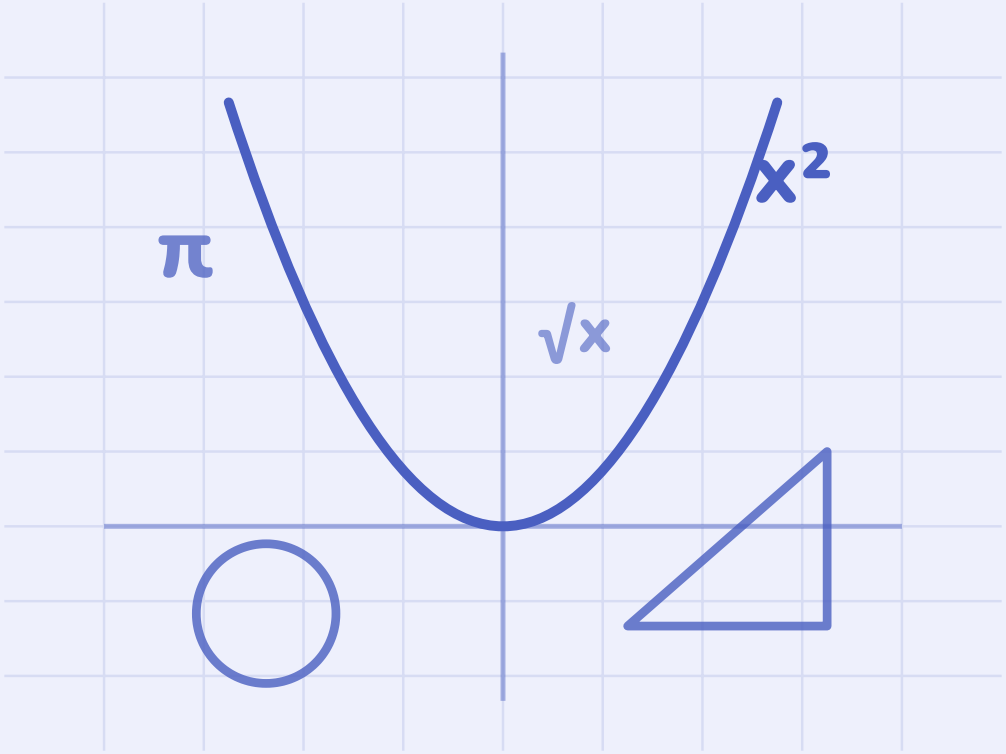
MATH PICNIC · WORKBOOK

Calculus Foundation Workbook

22 lessons · Ages 17–18 · answer key at the back



Name _____ Date _____

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1-1

The Power Rule

WORKED EXAMPLE

$f(x) = 3x^2 - 3$

$f'(x) = 2 \cdot 3x$

$= 6x$

1. Derivatives

Find f'(x). Bring each power down in front, and take one off the power.

1 $f(x) = -3x^2 - x$

2 $f(x) = -3x^2 - 1$

3 $f(x) = 3x^2 - x - 1$

4 $f(x) = 2x^2 - 2$

5 $f(x) = -3x^2$

6 $f(x) = x^2 + 2x + 1$

7 $f(x) = 2x^2 - 2x + 1$

8 $f(x) = -2x^2 + x - 1$

9 $f(x) = -x^2 - x - 1$

10 $f(x) = -x^2 + x - 1$

11 $f(x) = 3x^2 + x + 2$

12 $f(x) = -x^2$

The Product Rule

WORKED EXAMPLE

f(x) = (2x + 1)(−2x + 2)

u = 2x + 1, v = −2x + 2
u' = 2, v' = −2
f'(x) = 2(−2x + 2) + (2x + 1)(−2)
= −4x + 4 − 4x − 2
= −8x + 2

2. Derivatives

Find f'(x) with the product rule: u'v + uv'.

1 **f(x) = (x − 1)(−2x − 3)**

2 **f(x) = (−2x − 1)(−x + 1)**

3 **f(x) = (3x + 1)(2x + 3)**

4 **f(x) = (x − 3)(−2x + 1)**

5 **f(x) = (2x + 1)(3x − 1)**

6 **f(x) = (−3x + 1)(x − 2)**

7 **f(x) = (3x + 1)(−3x + 1)**

8 **f(x) = (−x + 1)(3x + 2)**

9 **f(x) = (−3x − 1)(x + 2)**

10 **f(x) = (−x − 2)(−x + 2)**

The Quotient Rule

WORKED EXAMPLE

$$f(x) = \frac{-3x - 1}{x - 3}$$

$u = -3x - 1$, $v = x - 3$

$u' = -3$, $v' = 1$

$$f'(x) = \frac{-3(x - 3) - (-3x - 1)(1)}{(x - 3)^2}$$

$$= \frac{-3x + 9 - (-3x - 1)}{(x - 3)^2}$$

$$= \frac{10}{(x - 3)^2}$$

3. Derivatives

Find f'(x) with the quotient rule: u'v − uv', all over v².

1 $f(x) = \frac{-3x}{2x - 2}$

2 $f(x) = \frac{2x + 3}{x - 1}$

3 $f(x) = \frac{3x - 1}{2x - 2}$

4 $f(x) = \frac{-2x - 2}{x - 1}$

5 $f(x) = \frac{-x}{x - 1}$

6 $f(x) = \frac{2x}{3x - 2}$

7 $f(x) = \frac{-3x + 2}{x - 2}$

8 $f(x) = \frac{-x - 2}{x - 1}$

The Chain Rule

WORKED EXAMPLE

$f(x) = (-2x - 1)^2$

$u = -2x - 1$, $u' = -2$

$f'(x) = 2(-2x - 1) \cdot (-2)$

$= -4(-2x - 1)$

4. Derivatives

Find f'(x) with the chain rule: the derivative of the outside, times u'.

1 **$f(x) = (-3x + 2)^2$**

2 **$f(x) = (x + 3)^2$**

3 **$f(x) = (-3x - 1)^3$**

4 **$f(x) = (2x - 3)^2$**

5 **$f(x) = (x - 2)^2$**

6 **$f(x) = (-3x - 1)^2$**

7 **$f(x) = (x - 1)^3$**

8 **$f(x) = (3x + 1)^2$**

9 **$f(x) = (-x + 2)^2$**

10 **$f(x) = (-2x - 2)^2$**

5. Derivatives

Find f'(x). Bring each power down in front, and take one off the power.

11 **$f(x) = x^2 - 3x$**

12 **$f(x) = -x^2 - x$**

13 **$f(x) = 2x^2 + 2x + 1$**

14 **$f(x) = -3x^2 + x$**

15 **$f(x) = x^2 - x$**

16 **$f(x) = -3x^2$**

Choosing the Rule

WORKED EXAMPLE

$f(x) = x^2 + x - 2$

$f'(x) = 2x + 1$

6. Derivatives

Find f'(x). Decide which rule the function needs before you start.

1 $f(x) = -2x^2 + 1$

2 $f(x) = (-2x + 1)^2$

3 $f(x) = (-x + 2)(-3x - 1)$

4 $f(x) = 2x^2 - 2x - 2$

5 $f(x) = (-2x - 3)^2$

6 $f(x) = -3x^2 - 2x + 1$

7 $f(x) = (-2x + 1)(-x + 1)$

8 $f(x) = (x + 3)^2$

9 $f(x) = x^2 - 3$

10 $f(x) = -2x^2 - 2x - 3$

7. Derivatives

Find f'(x) with the product rule: u'v + uv'.

11 $f(x) = (3x - 1)(x + 1)$

12 $f(x) = (-x + 1)(x - 1)$

13 $f(x) = (2x - 2)(-2x + 2)$

14 $f(x) = (-x - 1)(x - 1)$

15 $f(x) = (-x - 3)(-2x)$

16 $f(x) = (3x - 1)(-x)$

Derivatives of sin, cos, e^x and ln x

WORKED EXAMPLE

$$f(x) = -e^x - \ln x$$

$$f'(x) = -e^x - \frac{1}{x}$$

8. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

1 $f(x) = 2\sin x + \cos x$

2 $f(x) = -3\cos x + 2e^x$

3 $f(x) = 2\sin x - e^x + 3\ln x$

4 $f(x) = 4\sin x + 2\ln x$

5 $f(x) = 2\sin x + 3\cos x$

6 $f(x) = e^x - 4\ln x$

7 $f(x) = -\sin x + 3e^x + \ln x$

8 $f(x) = 3\sin x + e^x$

9 $f(x) = 2\sin x + 4e^x$

10 $f(x) = -3\sin x + 3e^x$

11 $f(x) = -\sin x + 2\cos x - 4\ln x$

12 $f(x) = 4\cos x + e^x$

9. Derivatives

Find f'(x) with the quotient rule: u'v − uv', all over v^2.

13 $f(x) = \dfrac{2x + 2}{2x + 1}$

14 $f(x) = \dfrac{x - 1}{2x - 1}$

15 $f(x) = \dfrac{3x + 1}{2x + 2}$

16 $f(x) = \dfrac{x}{3x - 1}$

17 $f(x) = \dfrac{-2x + 1}{x + 2}$

18 $f(x) = \dfrac{-x}{2x + 2}$

The Chain Rule with sin, cos, e^x and ln

WORKED EXAMPLE

$f(x) = -\ln(-x - 5)$

$u = -x - 5,\ u' = -1$

$$f'(x) = -\frac{1}{-x-5} \cdot (-1)$$

$$= \frac{1}{-x-5}$$

10. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

1 $f(x) = 5e^{-2x+1}$

2 $f(x) = -2e^{2x+1}$

3 $f(x) = e^{3x+4}$

4 $f(x) = 3\sin(2x - 2)$

5 $f(x) = 2e^{-x}$

6 $f(x) = -3\cos(2x + 3)$

7 $f(x) = 3\sin(4x + 1)$

8 $f(x) = -4e^{2x-1}$

9 $f(x) = -2e^{-2x-4}$

10 $f(x) = 4\sin(x - 3)$

11. Derivatives

Find f'(x) with the chain rule: the derivative of the outside, times u'.

11 $f(x) = (-2x + 2)^2$

12 $f(x) = (x + 3)^2$

13 $f(x) = (-x - 2)^3$

14 $f(x) = (2x + 1)^2$

15 $f(x) = (-3x - 2)^2$

16 $f(x) = (2x + 2)^2$

Product and Quotient Rules with Functions

WORKED EXAMPLE

$f(x) = 2e^x \cos x$

$u = 2e^x$, $v = \cos x$
$u' = 2e^x$, $v' = -\sin x$
$f'(x) = 2e^x \cos x - 2e^x \sin x$
$= 2e^x(\cos x - \sin x)$

12. Derivatives

Find f'(x). Decide whether f is a product, a quotient or a chain, then take u and v.

1 **$f(x) = 4e^x \cos x$**

2 **$f(x) = 2e^x \sin x$**

3 **$f(x) = \dfrac{\sin x}{x}$**

4 **$f(x) = 4e^x \sin x$**

5 **$f(x) = -4e^x \sin x$**

6 **$f(x) = 3e^x \cos x$**

7 **$f(x) = \dfrac{\cos x}{x}$**

8 **$f(x) = -3e^x \cos x$**

13. Derivatives

Find f'(x). Decide which rule the function needs before you start.

9 **$f(x) = -3x^2 + 2x$**

10 **$f(x) = -2x^2 - 2x - 1$**

11 **$f(x) = (x + 2)^3$**

12 **$f(x) = (-x + 1)^2$**

13 **$f(x) = (-2x - 1)^2$**

14 **$f(x) = -3x^2 + 3x + 3$**

The Slope of a Curve

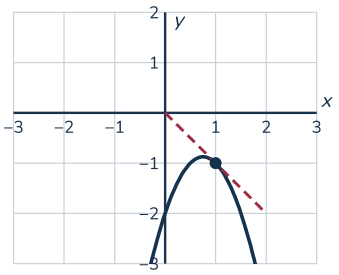
WORKED EXAMPLE

f(x) = −2x² + 3x − 2, x = 1

f'(x) = −4x + 3

f'(1) = −4(1) + 3

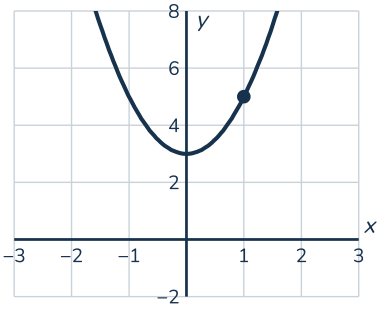
= −1



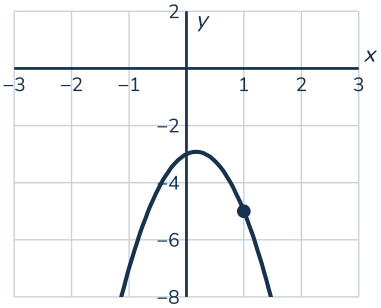
14. Using derivatives

Find the slope of each curve at the point: find f'(x), then put x in.

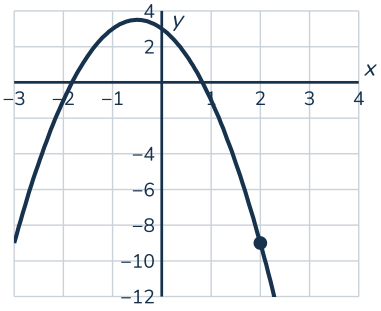
1 **f(x) = 2x² + 3, x = 1**



2 **f(x) = −3x² + x − 3, x = 1**



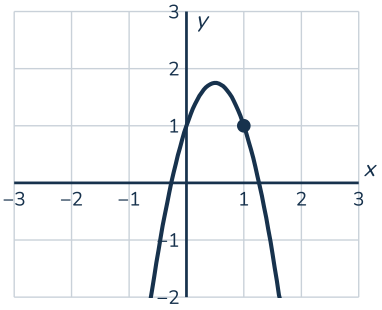
3 **f(x) = −2x² − 2x + 3, x = 2**



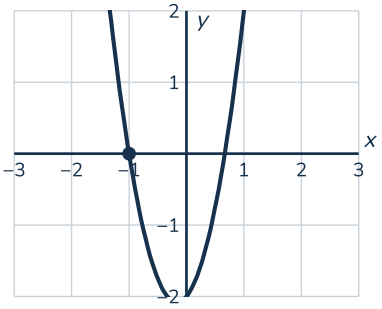
14. Using derivatives (continued)

Find the slope of each curve at the point: find f'(x), then put x in.

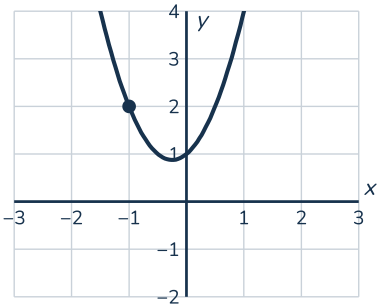
4 **$f(x) = -3x^2 + 3x + 1,\ x = 1$**



5 **$f(x) = 3x^2 + x - 2,\ x = -1$**



6 **$f(x) = 2x^2 + x + 1,\ x = -1$**



15. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

7 **$f(x) = 2 \sin x + 4e^x$**

8 **$f(x) = 5 \sin x + \ln x$**

9 **$f(x) = -\sin x + 2 \cos x - 4 \ln x$**

10 **$f(x) = 4 \cos x - 2e^x$**

11 **$f(x) = -2 \cos x - 4 \ln x$**

12 **$f(x) = -\sin x - 4 \cos x$**

Tangent Lines

WORKED EXAMPLE

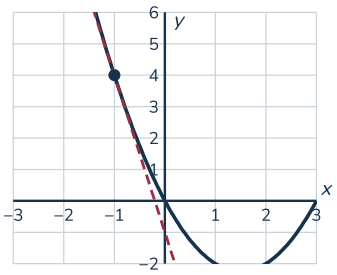
$f(x) = x^2 - 3x,\ x = -1$

$f'(x) = 2x - 3$

$m = f'(-1) = -5,\ f(-1) = 4$

$y - 4 = -5(x + 1)$

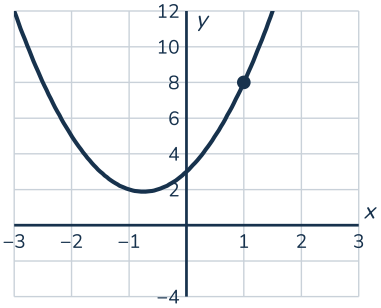
$y = -5x - 1$



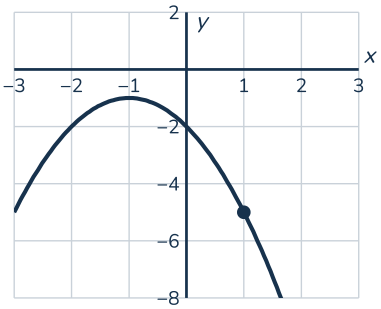
16. Using derivatives

Find the equation of the tangent line at the point given.

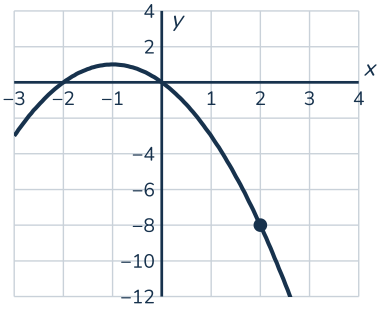
1 **$f(x) = 2x^2 + 3x + 3,\ x = 1$**



2 **$f(x) = -x^2 - 2x - 2,\ x = 1$**



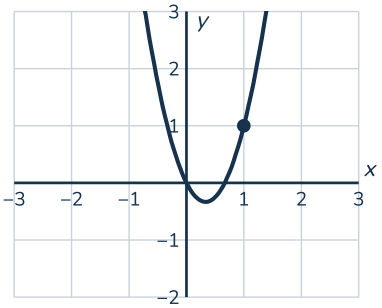
3 **$f(x) = -x^2 - 2x,\ x = 2$**



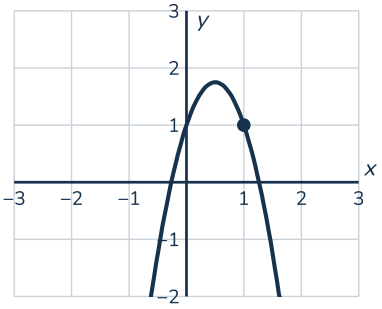
16. Using derivatives (continued)

Find the equation of the tangent line at the point given.

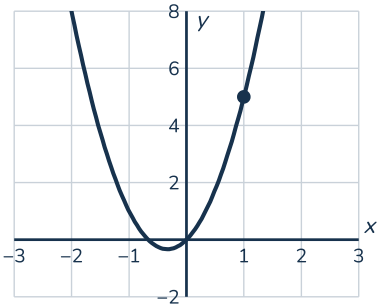
4 $f(x) = 3x^2 - 2x,\ x = 1$



5 $f(x) = -3x^2 + 3x + 1,\ x = 1$



6 $f(x) = 3x^2 + 2x,\ x = 1$



17. Derivatives

Find f'(x). Use the derivative of each function, and the chain rule when there is an inside.

7 $f(x) = -3e^{2x}$

8 $f(x) = 2\sin(4x)$

9 $f(x) = -5e^{3x-1}$

10 $f(x) = 2e^{2x+3}$

11 $f(x) = -2\cos(x + 2)$

12 $f(x) = 2\cos(x + 5)$

Stationary Points

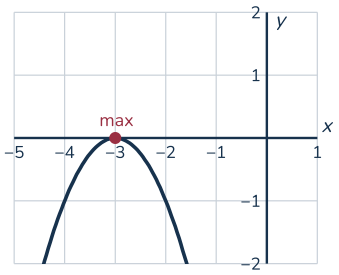
WORKED EXAMPLE

$f(x) = -x^2 - 6x - 9$

$f'(x) = -2x - 6 = 0$

$x = -3$, $f(-3) = 0$

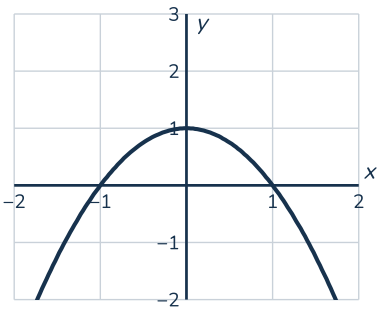
$f''(x) = -2 < 0$, so a maximum



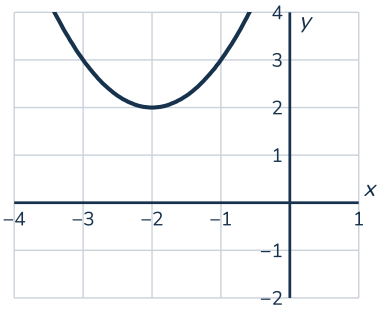
18. Using derivatives

Find where $f'(x) = 0$ and say if each point is a maximum or a minimum.

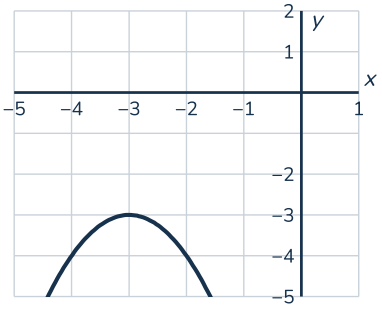
1 **$f(x) = -x^2 + 1$**



2 **$f(x) = x^2 + 4x + 6$**



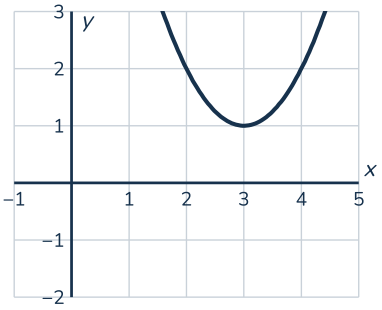
3 **$f(x) = -x^2 - 6x - 12$**



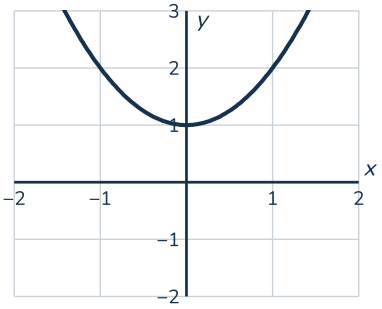
18. Using derivatives (continued)

Find where f'(x) = 0 and say if each point is a maximum or a minimum.

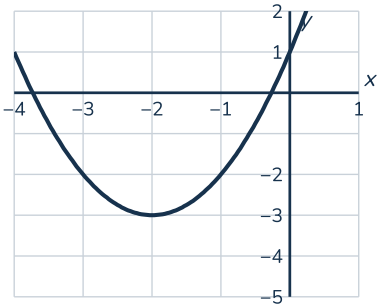
4 **f(x) = x² − 6x + 10**



5 **f(x) = x² + 1**



6 **f(x) = x² + 4x + 1**



19. Derivatives

Find f'(x). Decide whether f is a product, a quotient or a chain, then take u and v.

7 **f(x) = −4eˣ sin x**

8 **f(x) = −eˣ cos x**

9 $f(x) = \frac{2 \ln x}{x}$

10 **f(x) = −3eˣ sin x**

Indefinite Integrals

WORKED EXAMPLE

$$\int (4x - 2)\,dx$$
$$= \frac{4x^2}{2} - 2x + C$$
$$= 2x^2 - 2x + C$$

20. Integrals

Raise each power by one and divide by the new power. Don't forget + C.

1 $\int (-4x)\,dx$

2 $\int (2x - 1)\,dx$

3 $\int (-4x + 2)\,dx$

4 $\int (6x)\,dx$

5 $\int (-4x - 1)\,dx$

6 $\int (-2x - 2)\,dx$

7 $\int (2x + 3)\,dx$

8 $\int (4x)\,dx$

9 $\int (4x - 1)\,dx$

10 $\int (-4x + 1)\,dx$

11 $\int (-4x - 2)\,dx$

12 $\int (-2x + 1)\,dx$

21. Derivatives

Find f'(x). Bring each power down in front, and take one off the power.

13 $f(x) = -2x^2 - 3$

14 $f(x) = x^2 - x - 2$

15 $f(x) = -3x^2 + x - 2$

16 $f(x) = -x^2 - 2$

17 $f(x) = -2x^2 + 1$

18 $f(x) = -x^2 + 3x$

Definite Integrals

WORKED EXAMPLE

$$\int_0^1 (-4x)\,dx$$
$$= [-2x^2]_0^1$$
$$= (-2(1)^2) - 0$$
$$= -2 - 0$$
$$= -2$$

22. Integrals

Find the antiderivative, then take its value at the bottom from the top.

1 $\int_{-1}^{0} (4x)\,dx$

2 $\int_0^2 (4x)\,dx$

3 $\int_{-1}^{0} (2x - 2)\,dx$

4 $\int_0^1 (4x)\,dx$

5 $\int_0^2 (-4x - 2)\,dx$

6 $\int_0^1 (4x + 2)\,dx$

7 $\int_{-1}^{1} (-4x + 2)\,dx$

8 $\int_0^2 (-2x)\,dx$

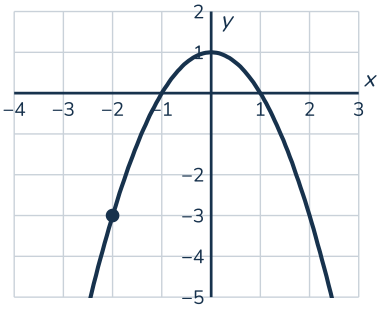
9 $\int_1^2 (-4x - 1)\,dx$

10 $\int_0^1 (-2x + 2)\,dx$

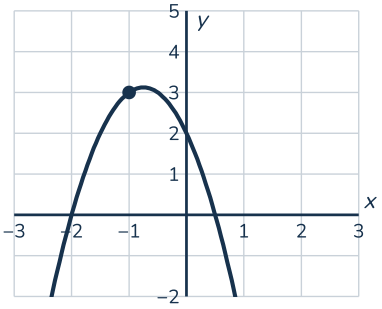
23. Using derivatives

Find the slope of each curve at the point: find f'(x), then put x in.

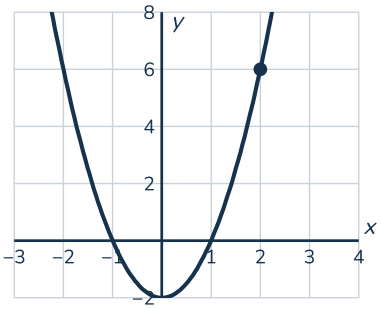
11 **$f(x) = -x^2 + 1,\ x = -2$**



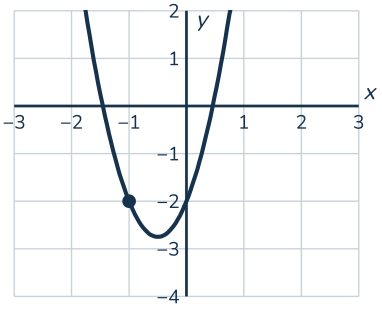
12 **$f(x) = -2x^2 - 3x + 2,\ x = -1$**



13 **$f(x) = 2x^2 - 2,\ x = 2$**



14 **$f(x) = 3x^2 + 3x - 2,\ x = -1$**



Integration by Substitution

WORKED EXAMPLE

$\int -3(-x+3)^2\,dx$

$u = -x + 3,\ du = -\,dx$

$= \int 3u^2\,du$

$= u^3 + C$

$= (-x+3)^3 + C$

24. Integrals

Find each integral by substitution: let u be the bracket.

1 $\int 6(2x-2)^2\,dx$

2 $\int 6(2x-3)^2\,dx$

3 $\int -9(-3x+3)^2\,dx$

4 $\int 3(x+1)^2\,dx$

5 $\int 6(2x+1)^2\,dx$

6 $\int -6(-2x-2)^2\,dx$

7 $\int 8(2x+1)^3\,dx$

8 $\int -6(-2x-3)^2\,dx$

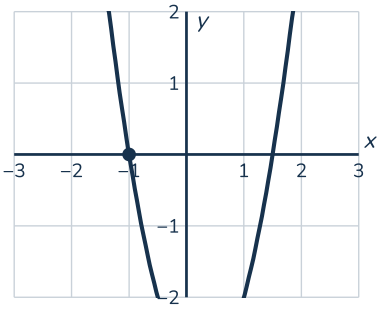
9 $\int -9(-3x-1)^2\,dx$

10 $\int -6(-2x+1)^2\,dx$

25. Using derivatives

Find the equation of the tangent line at the point given.

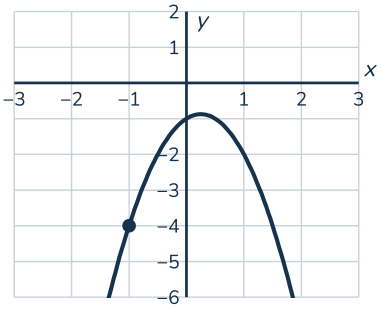
11 $f(x) = 2x^2 - x - 3,\ x = -1$



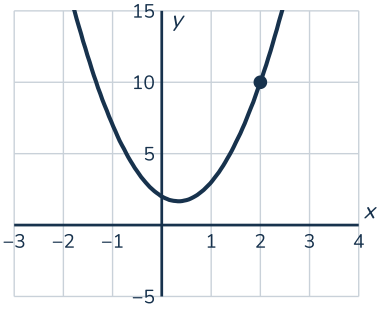
25. Using derivatives (continued)

Find the equation of the tangent line at the point given.

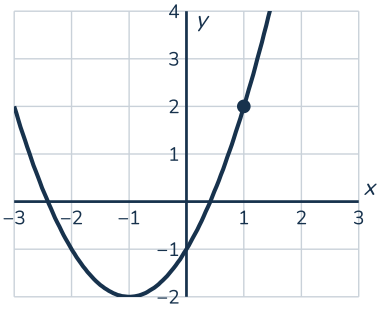
12 $f(x) = -2x^2 + x - 1,\ x = -1$



13 $f(x) = 3x^2 - 2x + 2,\ x = 2$



14 $f(x) = x^2 + 2x - 1,\ x = 1$



Integrals of sin, cos, e^x and 1/x

WORKED EXAMPLE

$$\int \left(-5 \sin x + \frac{1}{x}\right) dx$$
$$= 5 \cos x + \ln|x| + C$$

26. Integrals

Find each integral. Use the integral of each function, and divide by what the inside multiplies x by.

1 $\int \left(-4 \cos x - \frac{1}{x}\right) dx$

2 $\int (3 \sin x + 2e^x)\, dx$

3 $\int \left(-\cos x - \sin x - \frac{1}{x}\right) dx$

4 $\int (-3 \cos x + e^x)\, dx$

5 $\int \left(\cos x - \frac{1}{x}\right) dx$

6 $\int (\sin x - 5e^x)\, dx$

7 $\int (-3 \sin x + 4e^x)\, dx$

8 $\int (-\cos x - 2 \sin x)\, dx$

9 $\int (\cos x + 4e^x)\, dx$

10 $\int (-3 \cos x + \sin x)\, dx$

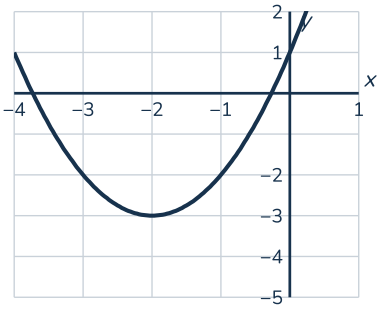
11 $\int (\cos x + 2 \sin x + 2e^x)\, dx$

12 $\int (-\cos x + 5 \sin x)\, dx$

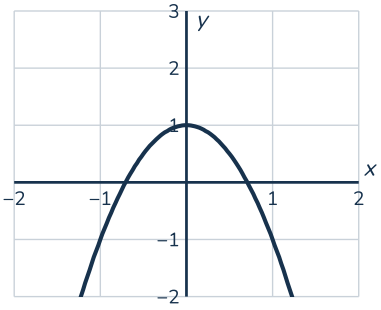
27. Using derivatives

Find where f'(x) = 0 and say if each point is a maximum or a minimum.

13 $f(x) = x^2 + 4x + 1$



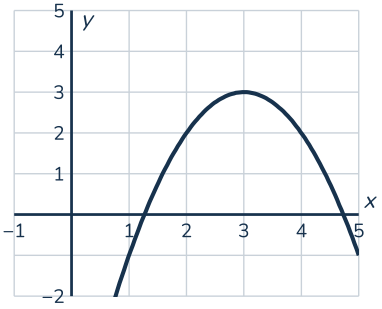
14 $f(x) = -2x^2 + 1$



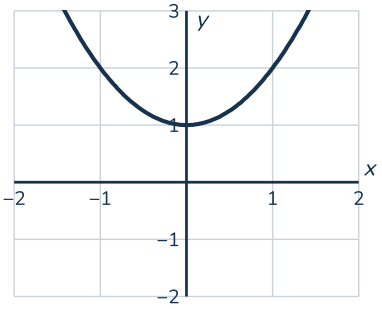
27. Using derivatives (continued)

Find where f'(x) = 0 and say if each point is a maximum or a minimum.

15 **$f(x) = -x^2 + 6x - 6$**



16 **$f(x) = x^2 + 1$**



Integration by Parts

WORKED EXAMPLE

$\int -2x \sin x \, dx$

$u = -2x,\ dv = \sin x \, dx$

$du = -2 \, dx,\ v = -\cos x$

$= 2x \cos x - \int 2 \cos x \, dx$

$= 2x \cos x - 2 \sin x + C$

28. Integrals

Find each integral by parts: $\int u \, dv = uv - \int v \, du$.

1 $\int -x \cos x \, dx$

2 $\int -x \sin x \, dx$

3 $\int -4x \cos x \, dx$

4 $\int 2x \cos x \, dx$

5 $\int 2xe^x \, dx$

6 $\int -2xe^x \, dx$

7 $\int -4xe^x \, dx$

8 $\int x \sin x \, dx$

29. Derivatives

Find f'(x) with the product rule: u'v + uv'.

9 $f(x) = (-x + 2)(2x + 1)$

10 $f(x) = (-x + 2)(-3x - 1)$

11 $f(x) = (-2x + 3)(x + 3)$

12 $f(x) = (-2x + 1)(-x - 3)$

13 $f(x) = (x + 1)(-2x - 3)$

14 $f(x) = (-x + 1)(-x - 2)$

The nth-Term Test

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} \frac{2}{n^2 - n + 3}$$

$$\lim_{n\to\infty} \frac{2}{n^2 - n + 3} = \lim_{n\to\infty} \frac{2/n^2}{1 - 1/n + 3/n^2}$$

$$= \frac{0}{1 + 0 + 0} = 0$$

$\lim a_n = 0$: the test says nothing

30. Series: converge or diverge?

Find $\lim a_n$. Not 0: the series diverges. 0: the test says nothing.

1 $$\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 3}$$

2 $$\sum_{n=1}^{\infty} \frac{-2n - 2}{2n^2 + 3}$$

3 $$\sum_{n=1}^{\infty} \frac{-n - 3}{n^2 + 1}$$

4 $$\sum_{n=1}^{\infty} \frac{-2n^2 + 1}{n^2 + 2}$$

5 $$\sum_{n=1}^{\infty} \frac{-n^2 - 3}{2n^2 + 1}$$

6 $$\sum_{n=1}^{\infty} \frac{2n - 2}{n^2 + 1}$$

7 $$\sum_{n=1}^{\infty} \frac{2n - 2}{2n + 3}$$

8 $$\sum_{n=1}^{\infty} \frac{2}{2n^2 + n - 1}$$

31. Integrals

Find the antiderivative, then take its value at the bottom from the top.

9 $\int_{1}^{2} (4x - 1)\,dx$

10 $\int_{0}^{1} (2x - 2)\,dx$

11 $\int_{-1}^{1} (2x)\,dx$

12 $\int_{0}^{2} (-2x + 2)\,dx$

Geometric and Telescoping Series

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} 3(-4)^{n-1}$$

$a = 3$ and $r = -4$

$|r| = 4 > 1$, so the series diverges

32. Series: converge or diverge?

Does each series converge? If it does, find its sum.

1 $$\sum_{n=1}^{\infty} -2(-4)^{n-1}$$

2 $$\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^{n}$$

3 $$\sum_{n=1}^{\infty} -2\left(-\frac{1}{4}\right)^{n-1}$$

4 $$\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{n}$$

5 $$\sum_{n=0}^{\infty} -2\left(\frac{2}{3}\right)^{n}$$

6 $$\sum_{n=0}^{\infty} 2 \cdot 4^{n}$$

7 $$\sum_{n=1}^{\infty} 3(-2)^{n}$$

8 $$\sum_{n=0}^{\infty} 3 \cdot 4^{n}$$

33. Integrals

Find each integral by substitution: let u be the bracket.

9 $\int -6(-2x + 2)^2 \, dx$

10 $\int -9(-3x + 2)^2 \, dx$

11 $\int -8(-2x + 1)^3 \, dx$

12 $\int 3(x + 3)^2 \, dx$

p-Series and the Comparison Test

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} \frac{n+3}{2n^3+3}$$

Compare with the p-series $\sum \frac{1}{n^2}$ with $p = 2$

$\frac{a_n}{b_n} = \frac{n^3+3n^2}{2n^3+3} \to \frac{1}{2} > 0$, and finite

$\sum \frac{1}{n^2}$ converges, so this series converges

34. Series: converge or diverge?

Does each series converge? Compare it with a p-series, $\sum 1/n^p$.

1 $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}$

2 $\sum_{n=1}^{\infty} \frac{2}{n}$

3 $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

4 $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}$

5 $\sum_{n=1}^{\infty} \frac{1}{n}$

6 $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$

7 $\sum_{n=1}^{\infty} \frac{1}{n^2\sqrt{n}}$

8 $\sum_{n=1}^{\infty} \frac{3}{n}$

35. Integrals

Find each integral. Use the integral of each function, and divide by what the inside multiplies x by.

9 $\int (-2\sin x + e^x)\,dx$

10 $\int (3\cos x - \sin x)\,dx$

11 $\int (5\sin x + \frac{2}{x})\,dx$

12 $\int (-\cos x + \frac{1}{x})\,dx$

The Ratio Test

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} \frac{4^n}{n^2}$$

$$\frac{a_{n+1}}{a_n} = \frac{4^{n+1}}{(n+1)^2} \cdot \frac{n^2}{4^n}$$

$$= 4 \cdot \frac{n^2}{(n+1)^2}$$

$$L = \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 4 \cdot 1 = 4$$

$L > 1$, so the series diverges

36. Series: converge or diverge?

Does each series converge? Use the ratio test: $L = \lim |a_{n+1} \div a_n|$.

1 $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

2 $\sum_{n=1}^{\infty} \frac{n}{3^n}$

3 $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$

4 $\sum_{n=1}^{\infty} \frac{3^n}{n}$

5 $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

6 $\sum_{n=1}^{\infty} \frac{n!}{2^n}$

7 $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

8 $\sum_{n=1}^{\infty} \frac{n}{2^n}$

37. Derivatives

Find $f'(x)$. Use the derivative of each function, and the chain rule when there is an inside.

9 $f(x) = -3\cos x - \ln x$

10 $f(x) = 2e^x - 3\ln x$

11 $f(x) = 5\sin x - 5e^x$

12 $f(x) = -\cos x - 2e^x$

13 $f(x) = 4e^x + 2\ln x$

14 $f(x) = 2\cos x - 3\ln x$

Alternating Series

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n}$$

$\sum |a_n| = \sum \frac{2}{n}$: $p = 1 \leq 1$, diverges

$b_n = \frac{2}{n}$ decreases, and $\lim_{n \to \infty} b_n = 0$

Alternating series test: converges

The series converges conditionally

38. Series: converge or diverge?

Does each series converge absolutely, converge conditionally, or diverge?

1 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n+1}$$

2 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$$

3 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$$

4 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{2n+1}$$

5 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n\sqrt{n}}$$

6 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+3}$$

7 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2\sqrt{n}}$$

8 $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n\sqrt{n}}$$

39. Integrals

Raise each power by one and divide by the new power. Don't forget + C.

9 $\int (-2x + 1)\,dx$

10 $\int (2x - 1)\,dx$

11 $\int (-6x + 1)\,dx$

12 $\int (-2x)\,dx$

13 $\int (-2x + 2)\,dx$

14 $\int (4x - 1)\,dx$

Interval of Convergence

WORKED EXAMPLE

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{3^n}$$

$$\frac{|a_{n+1}|}{|a_n|} = \frac{|x+2|}{3}$$

$L = \frac{|x+2|}{3} < 1$: $R = 3$, $-5 < x < 1$

$x = 1$: $\sum 1$ diverges ($a_n \not\to 0$)

$x = -5$: $\sum (-1)^n$ diverges ($a_n \not\to 0$)

Interval: $(-5, 1)$

40. Series: converge or diverge?

Find the interval of convergence. Check both endpoints.

1 $$\sum_{n=1}^{\infty} \frac{(x-1)^n}{3^n}$$

2 $$\sum_{n=1}^{\infty} \frac{(x+3)^n}{3^n}$$

3 $$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n \cdot 2^n}$$

4 $$\sum_{n=1}^{\infty} \frac{(x-2)^n}{2^n}$$

41. Derivatives

Find f'(x) with the product rule: u'v + uv'.

5 $f(x) = (-x - 1)(-x + 2)$

6 $f(x) = (-2x - 1)(3x + 1)$

7 $f(x) = (3x + 2)(-3x + 1)$

8 $f(x) = (x + 3)x$

Answer Key (1 of 5)

The Power Rule

Derivatives

1. $f(x) = -3x^2 - x \rightarrow f'(x) = -6x - 1$
2. $f(x) = -3x^2 - 1 \rightarrow f'(x) = -6x$
3. $f(x) = 3x^2 - x - 1 \rightarrow f'(x) = 6x - 1$
4. $f(x) = 2x^2 - 2 \rightarrow f'(x) = 4x$
5. $f(x) = -3x^2 \rightarrow f'(x) = -6x$
6. $f(x) = x^2 + 2x + 1 \rightarrow f'(x) = 2x + 2$
7. $f(x) = 2x^2 - 2x + 1 \rightarrow f'(x) = 4x - 2$
8. $f(x) = -2x^2 + x - 1 \rightarrow f'(x) = -4x + 1$
9. $f(x) = -x^2 - x - 1 \rightarrow f'(x) = -2x - 1$
10. $f(x) = -x^2 + x - 1 \rightarrow f'(x) = -2x + 1$
11. $f(x) = 3x^2 + x + 2 \rightarrow f'(x) = 6x + 1$
12. $f(x) = -x^2 \rightarrow f'(x) = -2x$

The Product Rule

Derivatives

1. $f(x) = (x - 1)(-2x - 3) \rightarrow f'(x) = -4x - 1$
2. $f(x) = (-2x - 1)(-x + 1) \rightarrow f'(x) = 4x - 1$
3. $f(x) = (3x + 1)(2x + 3) \rightarrow f'(x) = 12x + 11$
4. $f(x) = (x - 3)(-2x + 1) \rightarrow f'(x) = -4x + 7$
5. $f(x) = (2x + 1)(3x - 1) \rightarrow f'(x) = 12x + 1$
6. $f(x) = (-3x + 1)(x - 2) \rightarrow f'(x) = -6x + 7$
7. $f(x) = (3x + 1)(-3x + 1) \rightarrow f'(x) = -18x$
8. $f(x) = (-x + 1)(3x + 2) \rightarrow f'(x) = -6x + 1$
9. $f(x) = (-3x - 1)(x + 2) \rightarrow f'(x) = -6x - 7$
10. $f(x) = (-x - 2)(-x + 2) \rightarrow f'(x) = 2x$

The Quotient Rule

Derivatives

1. $f(x) = \frac{-3x}{2x - 2} \rightarrow f'(x) = \frac{6}{2x - 2^2}$
2. $f(x) = \frac{2x + 3}{x - 1} \rightarrow f'(x) = -\frac{5}{x - 1^2}$
3. $f(x) = \frac{3x - 1}{2x - 2} \rightarrow f'(x) = -\frac{4}{2x - 2^2}$
4. $f(x) = \frac{-2x - 2}{x - 1} \rightarrow f'(x) = \frac{4}{x - 1^2}$
5. $f(x) = \frac{-x}{x - 1} \rightarrow f'(x) = \frac{1}{x - 1^2}$
6. $f(x) = \frac{2x}{3x - 2} \rightarrow f'(x) = -\frac{4}{3x - 2^2}$
7. $f(x) = \frac{-3x + 2}{x - 2} \rightarrow f'(x) = \frac{4}{x - 2^2}$
8. $f(x) = \frac{-x - 2}{x - 1} \rightarrow f'(x) = \frac{3}{x - 1^2}$

The Chain Rule

Derivatives

1. $f(x) = (-3x + 2)^2 \rightarrow f'(x) = -6(-3x + 2)$
2. $f(x) = (x + 3)^2 \rightarrow f'(x) = 2(x + 3)$
3. $f(x) = (-3x - 1)^3 \rightarrow f'(x) = -9(-3x - 1)^2$
4. $f(x) = (2x - 3)^2 \rightarrow f'(x) = 4(2x - 3)$
5. $f(x) = (x - 2)^2 \rightarrow f'(x) = 2(x - 2)$
6. $f(x) = (-3x - 1)^2 \rightarrow f'(x) = -6(-3x - 1)$
7. $f(x) = (x - 1)^3 \rightarrow f'(x) = 3(x - 1)^2$
8. $f(x) = (3x + 1)^2 \rightarrow f'(x) = 6(3x + 1)$

The Chain Rule (continued)

Derivatives (continued)

9. $f(x) = (-x + 2)^2 \rightarrow f'(x) = -2(-x + 2)$
10. $f(x) = (-2x - 2)^2 \rightarrow f'(x) = -4(-2x - 2)$

Derivatives

11. $f(x) = x^2 - 3x \rightarrow f'(x) = 2x - 3$
12. $f(x) = -x^2 - x \rightarrow f'(x) = -2x - 1$
13. $f(x) = 2x^2 + 2x + 1 \rightarrow f'(x) = 4x + 2$
14. $f(x) = -3x^2 + x \rightarrow f'(x) = -6x + 1$
15. $f(x) = x^2 - x \rightarrow f'(x) = 2x - 1$
16. $f(x) = -3x^2 \rightarrow f'(x) = -6x$

Choosing the Rule

Derivatives

1. $f(x) = -2x^2 + 1 \rightarrow f'(x) = -4x$
2. $f(x) = (-2x + 1)^2 \rightarrow f'(x) = -4(-2x + 1)$
3. $f(x) = (-x + 2)(-3x - 1) \rightarrow f'(x) = 6x - 5$
4. $f(x) = 2x^2 - 2x - 2 \rightarrow f'(x) = 4x - 2$
5. $f(x) = (-2x - 3)^2 \rightarrow f'(x) = -4(-2x - 3)$
6. $f(x) = -3x^2 - 2x + 1 \rightarrow f'(x) = -6x - 2$
7. $f(x) = (-2x + 1)(-x + 1) \rightarrow f'(x) = 4x - 3$
8. $f(x) = (x + 3)^2 \rightarrow f'(x) = 2(x + 3)$
9. $f(x) = x^2 - 3 \rightarrow f'(x) = 2x$
10. $f(x) = -2x^2 - 2x - 3 \rightarrow f'(x) = -4x - 2$

Derivatives

11. $f(x) = (3x - 1)(x + 1) \rightarrow f'(x) = 6x + 2$
12. $f(x) = (-x + 1)(x - 1) \rightarrow f'(x) = -2x + 2$
13. $f(x) = (2x - 2)(-2x + 2) \rightarrow f'(x) = -8x + 8$
14. $f(x) = (-x - 1)(x - 1) \rightarrow f'(x) = -2x$
15. $f(x) = (-x - 3)(-2x) \rightarrow f'(x) = 4x + 6$
16. $f(x) = (3x - 1)(-x) \rightarrow f'(x) = -6x + 1$

Derivatives of sin, cos, e^x and ln x

Derivatives

1. $f(x) = 2 \sin x + \cos x \rightarrow f'(x) = 2 \cos x - \sin x$
2. $f(x) = -3 \cos x + 2e^x \rightarrow f'(x) = 3 \sin x + 2e^x$
3. $f(x) = 2 \sin x - e^x + 3 \ln x \rightarrow f'(x) = 2 \cos x - e^x + \frac{3}{x}$
4. $f(x) = 4 \sin x + 2 \ln x \rightarrow f'(x) = 4 \cos x + \frac{2}{x}$
5. $f(x) = 2 \sin x + 3 \cos x \rightarrow f'(x) = 2 \cos x - 3 \sin x$
6. $f(x) = e^x - 4 \ln x \rightarrow f'(x) = e^x - \frac{4}{x}$
7. $f(x) = -\sin x + 3e^x + \ln x \rightarrow f'(x) = -\cos x + 3e^x + \frac{1}{x}$
8. $f(x) = 3 \sin x + e^x \rightarrow f'(x) = 3 \cos x + e^x$
9. $f(x) = 2 \sin x + 4e^x \rightarrow f'(x) = 2 \cos x + 4e^x$
10. $f(x) = -3 \sin x + 3e^x \rightarrow f'(x) = -3 \cos x + 3e^x$
11. $f(x) = -\sin x + 2 \cos x - 4 \ln x \rightarrow f'(x) = -\cos x - 2 \sin x - \frac{4}{x}$
12. $f(x) = 4 \cos x + e^x \rightarrow f'(x) = -4 \sin x + e^x$

Derivatives

13. $f(x) = \frac{2x + 2}{2x + 1} \rightarrow f'(x) = -\frac{2}{2x + 1^2}$

Answer Key (2 of 5)

Derivatives of sin, cos, e^x and ln x (continued)

Derivatives (continued)

14. $f(x) = \frac{x-1}{2x-1} \rightarrow f'(x) = \frac{1}{2x-1}^2$
15. $f(x) = \frac{3x+1}{2x+2} \rightarrow f'(x) = \frac{4}{2x+2}^2$
16. $f(x) = x/(3x-1) \rightarrow f'(x) = -\frac{1}{3x-1}^2$
17. $f(x) = \frac{-2x+1}{x+2} \rightarrow f'(x) = -\frac{5}{x+2}^2$
18. $f(x) = \frac{-x}{2x+2} \rightarrow f'(x) = -\frac{2}{2x+2}^2$

The Chain Rule with sin, cos, e^x and ln

Derivatives

1. $f(x) = 5e^{-2x+1} \rightarrow f'(x) = -10e^{-2x+1}$
2. $f(x) = -2e^{2x+1} \rightarrow f'(x) = -4e^{2x+1}$
3. $f(x) = e^{3x+4} \rightarrow f'(x) = 3e^{3x+4}$
4. $f(x) = 3\sin(2x-2) \rightarrow f'(x) = 6\cos(2x-2)$
5. $f(x) = 2e^{-x} \rightarrow f'(x) = -2e^{-x}$
6. $f(x) = -3\cos(2x+3) \rightarrow f'(x) = 6\sin(2x+3)$
7. $f(x) = 3\sin(4x+1) \rightarrow f'(x) = 12\cos(4x+1)$
8. $f(x) = -4e^{2x-1} \rightarrow f'(x) = -8e^{2x-1}$
9. $f(x) = -2e^{-2x-4} \rightarrow f'(x) = 4e^{-2x-4}$
10. $f(x) = 4\sin(x-3) \rightarrow f'(x) = 4\cos(x-3)$

Derivatives

11. $f(x) = (-2x+2)^2 \rightarrow f'(x) = -4(-2x+2)$
12. $f(x) = (x+3)^2 \rightarrow f'(x) = 2(x+3)$
13. $f(x) = (-x-2)^3 \rightarrow f'(x) = -3(-x-2)^2$
14. $f(x) = (2x+1)^2 \rightarrow f'(x) = 4(2x+1)$
15. $f(x) = (-3x-2)^2 \rightarrow f'(x) = -6(-3x-2)$
16. $f(x) = (2x+2)^2 \rightarrow f'(x) = 4(2x+2)$

Product and Quotient Rules with Functions

Derivatives

1. $f(x) = 4e^x \cos x \rightarrow f'(x) = 4e^x(\cos x - \sin x)$
2. $f(x) = 2e^x \sin x \rightarrow f'(x) = 2e^x(\sin x + \cos x)$
3. $f(x) = \frac{\sin x}{x} \rightarrow f'(x) = \frac{x\cos x - \sin x}{x^2}$
4. $f(x) = 4e^x \sin x \rightarrow f'(x) = 4e^x(\sin x + \cos x)$
5. $f(x) = -4e^x \sin x \rightarrow f'(x) = -4e^x(\sin x + \cos x)$
6. $f(x) = 3e^x \cos x \rightarrow f'(x) = 3e^x(\cos x - \sin x)$
7. $f(x) = \frac{\cos x}{x} \rightarrow f'(x) = \frac{-(x\sin x + \cos x)}{x^2}$
8. $f(x) = -3e^x \cos x \rightarrow f'(x) = -3e^x(\cos x - \sin x)$

Derivatives

9. $f(x) = -3x^2 + 2x \rightarrow f'(x) = -6x + 2$
10. $f(x) = -2x^2 - 2x - 1 \rightarrow f'(x) = -4x - 2$
11. $f(x) = (x+2)^3 \rightarrow f'(x) = 3(x+2)^2$
12. $f(x) = (-x+1)^2 \rightarrow f'(x) = -2(-x+1)$
13. $f(x) = (-2x-1)^2 \rightarrow f'(x) = -4(-2x-1)$
14. $f(x) = -3x^2 + 3x + 3 \rightarrow f'(x) = -6x + 3$

The Slope of a Curve

Using derivatives

1. $f(x) = 2x^2 + 3$; slope at $x = 1 \rightarrow f'(1) = 4$

The Slope of a Curve (continued)

Using derivatives (continued)

2. $f(x) = -3x^2 + x - 3$; slope at $x = 1 \rightarrow f'(1) = -5$
3. $f(x) = -2x^2 - 2x + 3$; slope at $x = 2 \rightarrow f'(2) = -10$
4. $f(x) = -3x^2 + 3x + 1$; slope at $x = 1 \rightarrow f'(1) = -3$
5. $f(x) = 3x^2 + x - 2$; slope at $x = -1 \rightarrow f'(-1) = -5$
6. $f(x) = 2x^2 + x + 1$; slope at $x = -1 \rightarrow f'(-1) = -3$

Derivatives

7. $f(x) = 2\sin x + 4e^x \rightarrow f'(x) = 2\cos x + 4e^x$
8. $f(x) = 5\sin x + \ln x \rightarrow f'(x) = 5\cos x + \frac{1}{x}$
9. $f(x) = -\sin x + 2\cos x - 4\ln x \rightarrow f'(x) = -\cos x - 2\sin x - \frac{4}{x}$
10. $f(x) = 4\cos x - 2e^x \rightarrow f'(x) = -4\sin x - 2e^x$
11. $f(x) = -2\cos x - 4\ln x \rightarrow f'(x) = 2\sin x - \frac{4}{x}$
12. $f(x) = -\sin x - 4\cos x \rightarrow f'(x) = -\cos x + 4\sin x$

Tangent Lines

Using derivatives

1. $f(x) = 2x^2 + 3x + 3$; tangent at $x = 1 \rightarrow y = 7x + 1$
2. $f(x) = -x^2 - 2x - 2$; tangent at $x = 1 \rightarrow y = -4x - 1$
3. $f(x) = -x^2 - 2x$; tangent at $x = 2 \rightarrow y = -6x + 4$
4. $f(x) = 3x^2 - 2x$; tangent at $x = 1 \rightarrow y = 4x - 3$
5. $f(x) = -3x^2 + 3x + 1$; tangent at $x = 1 \rightarrow y = -3x + 4$
6. $f(x) = 3x^2 + 2x$; tangent at $x = 1 \rightarrow y = 8x - 3$

Derivatives

7. $f(x) = -3e^{2x} \rightarrow f'(x) = -6e^{2x}$
8. $f(x) = 2\sin(4x) \rightarrow f'(x) = 8\cos(4x)$
9. $f(x) = -5e^{3x-1} \rightarrow f'(x) = -15e^{3x-1}$
10. $f(x) = 2e^{2x+3} \rightarrow f'(x) = 4e^{2x+3}$
11. $f(x) = -2\cos(x+2) \rightarrow f'(x) = 2\sin(x+2)$
12. $f(x) = 2\cos(x+5) \rightarrow f'(x) = -2\sin(x+5)$

Stationary Points

Using derivatives

1. $f(x) = -x^2 + 1$; stationary points → maximum (0, 1)
2. $f(x) = x^2 + 4x + 6$; stationary points → minimum (−2, 2)
3. $f(x) = -x^2 - 6x - 12$; stationary points → maximum (−3, −3)
4. $f(x) = x^2 - 6x + 10$; stationary points → minimum (3, 1)
5. $f(x) = x^2 + 1$; stationary points → minimum (0, 1)
6. $f(x) = x^2 + 4x + 1$; stationary points → minimum (−2, −3)

Derivatives

7. $f(x) = -4e^x \sin x \rightarrow f'(x) = -4e^x(\sin x + \cos x)$
8. $f(x) = -e^x \cos x \rightarrow f'(x) = -e^x(\cos x - \sin x)$
9. $f(x) = \frac{2\ln x}{x} \rightarrow f'(x) = \frac{2(1-\ln x)}{x^2}$
10. $f(x) = -3e^x \sin x \rightarrow f'(x) = -3e^x(\sin x + \cos x)$

Indefinite Integrals

Integrals

1. $\int(-4x)\,dx = -2x^2 + C$
2. $\int(2x-1)\,dx = x^2 - x + C$

Answer Key (3 of 5)

Indefinite Integrals (continued)

Integrals (continued)

3. $\int(-4x + 2)\,dx = -2x^2 + 2x + C$
4. $\int(6x)\,dx = 3x^2 + C$
5. $\int(-4x - 1)\,dx = -2x^2 - x + C$
6. $\int(-2x - 2)\,dx = -x^2 - 2x + C$
7. $\int(2x + 3)\,dx = x^2 + 3x + C$
8. $\int(4x)\,dx = 2x^2 + C$
9. $\int(4x - 1)\,dx = 2x^2 - x + C$
10. $\int(-4x + 1)\,dx = -2x^2 + x + C$
11. $\int(-4x - 2)\,dx = -2x^2 - 2x + C$
12. $\int(-2x + 1)\,dx = -x^2 + x + C$

Derivatives

13. $f(x) = -2x^2 - 3 \rightarrow f'(x) = -4x$
14. $f(x) = x^2 - x - 2 \rightarrow f'(x) = 2x - 1$
15. $f(x) = -3x^2 + x - 2 \rightarrow f'(x) = -6x + 1$
16. $f(x) = -x^2 - 2 \rightarrow f'(x) = -2x$
17. $f(x) = -2x^2 + 1 \rightarrow f'(x) = -4x$
18. $f(x) = -x^2 + 3x \rightarrow f'(x) = -2x + 3$

Definite Integrals

Integrals

1. $\int$ from -1 to 0 of $(4x)\,dx = -2$
2. $\int$ from 0 to 2 of $(4x)\,dx = 8$
3. $\int$ from -1 to 0 of $(2x - 2)\,dx = -3$
4. $\int$ from 0 to 1 of $(4x)\,dx = 2$
5. $\int$ from 0 to 2 of $(-4x - 2)\,dx = -12$
6. $\int$ from 0 to 1 of $(4x + 2)\,dx = 4$
7. $\int$ from -1 to 1 of $(-4x + 2)\,dx = 4$
8. $\int$ from 0 to 2 of $(-2x)\,dx = -4$
9. $\int$ from 1 to 2 of $(-4x - 1)\,dx = -7$
10. $\int$ from 0 to 1 of $(-2x + 2)\,dx = 1$

Using derivatives

11. $f(x) = -x^2 + 1$; slope at $x = -2 \rightarrow f'(-2) = 4$
12. $f(x) = -2x^2 - 3x + 2$; slope at $x = -1 \rightarrow f'(-1) = 1$
13. $f(x) = 2x^2 - 2$; slope at $x = 2 \rightarrow f'(2) = 8$
14. $f(x) = 3x^2 + 3x - 2$; slope at $x = -1 \rightarrow f'(-1) = -3$

Integration by Substitution

Integrals

1. $\int 6(2x - 2)^2\,dx = (2x - 2)^3 + C$
2. $\int 6(2x - 3)^2\,dx = (2x - 3)^3 + C$
3. $\int -9(-3x + 3)^2\,dx = (-3x + 3)^3 + C$
4. $\int 3(x + 1)^2\,dx = (x + 1)^3 + C$
5. $\int 6(2x + 1)^2\,dx = (2x + 1)^3 + C$
6. $\int -6(-2x - 2)^2\,dx = (-2x - 2)^3 + C$
7. $\int 8(2x + 1)^3\,dx = (2x + 1)^4 + C$
8. $\int -6(-2x - 3)^2\,dx = (-2x - 3)^3 + C$
9. $\int -9(-3x - 1)^2\,dx = (-3x - 1)^3 + C$
10. $\int -6(-2x + 1)^2\,dx = (-2x + 1)^3 + C$

Integration by Substitution (continued)

Using derivatives

11. $f(x) = 2x^2 - x - 3$; tangent at $x = -1 \rightarrow y = -5x - 5$
12. $f(x) = -2x^2 + x - 1$; tangent at $x = -1 \rightarrow y = 5x + 1$
13. $f(x) = 3x^2 - 2x + 2$; tangent at $x = 2 \rightarrow y = 10x - 10$
14. $f(x) = x^2 + 2x - 1$; tangent at $x = 1 \rightarrow y = 4x - 2$

Integrals of sin, cos, e^x and 1/x

Integrals

1. $\int (-4\cos x - 1/x)\,dx = -4\sin x - \ln|x| + C$
2. $\int (3\sin x + 2e^x)\,dx = -3\cos x + 2e^x + C$
3. $\int (-\cos x - \sin x - 1/x)\,dx = -\sin x + \cos x - \ln|x| + C$
4. $\int (-3\cos x + e^x)\,dx = -3\sin x + e^x + C$
5. $\int (\cos x - 1/x)\,dx = \sin x - \ln|x| + C$
6. $\int (\sin x - 5e^x)\,dx = -\cos x - 5e^x + C$
7. $\int (-3\sin x + 4e^x)\,dx = 3\cos x + 4e^x + C$
8. $\int (-\cos x - 2\sin x)\,dx = -\sin x + 2\cos x + C$
9. $\int (\cos x + 4e^x)\,dx = \sin x + 4e^x + C$
10. $\int (-3\cos x + \sin x)\,dx = -3\sin x - \cos x + C$
11. $\int (\cos x + 2\sin x + 2e^x)\,dx = \sin x - 2\cos x + 2e^x + C$
12. $\int (-\cos x + 5\sin x)\,dx = -\sin x - 5\cos x + C$

Using derivatives

13. $f(x) = x^2 + 4x + 1$; stationary points → minimum $(-2, -3)$
14. $f(x) = -2x^2 + 1$; stationary points → maximum $(0, 1)$
15. $f(x) = -x^2 + 6x - 6$; stationary points → maximum $(3, 3)$
16. $f(x) = x^2 + 1$; stationary points → minimum $(0, 1)$

Integration by Parts

Integrals

1. $\int -x\cos x\,dx = -x\sin x - \cos x + C$
2. $\int -x\sin x\,dx = x\cos x - \sin x + C$
3. $\int -4x\cos x\,dx = -4x\sin x - 4\cos x + C$
4. $\int 2x\cos x\,dx = 2x\sin x + 2\cos x + C$
5. $\int 2xe^x\,dx = 2xe^x - 2e^x + C$
6. $\int -2xe^x\,dx = -2xe^x + 2e^x + C$
7. $\int -4xe^x\,dx = -4xe^x + 4e^x + C$
8. $\int x\sin x\,dx = -x\cos x + \sin x + C$

Derivatives

9. $f(x) = (-x + 2)(2x + 1) \rightarrow f'(x) = -4x + 3$
10. $f(x) = (-x + 2)(-3x - 1) \rightarrow f'(x) = 6x - 5$
11. $f(x) = (-2x + 3)(x + 3) \rightarrow f'(x) = -4x - 3$
12. $f(x) = (-2x + 1)(-x - 3) \rightarrow f'(x) = 4x + 5$
13. $f(x) = (x + 1)(-2x - 3) \rightarrow f'(x) = -4x - 5$
14. $f(x) = (-x + 1)(-x - 2) \rightarrow f'(x) = 2x + 1$

The nth-Term Test

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 3}$ → the test says nothing

Answer Key (4 of 5)

The nth-Term Test (continued)

Series: converge or diverge? (continued)

2. $\sum_{n=1}^{\infty} \frac{-2n-2}{2n^2+3}$ → the test says nothing
3. $\sum_{n=1}^{\infty} \frac{-n-3}{n^2+1}$ → the test says nothing
4. $\sum_{n=1}^{\infty} \frac{-2n^2+1}{n^2+2}$ → diverges
5. $\sum_{n=1}^{\infty} \frac{-n^2-3}{2n^2+1}$ → diverges
6. $\sum_{n=1}^{\infty} \frac{2n-2}{n^2+1}$ → the test says nothing
7. $\sum_{n=1}^{\infty} \frac{2n-2}{2n+3}$ → diverges
8. $\sum_{n=1}^{\infty} \frac{2}{2n^2+n-1}$ → the test says nothing

Integrals

9. $\int$ from 1 to 2 of $(4x-1)\,dx = 5$
10. $\int$ from 0 to 1 of $(2x-2)\,dx = -1$
11. $\int$ from -1 to 1 of $(2x)\,dx = 0$
12. $\int$ from 0 to 2 of $(-2x+2)\,dx = 0$

Geometric and Telescoping Series

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} -2(-4)^{n-1}$ → diverges
2. $\sum_{n=0}^{\infty} 2(\frac{2}{3})^n$ → converges, S = 6
3. $\sum_{n=1}^{\infty} -2(-\frac{1}{4})^{n-1}$ → converges, S = $\frac{-8}{5}$
4. $\sum_{n=0}^{\infty} (\frac{2}{3})^n$ → converges, S = 3
5. $\sum_{n=0}^{\infty} -2(\frac{2}{3})^n$ → converges, S = −6
6. $\sum_{n=0}^{\infty} 2 \cdot 4^n$ → diverges
7. $\sum_{n=1}^{\infty} 3(-2)^n$ → diverges
8. $\sum_{n=0}^{\infty} 3 \cdot 4^n$ → diverges

Integrals

9. $\int -6(-2x+2)^2\,dx = (-2x+2)^3 + C$
10. $\int -9(-3x+2)^2\,dx = (-3x+2)^3 + C$
11. $\int -8(-2x+1)^3\,dx = (-2x+1)^4 + C$
12. $\int 3(x+3)^2\,dx = (x+3)^3 + C$

p-Series and the Comparison Test

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}$ → converges

p-Series and the Comparison Test (continued)

Series: converge or diverge? (continued)

2. $\sum_{n=1}^{\infty} \frac{2}{n}$ → diverges
3. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ → converges
4. $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}$ → diverges
5. $\sum_{n=1}^{\infty} \frac{1}{n}$ → diverges
6. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$ → converges
7. $\sum_{n=1}^{\infty} \frac{1}{n^2\sqrt{n}}$ → converges
8. $\sum_{n=1}^{\infty} \frac{3}{n}$ → diverges

Integrals

9. $\int (-2\sin x + e^x)\,dx = 2\cos x + e^x + C$
10. $\int (3\cos x - \sin x)\,dx = 3\sin x + \cos x + C$
11. $\int (5\sin x + 2/x)\,dx = -5\cos x + 2\ln|x| + C$
12. $\int (-\cos x + 1/x)\,dx = -\sin x + \ln|x| + C$

The Ratio Test

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ → converges
2. $\sum_{n=1}^{\infty} \frac{n}{3^n}$ → converges
3. $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$ → diverges
4. $\sum_{n=1}^{\infty} \frac{3^n}{n}$ → diverges
5. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ → diverges
6. $\sum_{n=1}^{\infty} \frac{n!}{2^n}$ → diverges
7. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ → converges
8. $\sum_{n=1}^{\infty} \frac{n}{2^n}$ → converges

Derivatives

9. $f(x) = -3\cos x - \ln x \rightarrow f'(x) = 3\sin x - \frac{1}{x}$
10. $f(x) = 2e^x - 3\ln x \rightarrow f'(x) = 2e^x - \frac{3}{x}$
11. $f(x) = 5\sin x - 5e^x \rightarrow f'(x) = 5\cos x - 5e^x$
12. $f(x) = -\cos x - 2e^x \rightarrow f'(x) = \sin x - 2e^x$
13. $f(x) = 4e^x + 2\ln x \rightarrow f'(x) = 4e^x + \frac{2}{x}$
14. $f(x) = 2\cos x - 3\ln x \rightarrow f'(x) = -2\sin x - \frac{3}{x}$

Answer Key (5 of 5)

Alternating Series

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n+1}$ → diverges
2. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$ → diverges
3. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ → converges conditionally
4. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{2n+1}$ → diverges
5. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n\sqrt{n}}$ → converges absolutely
6. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+3}$ → diverges
7. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2\sqrt{n}}$ → converges absolutely
8. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n\sqrt{n}}$ → converges absolutely

Integrals

9. $\int(-2x + 1)\,dx = -x^2 + x + C$
10. $\int(2x - 1)\,dx = x^2 - x + C$
11. $\int(-6x + 1)\,dx = -3x^2 + x + C$
12. $\int(-2x)\,dx = -x^2 + C$
13. $\int(-2x + 2)\,dx = -x^2 + 2x + C$
14. $\int(4x - 1)\,dx = 2x^2 - x + C$

Interval of Convergence

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{(x-1)^n}{3^n}$ → $(-2, 4)$
2. $\sum_{n=1}^{\infty} \frac{(x+3)^n}{3^n}$ → $(-6, 0)$
3. $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n \cdot 2^n}$ → $[-5, -1)$
4. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{2^n}$ → $(0, 4)$

Derivatives

5. $f(x) = (-x - 1)(-x + 2)$ → $f'(x) = 2x - 1$
6. $f(x) = (-2x - 1)(3x + 1)$ → $f'(x) = -12x - 5$
7. $f(x) = (3x + 2)(-3x + 1)$ → $f'(x) = -18x - 3$
8. $f(x) = (x + 3)x$ → $f'(x) = 2x + 3$

Solutions (1 of 3)

The Power Rule

Derivatives

1. **$f(x) = -3x^2 - x$**

$$f'(x) = -2 \cdot 3x - 1$$
$$= -6x - 1$$

A sum of powers of x: take the derivative term by term. For each term, bring the power down in front as a multiplier and take one off the power — the derivative of x^n is nx^{n-1}.

Multiply out each term. That is the derivative.

2. **$f(x) = -3x^2 - 1$**

$$f'(x) = -2 \cdot 3x$$
$$= -6x$$

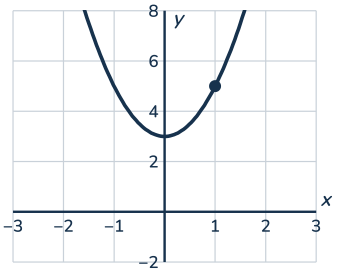
A sum of powers of x: take the derivative term by term. For each term, bring the power down in front as a multiplier and take one off the power — the derivative of x^n is nx^{n-1}. The constant −1 does not change as x moves, so its derivative is 0 and it is gone.

Multiply out each term. That is the derivative.

The Slope of a Curve

Using derivatives

1. **$f(x) = 2x^2 + 3,\ x = 1$**



$$f'(x) = 4x$$
$$f'(1) = 4(1)$$
$$= 4$$

The slope of a curve at a point is the derivative there. Find f'(x) first, term by term.

Put x = 1 into f'(x), not into f(x): f(x) is the height, f'(x) is the slope.

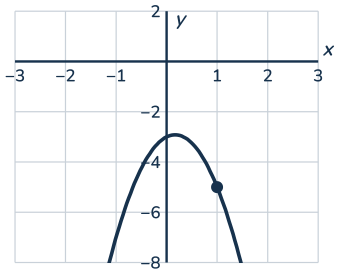
Work it out. That is the slope of the curve at that point.

Solutions (2 of 3)

The Slope of a Curve (continued)

Using derivatives (continued)

2. **$f(x) = -3x^2 + x - 3,\ x = 1$**



$$f'(x) = -6x + 1$$

$$f'(1) = -6(1) + 1$$

$$= -5$$

The slope of a curve at a point is the derivative there. Find f'(x) first, term by term.

Put x = 1 into f'(x), not into f(x): f(x) is the height, f'(x) is the slope.

Work it out. That is the slope of the curve at that point.

Indefinite Integrals

Integrals

1. **$\int (-4x)\, dx$**

$$\int (-4x)\, dx = -\frac{4x^2}{2} + C$$

$$= -2x^2 + C$$

Integrate term by term, running the power rule backwards: add one to each power, then divide by the new power. A number on its own becomes that number times x. Add + C, because any constant would have disappeared when the answer was differentiated.

Divide out each fraction. Check by differentiating: you should get back the question.

2. **$\int (2x - 1)\, dx$**

$$\int (2x - 1)\, dx = \frac{2x^2}{2} - 1x + C$$

$$= x^2 - x + C$$

Integrate term by term, running the power rule backwards: add one to each power, then divide by the new power. A number on its own becomes that number times x. Add + C, because any constant would have disappeared when the answer was differentiated.

Divide out each fraction. Check by differentiating: you should get back the question.

Solutions (3 of 3)

The nth-Term Test

Series: converge or diverge?

1. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 3}$

$$\lim_{n\to\infty} \frac{1}{n^2 + n + 3} = \lim_{n\to\infty} \frac{1/n^2}{1 + 1/n + 3/n^2}$$

$$= \frac{0}{1 + 0 + 0} = 0$$

$\lim a_n = 0$: the test says nothing

The nth-term test asks one thing: do the terms go to 0? Find the limit of a_n as n grows. The highest power on the bottom is n^2, so divide every term, top and bottom, by n^2.

Every term with n left on its bottom shrinks to 0, which leaves 0 on the top: the limit is 0.

The terms do go to 0, and that is NOT enough to say the series converges: the series of 1 over n has terms that go to 0 and still diverges. The test says nothing here, and another test is needed.

2. $\sum_{n=1}^{\infty} \frac{-2n - 2}{2n^2 + 3}$

$$\lim_{n\to\infty} \frac{-2n - 2}{2n^2 + 3} = \lim_{n\to\infty} \frac{-2/n - 2/n^2}{2 + 3/n^2}$$

$$= \frac{0 + 0}{2 + 0} = 0$$

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