1)\,dx = 0$
9. $\int$ from $-1$ to $2$ of $(2x - 2)\,dx = -3$
10. $\int$ from $-1$ to $2$ of $(-4x - 1)\,dx = -9$

Using derivatives

11. $f(x) = 2x^2 - 2$; slope at $x = 2 \rightarrow f'(2) = 8$
12. $f(x) = 3x^2 - 3x - 3$; slope at $x = 3 \rightarrow f'(3) = 15$
13. $f(x) = -x^2 + 2x + 1$; slope at $x = -2 \rightarrow f'(-2) = 6$
14. $f(x) = x^2 - 3x - 1$; slope at $x = -3 \rightarrow f'(-3) = -9$

## Integration by Substitution

Integrals

1. $\int -9(-3x + 3)^2\,dx = (-3x + 3)^3 + C$
2. $\int -12(-3x - 3)^3\,dx = (-3x - 3)^4 + C$
3. $\int 8(2x + 1)^3\,dx = (2x + 1)^4 + C$
4. $\int 12(3x + 3)^3\,dx = (3x + 3)^4 + C$
5. $\int -4(-x + 2)^3\,dx = (-x + 2)^4 + C$
6. $\int -12(-3x - 2)^3\,dx = (-3x - 2)^4 + C$
7. $\int 8(2x + 3)^3\,dx = (2x + 3)^4 + C$
8. $\int -4(-x - 1)^3\,dx = (-x - 1)^4 + C$

## Integration by Substitution (continued)

Integrals (continued)

9. $\int 12(3x + 2)^3\,dx = (3x + 2)^4 + C$
10. $\int -8(-2x - 3)^3\,dx = (-2x - 3)^4 + C$

Using derivatives

11. $f(x) = 3x^2 - 2x + 2$; tangent at $x = 2 \rightarrow y = 10x - 10$
12. $f(x) = -2x^2 - x + 2$; tangent at $x = 3 \rightarrow y = -13x + 20$
13. $f(x) = -2x^2 + 2x$; tangent at $x = 2 \rightarrow y = -6x + 8$
14. $f(x) = -x^2 - 3x + 2$; tangent at $x = -3 \rightarrow y = 3x + 11$

## Integrals of sin, cos, $e^x$ and 1/x

Integrals

1. $\int (-\cos x - \sin x - 1/x)\,dx = -\sin x + \cos x - \ln|x| + C$
2. $\int (\cos x - 2\sin x - 5/x)\,dx = \sin x + 2\cos x - 5\ln|x| + C$
3. $\int (-3\sin x + 4e^x)\,dx = 3\cos x + 4e^x + C$
4. $\int (-4\sin x + 2e^x - 5/x)\,dx = 4\cos x + 2e^x - 5\ln|x| + C$
5. $\int (\cos x + 2\sin x + 2e^x)\,dx = \sin x - 2\cos x + 2e^x + C$
6. $\int (-2\sin x - e^x + 5/x)\,dx = 2\cos x - e^x + 5\ln|x| + C$
7. $\int (-3\cos x - 3\sin x - 3e^x)\,dx = -3\sin x + 3\cos x - 3e^x + C$
8. $\int (2\sin x + 5e^x)\,dx = -2\cos x + 5e^x + C$
9. $\int (-4\cos x - 4\sin x + 3/x)\,dx = -4\sin x + 4\cos x + 3\ln|x| + C$
10. $\int (-\sin x + 4e^x + 3/x)\,dx = \cos x + 4e^x + 3\ln|x| + C$
11. $\int (-4\cos x + 2e^x + 1/x)\,dx = -4\sin x + 2e^x + \ln|x| + C$
12. $\int (4\cos x - 5\sin x + 3e^x)\,dx = 4\sin x + 5\cos x + 3e^x + C$

Using derivatives

13. $f(x) = -x^2 + 6x - 6$; stationary points → maximum $(3, 3)$
14. $f(x) = 3x^2 - 12x + 12$; stationary points → minimum $(2, 0)$
15. $f(x) = 2x^2 + 4x$; stationary points → minimum $(-1, -2)$
16. $f(x) = -2x^2 - 12x - 21$; stationary points → maximum $(-3, -3)$

## Integration by Parts

Integrals

1. $\int -4x\cos x\,dx = -4x\sin x - 4\cos x + C$
2. $\int 5x\sin x\,dx = -5x\cos x + 5\sin x + C$
3. $\int -4xe^x\,dx = -4xe^x + 4e^x + C$
4. $\int -5xe^x\,dx = -5xe^x + 5e^x + C$
5. $\int 3x\cos x\,dx = 3x\sin x + 3\cos x + C$
6. $\int -5x\cos x\,dx = -5x\sin x - 5\cos x + C$
7. $\int 5xe^x\,dx = 5xe^x - 5e^x + C$
8. $\int 4x\sin x\,dx = -4x\cos x + 4\sin x + C$

Derivatives

9. $f(x) = (-2x + 3)(x + 3) \rightarrow f'(x) = -4x - 3$
10. $f(x) = (3x - 3)(-2x - 2) \rightarrow f'(x) = -12x$
11. $f(x) = (-2x - 2)(-x - 3) \rightarrow f'(x) = 4x + 8$
12. $f(x) = (-2x - 2)(3x - 3) \rightarrow f'(x) = -12x$
13. $f(x) = (2x + 2)(3x - 1) \rightarrow f'(x) = 12x + 4$
14. $f(x) = (2x - 3)(3x + 3) \rightarrow f'(x) = 12x - 3$

# Answer Key (4 of 5)

## The nth-Term Test

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{-n^2-3}{2n^2+1}$ → diverges
2. $\sum_{n=1}^{\infty} \frac{1}{n^2+n+3}$ → the test says nothing
3. $\sum_{n=1}^{\infty} \frac{n^2-3}{2n^2+1}$ → diverges
4. $\sum_{n=1}^{\infty} \frac{-n^2-2}{2n^2-1}$ → diverges
5. $\sum_{n=1}^{\infty} \frac{-n^2}{2n^2+3}$ → diverges
6. $\sum_{n=1}^{\infty} \frac{-2n-2}{2n^2+3}$ → the test says nothing
7. $\sum_{n=1}^{\infty} \frac{n^2-1}{2n^2+2}$ → diverges
8. $\sum_{n=1}^{\infty} \frac{-n^2+3}{2n^2-1}$ → diverges

Integrals

9. $\int$ from $-1$ to $1$ of $(2x)\,dx = 0$
10. $\int$ from $-2$ to $2$ of $(2x)\,dx = 0$
11. $\int$ from $-2$ to $1$ of $(-2x+1)\,dx = 6$
12. $\int$ from $-2$ to $-1$ of $(4x-1)\,dx = -7$

## Geometric and Telescoping Series

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} -2(-\frac{1}{4})^{n-1}$ → converges, S = $\frac{-8}{5}$
2. $\sum_{n=1}^{\infty} \frac{2}{(n+2)(n+3)}$ → converges, S = $\frac{2}{3}$
3. $\sum_{n=1}^{\infty} 3(-2)^n$ → diverges
4. $\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+3)}$ → converges, S = $\frac{5}{6}$
5. $\sum_{n=1}^{\infty} 3(-\frac{1}{4})^{n-1}$ → converges, S = $\frac{12}{5}$
6. $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$ → converges, S = $\frac{1}{3}$
7. $\sum_{n=1}^{\infty} \frac{4}{(n+1)(n+3)}$ → converges, S = $\frac{5}{3}$
8. $\sum_{n=1}^{\infty} (-\frac{1}{2})^{n-1}$ → converges, S = $\frac{2}{3}$

Integrals

9. $\int -8(-2x+1)^3\,dx = (-2x+1)^4 + C$
10. $\int 12(3x-2)^3\,dx = (3x-2)^4 + C$
11. $\int -8(-2x+2)^3\,dx = (-2x+2)^4 + C$
12. $\int 8(2x+3)^3\,dx = (2x+3)^4 + C$

## p-Series and the Comparison Test

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ → converges
2. $\sum_{n=1}^{\infty} \frac{n+1}{2n^3+1}$ → converges
3. $\sum_{n=1}^{\infty} \frac{1}{n^2\sqrt{n}}$ → converges
4. $\sum_{n=1}^{\infty} \frac{n+2}{n^3+2}$ → converges
5. $\sum_{n=1}^{\infty} \frac{n+2}{n^2+2}$ → diverges
6. $\sum_{n=1}^{\infty} \frac{n+3}{2n^3+3}$ → converges
7. $\sum_{n=1}^{\infty} \frac{2}{n+2}$ → diverges
8. $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}$ → converges

Integrals

9. $\int (5\sin x + 2/x)\,dx = -5\cos x + 2\ln|x| + C$
10. $\int (3\sin x - 3e^x + 4/x)\,dx = -3\cos x - 3e^x + 4\ln|x| + C$
11. $\int (-5\cos x + 2/x)\,dx = -5\sin x + 2\ln|x| + C$
12. $\int (3\cos x + 4\sin x + 4/x)\,dx = 3\sin x - 4\cos x + 4\ln|x| + C$

## The Ratio Test

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$ → diverges
2. $\sum_{n=1}^{\infty} \frac{4^n}{n!}$ → converges
3. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ → converges
4. $\sum_{n=1}^{\infty} \frac{n^2}{4^n}$ → converges
5. $\sum_{n=1}^{\infty} \frac{n!}{3^n}$ → diverges
6. $\sum_{n=1}^{\infty} \frac{4^n}{n^2}$ → diverges
7. $\sum_{n=1}^{\infty} \frac{n!}{4^n}$ → diverges
8. $\sum_{n=1}^{\infty} \frac{4^n}{n}$ → diverges

Derivatives

9. $f(x) = 5\sin x - 5e^x \rightarrow f'(x) = 5\cos x - 5e^x$
10. $f(x) = -5\cos x + 3e^x - \ln x \rightarrow f'(x) = 5\sin x + 3e^x - \frac{1}{x}$
11. $f(x) = -5\sin x - 5\cos x \rightarrow f'(x) = -5\cos x + 5\sin x$
12. $f(x) = -3\sin x - 5\cos x - 2\ln x \rightarrow f'(x) = -3\cos x + 5\sin x - \frac{2}{x}$
13. $f(x) = 2\sin x - e^x + 3\ln x \rightarrow f'(x) = 2\cos x - e^x + \frac{3}{x}$
14. $f(x) = -3\sin x + 4\cos x + 5\ln x \rightarrow f'(x) = -3\cos x - 4\sin x + \frac{5}{x}$

# Answer Key (5 of 5)

## Alternating Series

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{n}$ → converges conditionally
2. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{3}{n^2\sqrt{n}}$ → converges absolutely
3. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{n^2\sqrt{n}}$ → converges absolutely
4. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{3}{n}$ → converges conditionally
5. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{2}{\sqrt{n}}$ → converges conditionally
6. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{2}{n^2\sqrt{n}}$ → converges absolutely
7. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{2}{n}$ → converges conditionally
8. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{2n}{n+3}$ → diverges

Integrals

9. $\int(-6x + 1)\,dx = -3x^2 + x + C$
10. $\int(4x - 3)\,dx = 2x^2 - 3x + C$
11. $\int(6x - 1)\,dx = 3x^2 - x + C$
12. $\int(-6x + 2)\,dx = -3x^2 + 2x + C$
13. $\int(2x - 3)\,dx = x^2 - 3x + C$
14. $\int(-6x + 3)\,dx = -3x^2 + 3x + C$

## Interval of Convergence

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n \cdot 2^n}$ → $[-5, -1)$
2. $\sum_{n=1}^{\infty} \frac{(-1)^n(x+1)^n}{n \cdot 3^n}$ → $(-4, 2]$
3. $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n^2}$ → $[-3, -1]$
4. $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n^2 \cdot 3^n}$ → $[-4, 2]$

Derivatives

5. $f(x) = (3x + 2)(-3x + 1)$ → $f'(x) = -18x - 3$
6. $f(x) = (3x + 3)(x - 3)$ → $f'(x) = 6x - 6$
7. $f(x) = (x - 2)(3x - 2)$ → $f'(x) = 6x - 8$
8. $f(x) = (-3x - 3)(2x + 3)$ → $f'(x) = -12x - 15$

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# Solutions (1 of 3)

## The Power Rule

Derivatives

1. **$f(x) = 3x^2 - x - 1$**

$$f'(x) = 2 \cdot 3x - 1$$

$$= 6x - 1$$

A sum of powers of x: take the derivative term by term. For each term, bring the power down in front as a multiplier and take one off the power — the derivative of $x^n$ is $nx^{n-1}$. The constant −1 does not change as x moves, so its derivative is 0 and it is gone.

Multiply out each term. That is the derivative.

2. **$f(x) = -3x^2 + x - 3$**

$$f'(x) = -2 \cdot 3x + 1$$

$$= -6x + 1$$

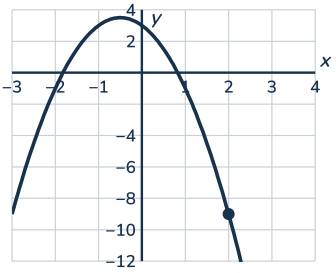
A sum of powers of x: take the derivative term by term. For each term, bring the power down in front as a multiplier and take one off the power — the derivative of $x^n$ is $nx^{n-1}$. The constant −3 does not change as x moves, so its derivative is 0 and it is gone.

Multiply out each term. That is the derivative.

## The Slope of a Curve

Using derivatives

1. **$f(x) = -2x^2 - 2x + 3,\ x = 2$**




$$f'(x) = -4x - 2$$

$$f'(2) = -4(2) - 2$$

$$= -10$$

The slope of a curve at a point is the derivative there. Find f'(x) first, term by term.

Put x = 2 into f'(x), not into f(x): f(x) is the height, f'(x) is the slope.

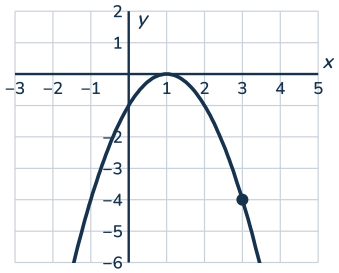
Work it out. That is the slope of the curve at that point.

# Solutions (2 of 3)

**The Slope of a Curve** (continued)

Using derivatives (continued)

2. **$f(x) = -x^2 + 2x - 1,\ x = 3$**




$$f'(x) = -2x + 2$$

$$f'(3) = -2(3) + 2$$

$$= -4$$

The slope of a curve at a point is the derivative there. Find f'(x) first, term by term.

Put x = 3 into f'(x), not into f(x): f(x) is the height, f'(x) is the slope.

Work it out. That is the slope of the curve at that point.

**Indefinite Integrals**

Integrals

1. **$\int (-4x + 2)\, dx$**

$$\int (-4x + 2)\, dx = -\frac{4x^2}{2} + 2x + C$$

$$= -2x^2 + 2x + C$$

Integrate term by term, running the power rule backwards: add one to each power, then divide by the new power. A number on its own becomes that number times x. Add + C, because any constant would have disappeared when the answer was differentiated.

Divide out each fraction. Check by differentiating: you should get back the question.

2. **$\int (-6x - 2)\, dx$**

$$\int (-6x - 2)\, dx = -\frac{6x^2}{2} - 2x + C$$

$$= -3x^2 - 2x + C$$

Integrate term by term, running the power rule backwards: add one to each power, then divide by the new power. A number on its own becomes that number times x. Add + C, because any constant would have disappeared when the answer was differentiated.

Divide out each fraction. Check by differentiating: you should get back the question.

# Solutions (3 of 3)

**The nth-Term Test**

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{-n^2 - 3}{2n^2 + 1}$

$$\lim_{n\to\infty} \frac{-n^2 - 3}{2n^2 + 1} = \lim_{n\to\infty} \frac{-1 - 3/n^2}{2 + 1/n^2}$$

$$= \frac{-1 + 0}{2 + 0} = \frac{-1}{2}$$

**$\lim a_n \neq 0$, so the series diverges**

The nth-term test asks one thing: do the terms go to 0? Find the limit of $a_n$ as n grows. The highest power on the bottom is $n^2$, so divide every term, top and bottom, by $n^2$.

Every term with n left on its bottom shrinks to 0, which leaves $-1$ over 2: the limit is $-\frac{1}{2}$.

The terms settle at $-\frac{1}{2}$, not at 0. Adding infinitely many numbers near $-\frac{1}{2}$ cannot give a finite total, so the series diverges.

2. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 3}$

$$\lim_{n\to\infty} \frac{1}{n^2 + n + 3} = \lim_{n\to\infty} \frac{1/n^2}{1 + 1/n + 3/n^2}$$

$$= \frac{0}{1 + 0 + 0} = 0$$

**$\lim a_n = 0$: the test says nothing**

The nth-term test asks one thing: do the terms go to 0? Find the limit of $a_n$ as n grows. The highest power on the bottom is $n^2$, so divide every term, top and bottom, by $n^2$.

Every term with n left on its bottom shrinks to 0, which leaves 0 on the top: the limit is 0.

The terms do go to 0, and that is NOT enough to say the series converges: the series of 1 over n has terms that go to 0 and still diverges. The test says nothing here, and another test is needed.