Geometry Advanced Workbook

21 lessons · printable · answer key at the back

Name _____ Date _____

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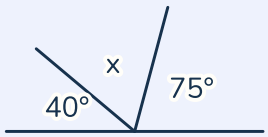
Angle Pairs

WORKED EXAMPLE

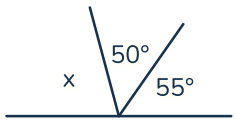


65 degrees

1. Angles on a line and around a point

Find the angle marked x. The angles on each line add to 180°.

1

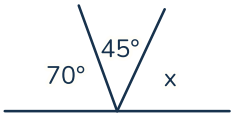


_____ degrees

2

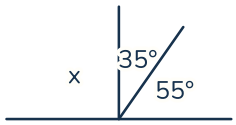
_____ degrees

3



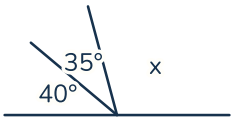
_____ degrees

4



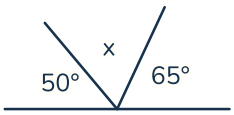
_____ degrees

5



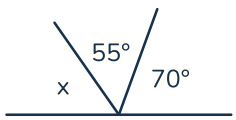
_____ degrees

6



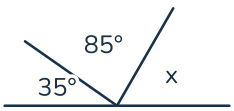
_____ degrees

7



_____ degrees

8

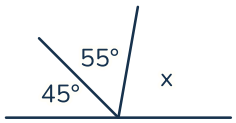


_____ degrees

9

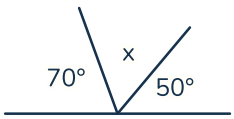
_____ degrees

10



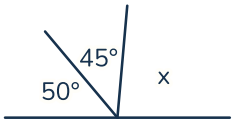
_____ degrees

11



_____ degrees

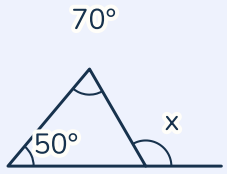
12



_____ degrees

Exterior Angles

WORKED EXAMPLE

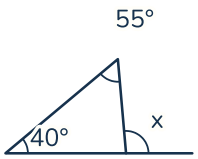


120 degrees

2. The exterior angle of a triangle

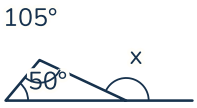
Find the angle marked x. Give each answer in degrees.

1



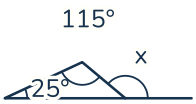
_____ degrees

2



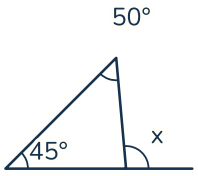
_____ degrees

3



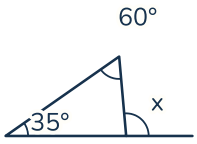
_____ degrees

4



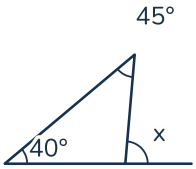
_____ degrees

5



_____ degrees

6

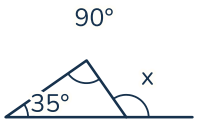


_____ degrees

7

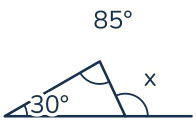
_____ degrees

8



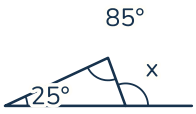
_____ degrees

9



_____ degrees

10

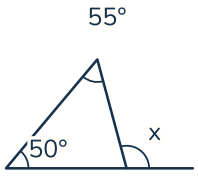


_____ degrees

11

_____ degrees

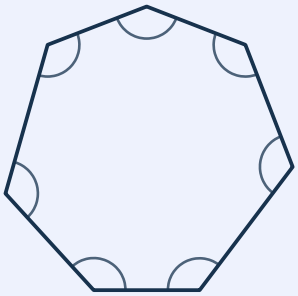
12



_____ degrees

Angles in Polygons

WORKED EXAMPLE



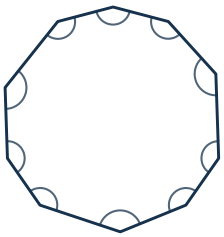
All the angles together

900 degrees

3. Angles inside a polygon

Work out the angle each shape is asking for. Give every answer in degrees.

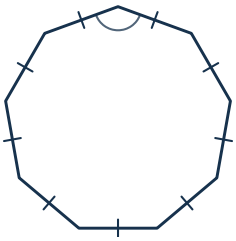
1



All the angles together

_____ degrees

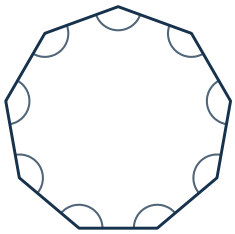
2



Each of its angles

_____ degrees

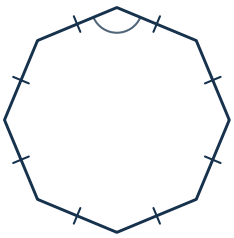
3



All the angles together

_____ degrees

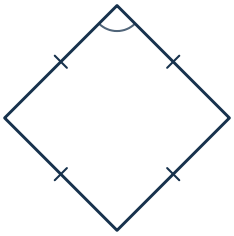
4



Each of its angles

_____ degrees

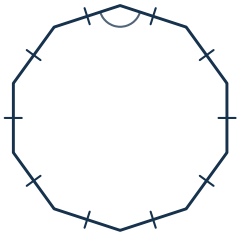
5



Each of its angles

_____ degrees

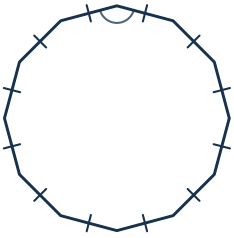
6



Each of its angles

_____ degrees

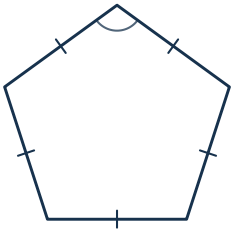
7



Each of its angles

_____ degrees

8



Each of its angles

_____ degrees

The Triangle Inequality

WORKED EXAMPLE

6

12

12

Yes No

4. Can these be the sides of a triangle?

Can these three lengths form a triangle? Answer Yes or No.

1

4

6

12

Yes No

2

3

9

11

Yes No

3

3

11

16

Yes No

4

4

6

10

Yes No

5

3

15

15

Yes No

6

4

6

11

Yes No

7

4

12

15

Yes No

8

5

14

16

Yes No

9

3

7

10

Yes No

10

3

6

8

Yes No

11

6

7

16

Yes No

12

4

10

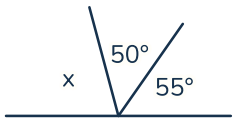
14

Yes No

5. Angles on a line and around a point

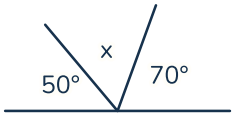
Find the angle marked x. The angles on each line add to 180°.

13



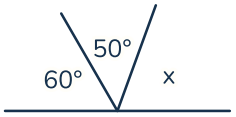
_____ degrees

14



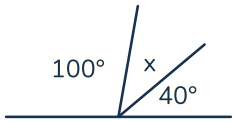
_____ degrees

15



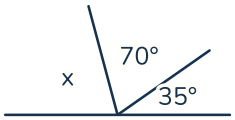
_____ degrees

16



_____ degrees

17



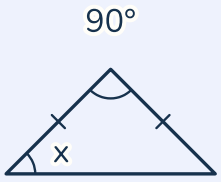
_____ degrees

18

_____ degrees

Isosceles Triangles

WORKED EXAMPLE



45 degrees

6. Angles in an isosceles triangle

Find the angle marked x. The ticks mark the two equal sides.

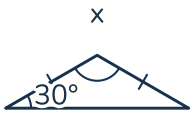
1

_____ degrees

2

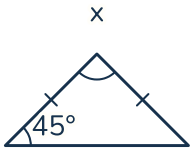
_____ degrees

3



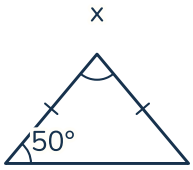
_____ degrees

4



_____ degrees

5

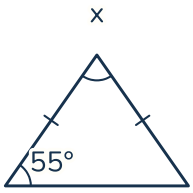


_____ degrees

6

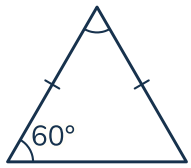
_____ degrees

7



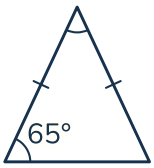
_____ degrees

8



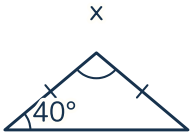
_____ degrees

9



_____ degrees

10

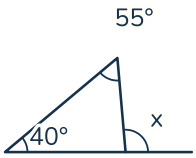


_____ degrees

7. The exterior angle of a triangle

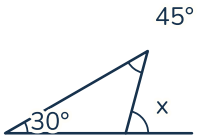
Find the angle marked x. Give each answer in degrees.

11



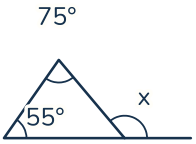
_____ degrees

12



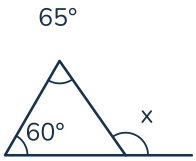
_____ degrees

13



_____ degrees

14

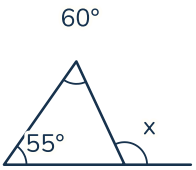


_____ degrees

15

_____ degrees

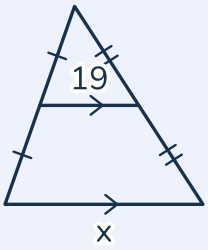
16



_____ degrees

Midsegments

WORKED EXAMPLE

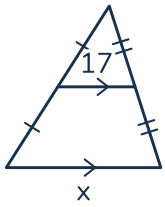


38 cm

8. The midsegment of a triangle

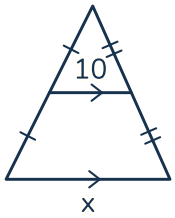
Find the length marked x. The dashes mark equal halves.

1



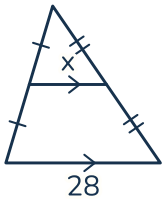
_____ cm

2



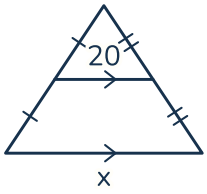
_____ cm

3



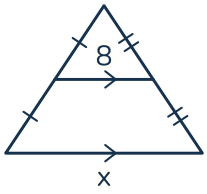
_____ cm

4



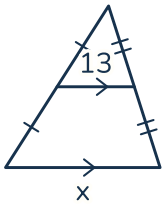
_____ cm

5



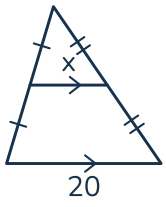
_____ cm

6



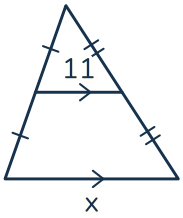
_____ cm

7



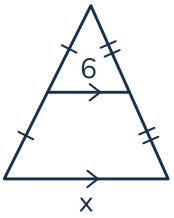
_____ cm

8



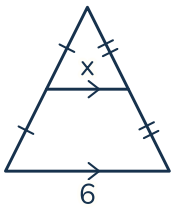
_____ cm

9



_____ cm

10

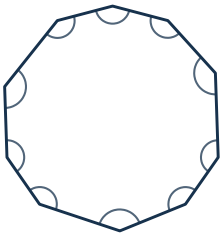


_____ cm

9. Angles inside a polygon

Work out the angle each shape is asking for. Give every answer in degrees.

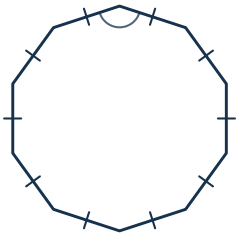
11



All the angles together

_____ degrees

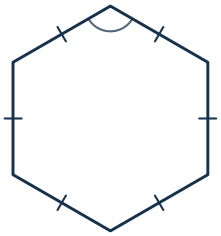
12



Each of its angles

_____ degrees

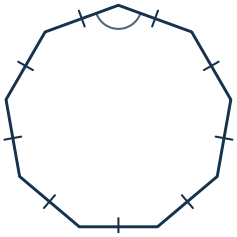
13



Each of its angles

_____ degrees

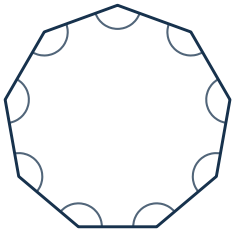
14



Each of its angles

_____ degrees

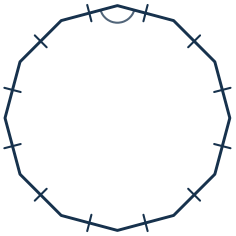
15



All the angles together

_____ degrees

16

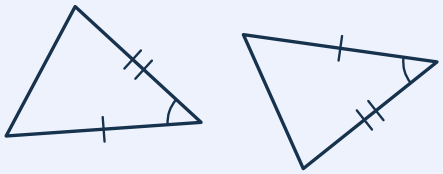


Each of its angles

_____ degrees

Congruence Criteria

WORKED EXAMPLE

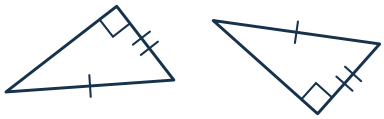


SSS **SAS** ASA AAS HL

10. Which congruence rule (SSS, SAS, ASA, AAS, HL)

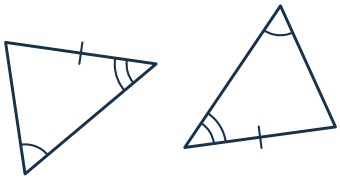
Which rule shows each pair of triangles is congruent?

1



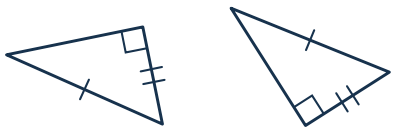
SSS SAS ASA AAS HL

2



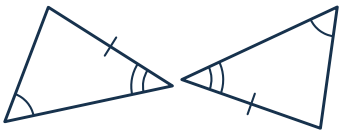
SSS SAS ASA AAS HL

3



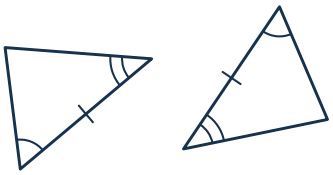
SSS SAS ASA AAS HL

4



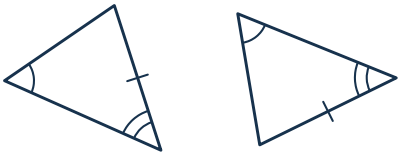
SSS SAS ASA AAS HL

5



SSS SAS ASA AAS HL

6

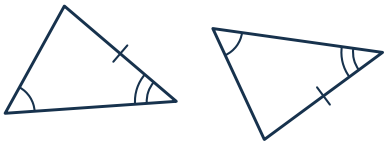


SSS SAS ASA AAS HL

10. Which congruence rule (SSS, SAS, ASA, AAS, HL) (continued)

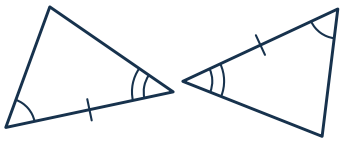
Which rule shows each pair of triangles is congruent?

7



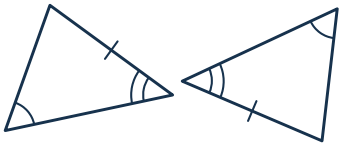
SSS SAS ASA AAS HL

8



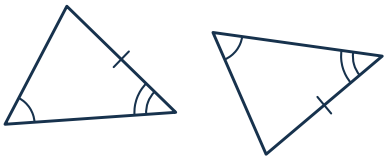
SSS SAS ASA AAS HL

9



SSS SAS ASA AAS HL

10



SSS SAS ASA AAS HL

11. Can these be the sides of a triangle?

Can these three lengths form a triangle? Answer Yes or No.

11

6

8

12

Yes No

12

4

4

8

Yes No

13

6

13

15

Yes No

14

3

6

10

Yes No

15

3

6

12

Yes No

16

6

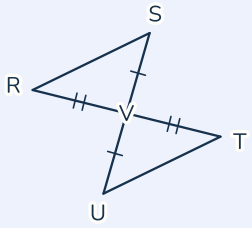
11

16

Yes No

Proofs with Triangles

WORKED EXAMPLE



Given V is the midpoint of SU; RV ≅ TV

Prove ΔRVS ≅ ΔTVU

Statements	Reasons
1. SV ≅ UV	Definition of Midpoint
2. ∠RVS ≅ ∠TVU	?
3. RV ≅ TV	Given
4. ΔRVS ≅ ΔTVU	SAS

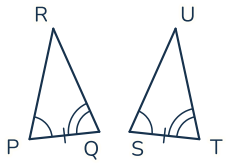
Reason for step 2:

Given | Reflexive Property | **Vertical Angles** | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

12. Complete the triangle congruence proof

Read each proof and choose the missing reason.

1



Given ∠P ≅ ∠S; PQ ≅ ST; ∠Q ≅ ∠T

Prove ΔPQR ≅ ΔSTU

Statements	Reasons
1. ∠P ≅ ∠S	Given
2. PQ ≅ ST	Given
3. ∠Q ≅ ∠T	Given
4. ΔPQR ≅ ΔSTU	?

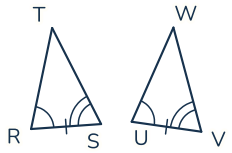
Reason for step 4:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

12. Complete the triangle congruence proof (continued)

Read each proof and choose the missing reason.

2



Given ∠R ≅ ∠U; RS ≅ UV; ∠S ≅ ∠V

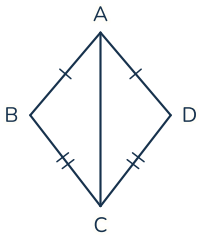
Prove ΔRST ≅ ΔUVW

Statements	Reasons
1. ∠R ≅ ∠U	Given
2. RS ≅ UV	Given
3. ∠S ≅ ∠V	Given
4. ΔRST ≅ ΔUVW	?

Reason for step 4:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

3



Given AB ≅ AD; CB ≅ CD

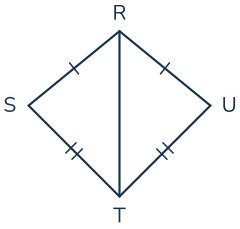
Prove ΔABC ≅ ΔADC

Statements	Reasons
1. AB ≅ AD	Given
2. CB ≅ CD	Given
3. AC ≅ AC	?
4. ΔABC ≅ ΔADC	SSS

Reason for step 3:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

4



Given RS ≅ RU; TS ≅ TU

Prove ΔRST ≅ ΔRUT

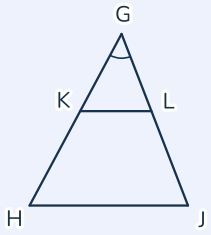
Statements	Reasons
1. RS ≅ RU	Given
2. TS ≅ TU	Given
3. RT ≅ RT	Reflexive Property
4. ΔRST ≅ ΔRUT	?

Reason for step 4:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

Proofs with Similar Triangles

WORKED EXAMPLE



Given GK/GH = GL/GJ

Prove ΔGKL ~ ΔGHJ

Statements	Reasons
1. ∠G ≅ ∠G	?
2. GK/GH = GL/GJ	Given
3. ΔGKL ~ ΔGHJ	SAS ~

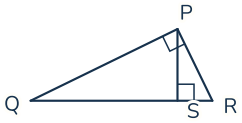
Reason for step 1:

Given | **Reflexive Property** | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

13. Complete the similar-triangle proof

Read each proof and choose the missing reason.

1



Given ∠QPR is a right angle; PS ⊥ QR

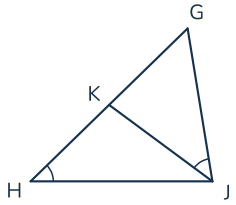
Prove ΔSPR ~ ΔPQR

Statements	Reasons
1. ∠R ≅ ∠R	Reflexive Property
2. ∠PSR ≅ ∠QPR	Right angles are congruent
3. ΔSPR ~ ΔPQR	?

Reason for step 3:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

2



Given ∠GJK ≅ ∠H

Prove ΔGJK ~ ΔGHJ

Statements	Reasons
1. ∠G ≅ ∠G	Reflexive Property
2. ∠GJK ≅ ∠H	Given
3. ΔGJK ~ ΔGHJ	?

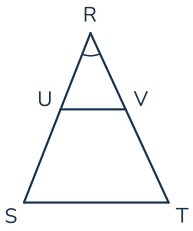
Reason for step 3:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

13. Complete the similar-triangle proof (continued)

Read each proof and choose the missing reason.

3



Given RU/RS = RV/RT

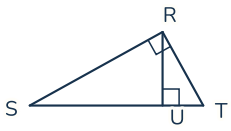
Prove ΔRUV ~ ΔRST

Statements	Reasons
1. ∠R ≅ ∠R	?
2. RU/RS = RV/RT	Given
3. ΔRUV ~ ΔRST	SAS ~

Reason for step 1:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

4



Given ∠SRT is a right angle; RU ⊥ ST

Prove ΔURT ~ ΔRST

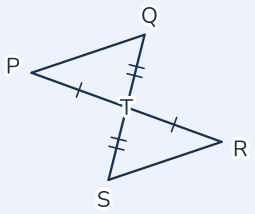
Statements	Reasons
1. ∠T ≅ ∠T	Reflexive Property
2. ∠RUT ≅ ∠SRT	Right angles are congruent
3. ΔURT ~ ΔRST	?

Reason for step 3:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

Writing a Proof

WORKED EXAMPLE



Given PT ≅ RT; QT ≅ ST

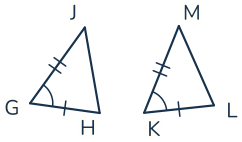
Prove ΔPTQ ≅ ΔRTS

Statements	Reasons
1. PT ≅ RT	Given
2. ∠PTQ ≅ ∠RTS	Vertical Angles
3. QT ≅ ST	Given
4. ΔPTQ ≅ ΔRTS	SAS

14. Write a triangle proof

Write a full two-column proof for each. Reasons may be used more than once.

1

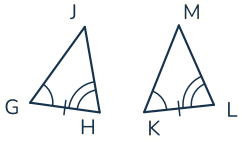


Given GH ≅ KL; ∠G ≅ ∠K; GJ ≅ KM

Prove ΔGHJ ≅ ΔKLM

Statements	Reasons
1.	
2.	
3.	
4.	

2



Given ∠G ≅ ∠K; GH ≅ KL; ∠H ≅ ∠L

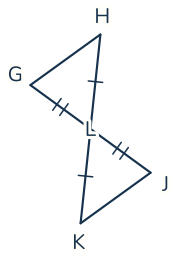
Prove ΔGHJ ≅ ΔKLM

Statements	Reasons
1.	
2.	
3.	
4.	

14. Write a triangle proof (continued)

Write a full two-column proof for each. Reasons may be used more than once.

3

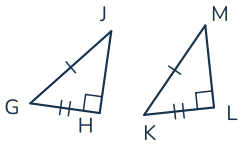


Given L is the midpoint of HK; GL ≅ JL

Prove ΔGLH ≅ ΔJLK

Statements	Reasons
1.	
2.	
3.	
4.	

4



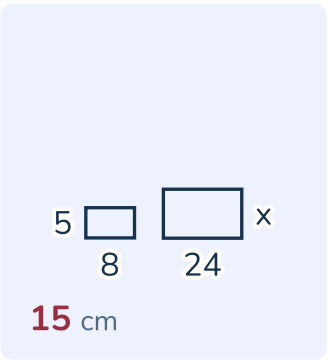
Given ∠H and ∠L are right angles; GJ ≅ KM; GH ≅ KL

Prove ΔGHJ ≅ ΔKLM

Statements	Reasons
1.	
2.	
3.	
4.	

Similar Figures

WORKED EXAMPLE



15 cm

15. Similar figures and scale

Each pair of shapes is the same shape at two sizes. Find the missing length.

1

_____ cm

2

_____ cm

3

_____ cm

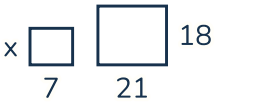
4

_____ cm

5

_____ cm

6

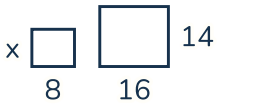


_____ cm

7

_____ cm

8

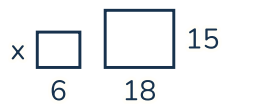


_____ cm

9

_____ cm

10

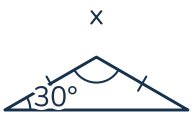


_____ cm

16. Angles in an isosceles triangle

Find the angle marked x. The ticks mark the two equal sides.

11



_____ degrees

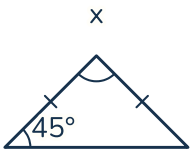
12

_____ degrees

13

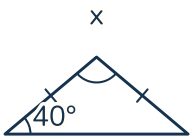
_____ degrees

14



_____ degrees

15



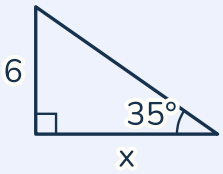
_____ degrees

16

_____ degrees

Right Triangle Trigonometry

WORKED EXAMPLE



8.6 cm

17. Right-triangle trigonometry

Find the side marked x, to one decimal place. Use a calculator.

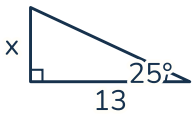
1

_____ cm

2

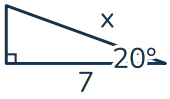
_____ cm

3



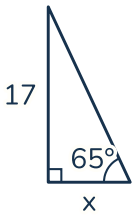
_____ cm

4



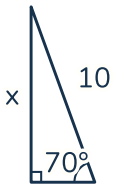
_____ cm

5



_____ cm

6

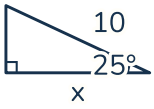


_____ cm

7

_____ cm

8



_____ cm

9

_____ cm

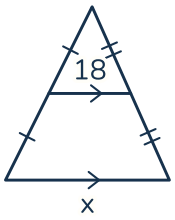
10

_____ cm

18. The midsegment of a triangle

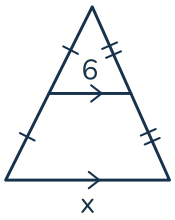
Find the length marked x. The dashes mark equal halves.

11



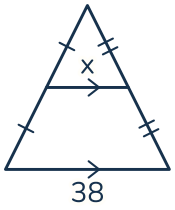
_____ cm

12



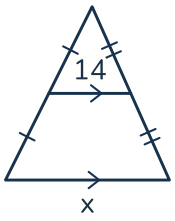
_____ cm

13



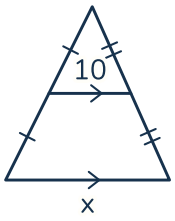
_____ cm

14



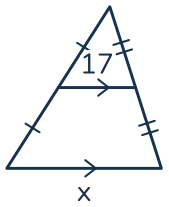
_____ cm

15



_____ cm

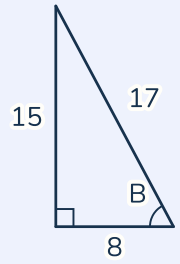
16



_____ cm

Trigonometric Ratios

WORKED EXAMPLE

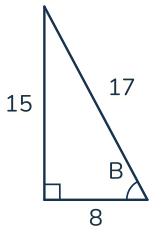


$$\cos B = \frac{8}{17}$$

19. Write a trigonometric ratio

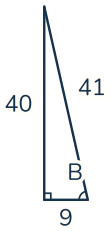
Write each ratio as a fraction. Read the sides straight off the triangle.

1



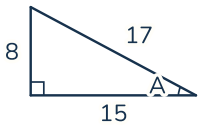
tan B = _____

2



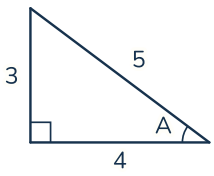
tan B = _____

3



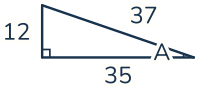
sin A = _____

4



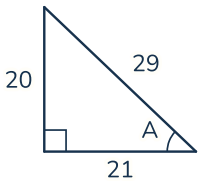
tan A = _____

5



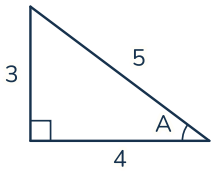
tan A = _____

6



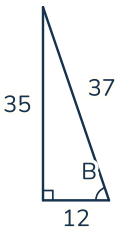
tan A = _____

7



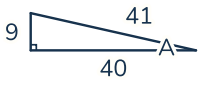
sin A = _____

8



tan B = _____

9

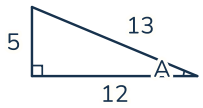


tan A = _____

19. Write a trigonometric ratio (continued)

Write each ratio as a fraction. Read the sides straight off the triangle.

10

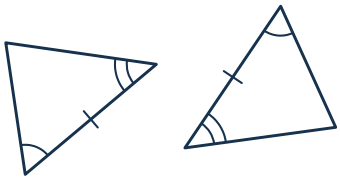


sin A = _____

20. Which congruence rule (SSS, SAS, ASA, AAS, HL)

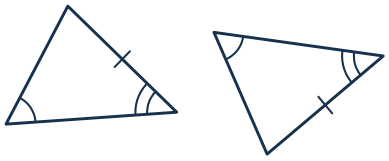
Which rule shows each pair of triangles is congruent?

11



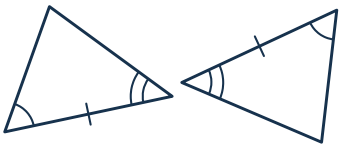
SSS SAS ASA AAS HL

12



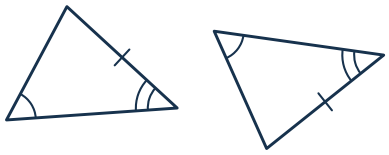
SSS SAS ASA AAS HL

13



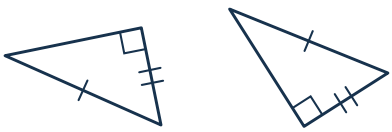
SSS SAS ASA AAS HL

14



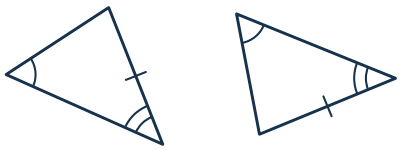
SSS SAS ASA AAS HL

15



SSS SAS ASA AAS HL

16



SSS SAS ASA AAS HL

The Laws of Sines and Cosines

WORKED EXAMPLE

In ΔABC, a = 7, ∠A = 50°, ∠B = 60°. Find b: **7.9**

21. Laws of sines and cosines

Use the law of sines. Round each length to one decimal place.

1 In ΔABC, a = 6, ∠A = 60°, ∠B = 45°. Find b:

_____

2 In ΔABC, a = 9, ∠A = 45°, ∠B = 55°. Find b:

_____

3 In ΔABC, a = 7, ∠A = 40°, ∠B = 60°. Find b:

_____

4 In ΔABC, a = 10, ∠A = 50°, ∠B = 70°. Find b:

_____

5 In ΔABC, a = 12, ∠A = 40°, ∠B = 50°. Find b:

_____

6 In ΔABC, a = 9, ∠A = 65°, ∠B = 40°. Find b:

_____

7 In ΔABC, a = 5, ∠A = 55°, ∠B = 65°. Find b:

_____

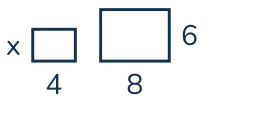
8 In ΔABC, a = 11, ∠A = 30°, ∠B = 75°. Find b:

_____

22. Similar figures and scale

Each pair of shapes is the same shape at two sizes. Find the missing length.

9

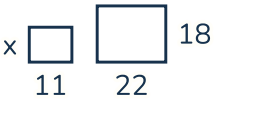


_____ cm

10

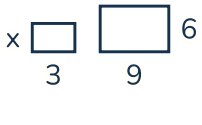
_____ cm

11



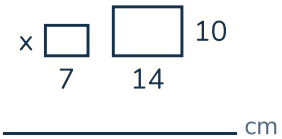
_____ cm

12



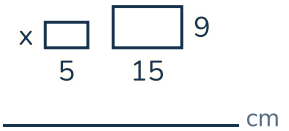
_____ cm

13



_____ cm

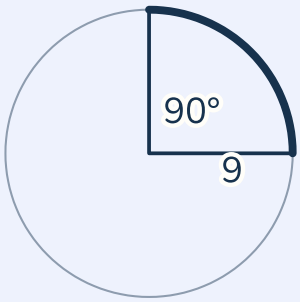
14



_____ cm

Parts of a Circle

WORKED EXAMPLE



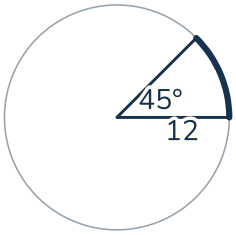
Length of the curved edge

14.13 cm

23. Arc length and sector area

Use π = 3.14. Each shape is part of a circle — find what the label asks for.

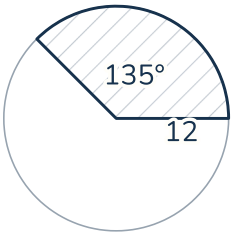
1



Length of the curved edge

_____ cm

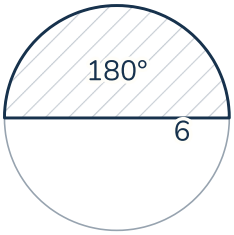
2



Area of the slice

_____ cm²

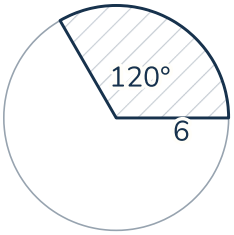
3



Area of the slice

_____ cm²

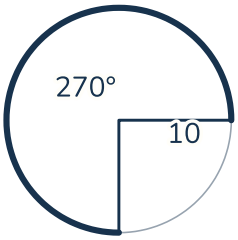
4



Area of the slice

_____ cm²

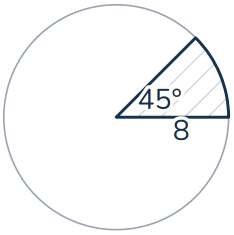
5



Length of the curved edge

_____ cm

6



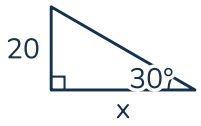
Area of the slice

_____ cm²

24. Right-triangle trigonometry

Find the side marked x, to one decimal place. Use a calculator.

7

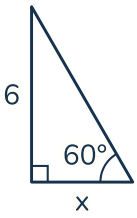


_____ cm

8

_____ cm

9

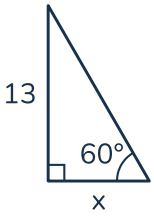


_____ cm

10

_____ cm

11



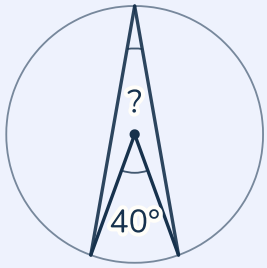
_____ cm

12

_____ cm

Inscribed Angles

WORKED EXAMPLE

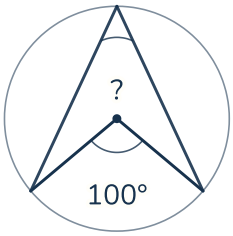


20 degrees

25. Inscribed angle in a circle

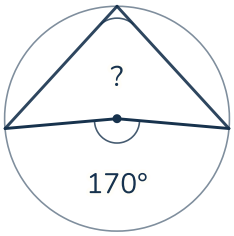
The angle at the center is given. Find the angle on the circle.

1



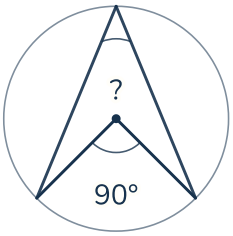
_____ degrees

2



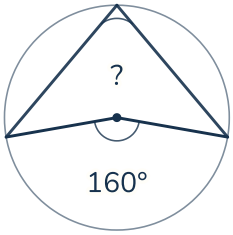
_____ degrees

3



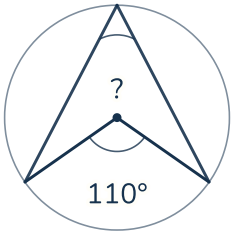
_____ degrees

4



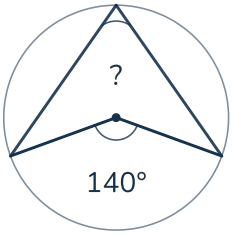
_____ degrees

5



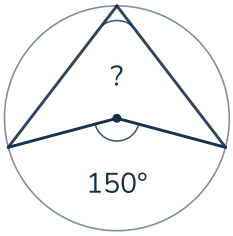
_____ degrees

6



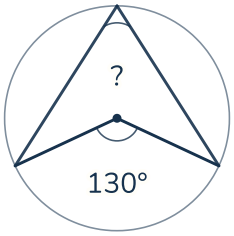
_____ degrees

7



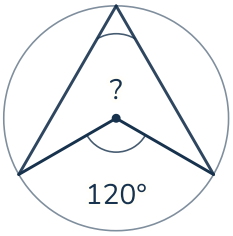
_____ degrees

8



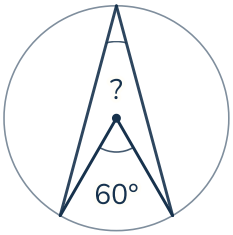
_____ degrees

9



_____ degrees

10

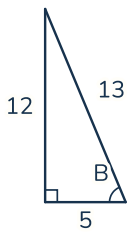


_____ degrees

26. Write a trigonometric ratio

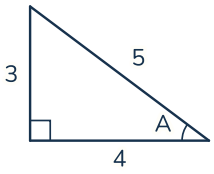
Write each ratio as a fraction. Read the sides straight off the triangle.

11



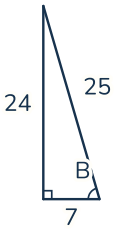
tan B = _____

12



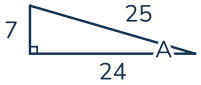
tan A = _____

13



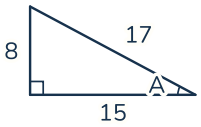
sin B = _____

14



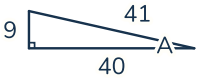
tan A = _____

15



tan A = _____

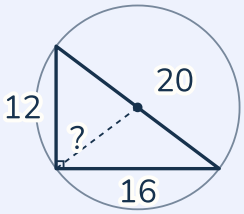
16



tan A = _____

Triangles and Circles

WORKED EXAMPLE

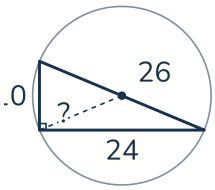


10 cm

27. Circles of a right triangle

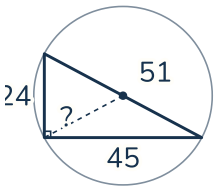
Each circle passes through all three corners. Find its radius.

1



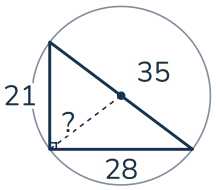
_____ cm

2



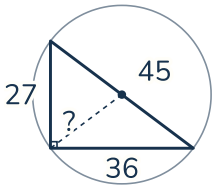
_____ cm

3



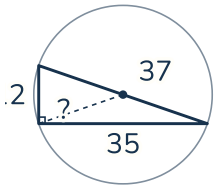
_____ cm

4



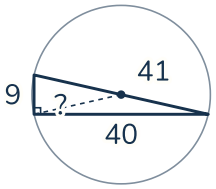
_____ cm

5



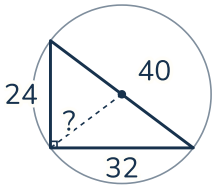
_____ cm

6



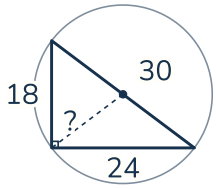
_____ cm

7



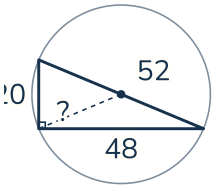
_____ cm

8



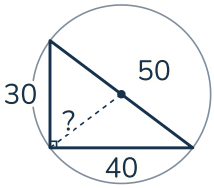
_____ cm

9



_____ cm

10



_____ cm

28. Laws of sines and cosines

Use the law of sines. Round each length to one decimal place.

11 In ΔABC, $a = 6$, $\angle A = 60°$, $\angle B = 45°$. Find b:

_____

12 In ΔABC, $a = 9$, $\angle A = 45°$, $\angle B = 55°$. Find b:

_____

13 In ΔABC, $a = 7$, $\angle A = 40°$, $\angle B = 60°$. Find b:

_____

14 In ΔABC, $a = 10$, $\angle A = 50°$, $\angle B = 70°$. Find b:

_____

15 In ΔABC, $a = 12$, $\angle A = 40°$, $\angle B = 50°$. Find b:

_____

16 In ΔABC, $a = 9$, $\angle A = 65°$, $\angle B = 40°$. Find b:

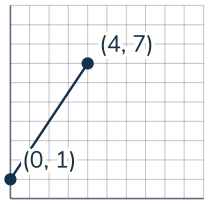
_____

Midpoints

29. Midpoint of a segment

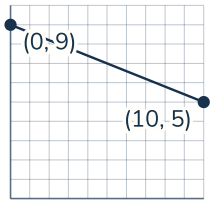
Find the midpoint of each segment.

1



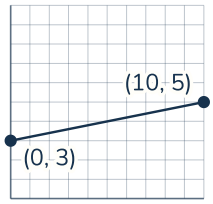
(____ , ____)

2



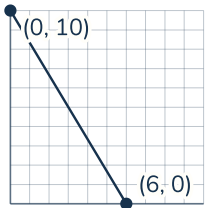
(____ , ____)

3



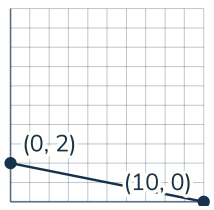
(____ , ____)

4



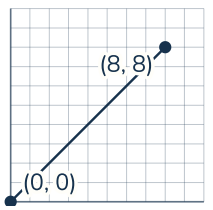
(____ , ____)

5



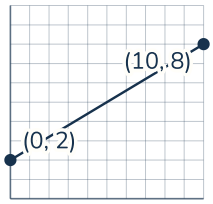
(____ , ____)

6



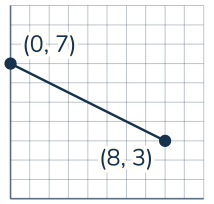
(____ , ____)

7



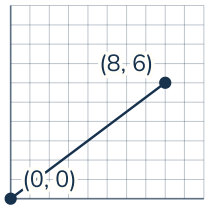
(____ , ____)

8



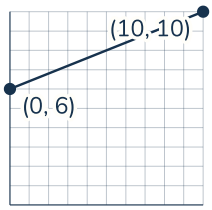
(____ , ____)

9



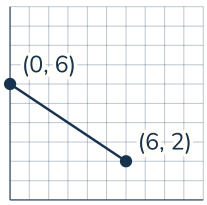
(____ , ____)

10



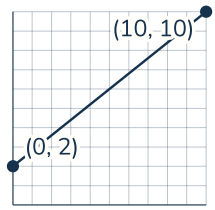
(____ , ____)

11



(____ , ____)

12

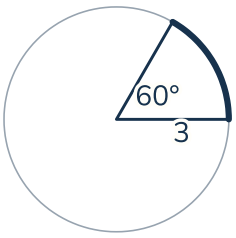


(____ , ____)

30. Arc length and sector area

Use π = 3.14. Each shape is part of a circle — find what the label asks for.

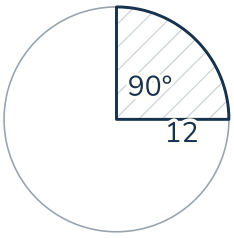
13



Length of the curved edge

_____ cm

14



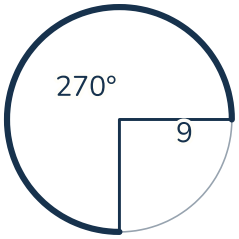
Area of the slice

_____ cm²

30. Arc length and sector area (continued)

Use π = 3.14. Each shape is part of a circle — find what the label asks for.

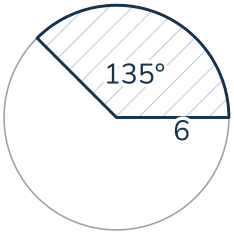
15



Length of the curved edge

_____ cm

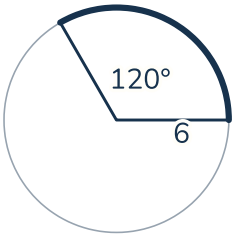
16



Area of the slice

_____ cm²

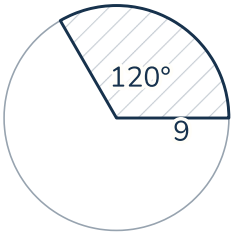
17



Length of the curved edge

_____ cm

18

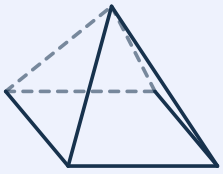


Area of the slice

_____ cm²

Parts of a Solid

WORKED EXAMPLE



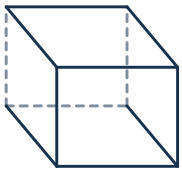
How many edges?

8

31. Faces, edges and vertices

Hidden edges are dashed. Answer the question under each solid.

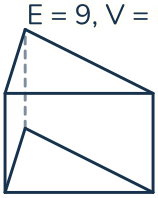
1



How many edges?

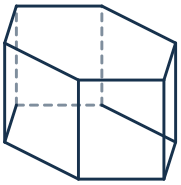
2

E = 9, V = 6



How many faces?

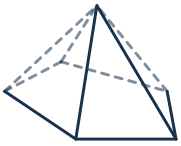
3



How many vertices?

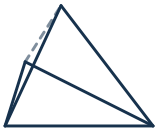
4

E = 10, V = 6



How many faces?

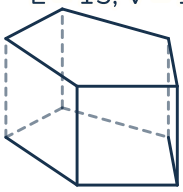
5



How many edges?

6

E = 15, V = 10

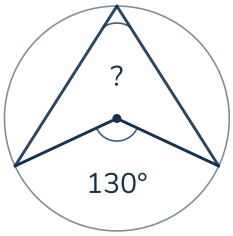


How many faces?

32. Inscribed angle in a circle

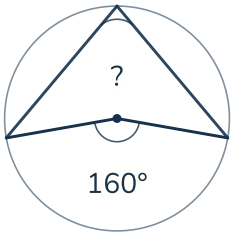
The angle at the center is given. Find the angle on the circle.

7



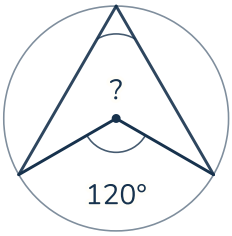
_____ degrees

8



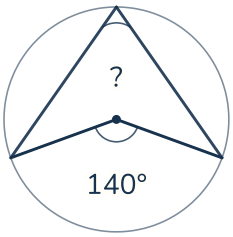
_____ degrees

9



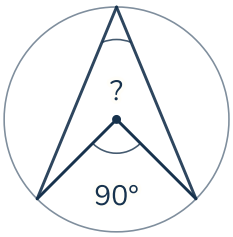
_____ degrees

10



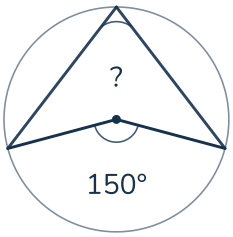
_____ degrees

11



_____ degrees

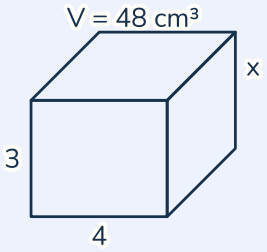
12



_____ degrees

Missing Edges

WORKED EXAMPLE

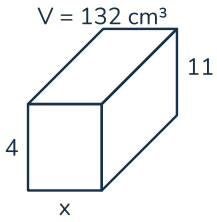


4 cm

33. Missing edge from the volume

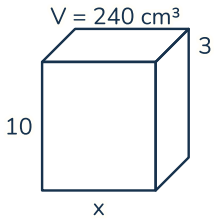
The volume of each box is shown. Find the missing edge, in centimeters.

1



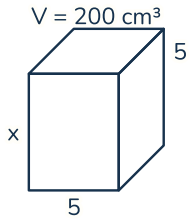
_____ cm

2



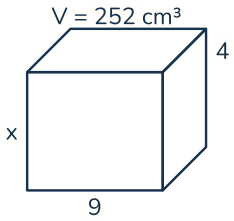
_____ cm

3



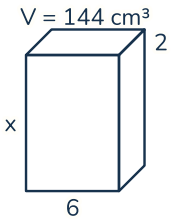
_____ cm

4



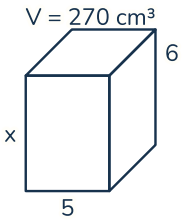
_____ cm

5



_____ cm

6

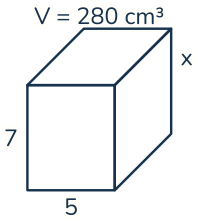


_____ cm

33. Missing edge from the volume (continued)

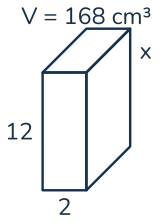
The volume of each box is shown. Find the missing edge, in centimeters.

7



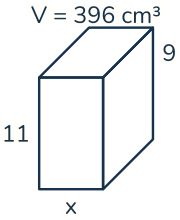
_____ cm

8



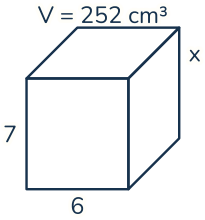
_____ cm

9



_____ cm

10

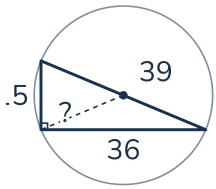


_____ cm

34. Circles of a right triangle

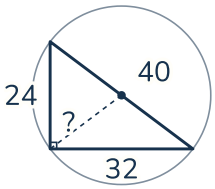
Each circle passes through all three corners. Find its radius.

11



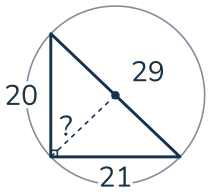
_____ cm

12



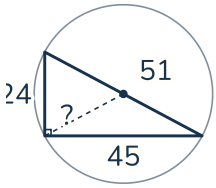
_____ cm

13



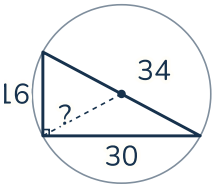
_____ cm

14



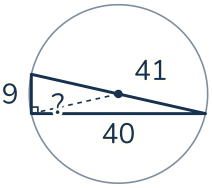
_____ cm

15



_____ cm

16



_____ cm

Surface Area of a Sphere

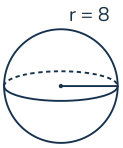
WORKED EXAMPLE

314 cm²

35. Surface area of a sphere

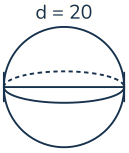
Find the surface area of each sphere. Use π = 3.14.

1



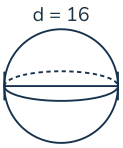
_____ cm²

2



_____ cm²

3

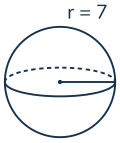


_____ cm²

4

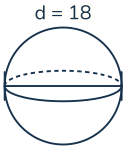
_____ cm²

5



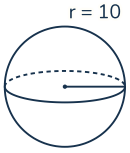
_____ cm²

6



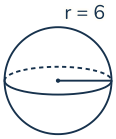
_____ cm²

7



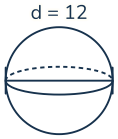
_____ cm²

8



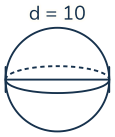
_____ cm²

9



_____ cm²

10

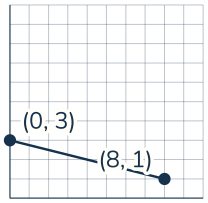


_____ cm²

36. Midpoint of a segment

Find the midpoint of each segment.

11



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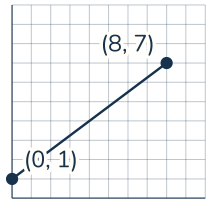
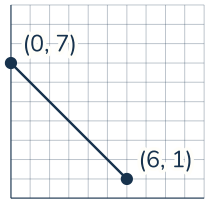
12



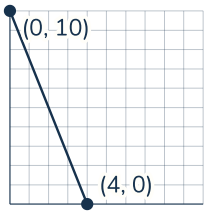
(_____ , _____)

13



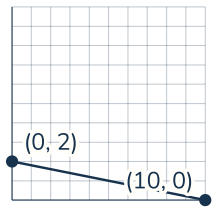
(_____ , _____)

14



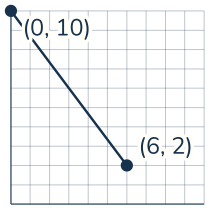
(_____ , _____)

15



(_____ , _____)

16



(_____ , _____)

Answer Key (1 of 5)

Angle Pairs

Angles on a line and around a point

1. 180° (55°, 50°, x): 75°
2. 180° (45°, x, 95°): 40°
3. 180° (x, 45°, 70°): 65°
4. 180° (55°, 35°, x): 90°
5. 180° (x, 35°, 40°): 105°
6. 180° (65°, x, 50°): 65°
7. 180° (70°, 55°, x): 55°
8. 180° (x, 85°, 35°): 60°
9. 180° (x, 75°, 45°): 60°
10. 180° (x, 55°, 45°): 80°
11. 180° (50°, x, 70°): 60°
12. 180° (x, 45°, 50°): 85°

Exterior Angles

The exterior angle of a triangle

1. 40° + 55°: 95°
2. 50° + 105°: 155°
3. 25° + 115°: 140°
4. 45° + 50°: 95°
5. 35° + 60°: 95°
6. 40° + 45°: 85°
7. 55° + 85°: 140°
8. 35° + 90°: 125°
9. 30° + 85°: 115°
10. 25° + 85°: 110°
11. 25° + 90°: 115°
12. 50° + 55°: 105°

Angles in Polygons

Angles inside a polygon

1. 10-sided, all the angles: 1440°
2. regular 9-sided, each angle: 140°
3. 9-sided, all the angles: 1260°
4. regular 8-sided, each angle: 135°
5. regular 4-sided, each angle: 90°
6. regular 10-sided, each angle: 144°
7. regular 12-sided, each angle: 150°
8. regular 5-sided, each angle: 108°

The Triangle Inequality

Can these be the sides of a triangle?

1. 4, 6, 12: No
2. 3, 9, 11: Yes
3. 3, 11, 16: No
4. 4, 6, 10: No

The Triangle Inequality (continued)

Can these be the sides of a triangle? (continued)

5. 3, 15, 15: Yes
6. 4, 6, 11: No
7. 4, 12, 15: Yes
8. 5, 14, 16: Yes
9. 3, 7, 10: No
10. 3, 6, 8: Yes
11. 6, 7, 16: No
12. 4, 10, 14: No

Angles on a line and around a point

13. 180° (55°, 50°, x): 75°
14. 180° (70°, x, 50°): 60°
15. 180° (x, 50°, 60°): 70°
16. 180° (40°, x, 100°): 40°
17. 180° (35°, 70°, x): 75°
18. 180° (35°, 55°, x): 90°

Isosceles Triangles

Angles in an isosceles triangle

1. base 70°: 40°
2. base 75°: 30°
3. base 30°: 120°
4. base 45°: 90°
5. base 50°: 80°
6. base 35°: 110°
7. base 55°: 70°
8. base 60°: 60°
9. base 65°: 50°
10. base 40°: 100°

The exterior angle of a triangle

11. 40° + 55°: 95°
12. 30° + 45°: 75°
13. 55° + 75°: 130°
14. 60° + 65°: 125°
15. 45° + 100°: 145°
16. 55° + 60°: 115°

Midsegments

The midsegment of a triangle

1. midsegment 17: 34 cm
2. midsegment 10: 20 cm
3. base 28: 14 cm
4. midsegment 20: 40 cm
5. midsegment 8: 16 cm
6. midsegment 13: 26 cm
7. base 20: 10 cm

Answer Key (2 of 5)

Midsegments (continued)

The midsegment of a triangle (continued)

8. midsegment 11: 22 cm
9. midsegment 6: 12 cm
10. base 6: 3 cm

Angles inside a polygon

11. 10-sided, all the angles: 1440°
12. regular 10-sided, each angle: 144°
13. regular 6-sided, each angle: 120°
14. regular 9-sided, each angle: 140°
15. 9-sided, all the angles: 1260°
16. regular 12-sided, each angle: 150°

Congruence Criteria

Which congruence rule (SSS, SAS, ASA, AAS, HL)

1. marked a right angle, 2 sides: HL
2. marked 1 side, 2 angles: AAS
3. marked a right angle, 2 sides: HL
4. marked 1 side, 2 angles: AAS
5. marked 1 side, 2 angles: ASA
6. marked 1 side, 2 angles: AAS
7. marked 1 side, 2 angles: AAS
8. marked 1 side, 2 angles: ASA
9. marked 1 side, 2 angles: AAS
10. marked 1 side, 2 angles: AAS

Can these be the sides of a triangle?

11. 6, 8, 12: Yes
12. 4, 4, 8: No
13. 6, 13, 15: Yes
14. 3, 6, 10: No
15. 3, 6, 12: No
16. 6, 11, 16: Yes

Proofs with Triangles

Complete the triangle congruence proof

1. $\triangle PQR \cong \triangle STU$ ($\angle P \cong \angle S$, $PQ \cong ST$, $\angle Q \cong \angle T$), step 4: ASA
2. $\triangle RST \cong \triangle UVW$ ($\angle R \cong \angle U$, $RS \cong UV$, $\angle S \cong \angle V$), step 4: ASA
3. $\triangle ABC \cong \triangle ADC$ ($AB \cong AD$, $CB \cong CD$), step 3: Reflexive Property
4. $\triangle RST \cong \triangle RUT$ ($RS \cong RU$, $TS \cong TU$), step 4: SSS

Proofs with Similar Triangles

Complete the similar-triangle proof

1. $\triangle SPR \sim \triangle PQR$ ($\angle QPR$ is a right angle, $PS \perp QR$), step 3: AA ~
2. $\triangle GJK \sim \triangle GHJ$ ($\angle GJK \cong \angle H$), step 3: AA ~

Proofs with Similar Triangles (continued)

Complete the similar-triangle proof (continued)

3. $\triangle RUV \sim \triangle RST$ ($RU/RS = RV/RT$), step 1: Reflexive Property
4. $\triangle URT \sim \triangle RST$ ($\angle SRT$ is a right angle, $RU \perp ST$), step 3: AA ~

Writing a Proof

Write a triangle proof

1. $\triangle GHJ \cong \triangle KLM$ — 1. $GH \cong KL$ — Given; 2. $\angle G \cong \angle K$ — Given; 3. $GJ \cong KM$ — Given; 4. $\triangle GHJ \cong \triangle KLM$ — SAS
2. $\triangle GHJ \cong \triangle KLM$ — 1. $\angle G \cong \angle K$ — Given; 2. $GH \cong KL$ — Given; 3. $\angle H \cong \angle L$ — Given; 4. $\triangle GHJ \cong \triangle KLM$ — ASA
3. $\triangle GLH \cong \triangle JLK$ — 1. $HL \cong KL$ — Definition of Midpoint; 2. $\angle GLH \cong \angle JLK$ — Vertical Angles; 3. $GL \cong JL$ — Given; 4. $\triangle GLH \cong \triangle JLK$ — SAS
4. $\triangle GHJ \cong \triangle KLM$ — 1. $\angle H \cong \angle L$ — Given; 2. $GJ \cong KM$ — Given; 3. $GH \cong KL$ — Given; 4. $\triangle GHJ \cong \triangle KLM$ — HL

Similar Figures

Similar figures and scale

1. 9 by x and 18 by 8: 4
2. 5 by x and 20 by 12: 3
3. 10 by x and 20 by 6: 3
4. 8 by x and 24 by 12: 4
5. 12 by x and 24 by 10: 5
6. 7 by x and 21 by 18: 6
7. 6 by x and 18 by 12: 4
8. 8 by x and 16 by 14: 7
9. 5 by x and 15 by 6: 2
10. 6 by x and 18 by 15: 5

Angles in an isosceles triangle

11. base 30°: 120°
12. base 75°: 30°
13. base 70°: 40°
14. base 45°: 90°
15. base 40°: 100°
16. base 65°: 50°

Right Triangle Trigonometry

Right-triangle trigonometry

1. 65°, opp 9 → hyp x = 9.9
2. 70°, adj 5 → hyp x = 14.6
3. 25°, adj 13 → opp x = 6.1
4. 20°, adj 7 → hyp x = 7.4
5. 65°, opp 17 → adj x = 7.9
6. 70°, hyp 10 → opp x = 9.4
7. 20°, hyp 10 → opp x = 3.4

Answer Key (3 of 5)

Right Triangle Trigonometry (continued)

Right-triangle trigonometry (continued)

8. 25°, hyp 10 → adj x = 9.1
9. 70°, opp 14 → hyp x = 14.9
10. 70°, hyp 18 → opp x = 16.9

The midsegment of a triangle

11. midsegment 18: 36 cm
12. midsegment 6: 12 cm
13. base 38: 19 cm
14. midsegment 14: 28 cm
15. midsegment 10: 20 cm
16. midsegment 17: 34 cm

Trigonometric Ratios

Write a trigonometric ratio

1. tan B (legs 15, 8, hyp 17) = 15/8
2. tan B (legs 40, 9, hyp 41) = 40/9
3. sin A (legs 8, 15, hyp 17) = 8/17
4. tan A (legs 3, 4, hyp 5) = 3/4
5. tan A (legs 12, 35, hyp 37) = 12/35
6. tan A (legs 20, 21, hyp 29) = 20/21
7. sin A (legs 3, 4, hyp 5) = 3/5
8. tan B (legs 35, 12, hyp 37) = 35/12
9. tan A (legs 9, 40, hyp 41) = 9/40
10. sin A (legs 5, 12, hyp 13) = 5/13

Which congruence rule (SSS, SAS, ASA, AAS, HL)

11. marked 1 side, 2 angles: ASA
12. marked 1 side, 2 angles: AAS
13. marked 1 side, 2 angles: ASA
14. marked 1 side, 2 angles: AAS
15. marked a right angle, 2 sides: HL
16. marked 1 side, 2 angles: AAS

The Laws of Sines and Cosines

Laws of sines and cosines

1. In ΔABC, a = 6, ∠A = 60°, ∠B = 45°. Find b: 4.9
2. In ΔABC, a = 9, ∠A = 45°, ∠B = 55°. Find b: 10.4
3. In ΔABC, a = 7, ∠A = 40°, ∠B = 60°. Find b: 9.4
4. In ΔABC, a = 10, ∠A = 50°, ∠B = 70°. Find b: 12.3
5. In ΔABC, a = 12, ∠A = 40°, ∠B = 50°. Find b: 14.3
6. In ΔABC, a = 9, ∠A = 65°, ∠B = 40°. Find b: 6.4
7. In ΔABC, a = 5, ∠A = 55°, ∠B = 65°. Find b: 5.5
8. In ΔABC, a = 11, ∠A = 30°, ∠B = 75°. Find b: 21.3

Similar figures and scale

9. 4 by x and 8 by 6: 3
10. 5 by x and 20 by 12: 3
11. 11 by x and 22 by 18: 9

The Laws of Sines and Cosines (continued)

Similar figures and scale (continued)

12. 3 by x and 9 by 6: 2
13. 7 by x and 14 by 10: 5
14. 5 by x and 15 by 9: 3

Parts of a Circle

Arc length and sector area

1. r = 12, 45°, arc: 9.42 cm
2. r = 12, 135°, area: 169.56 cm²
3. r = 6, 180°, area: 56.52 cm²
4. r = 6, 120°, area: 37.68 cm²
5. r = 10, 270°, arc: 47.1 cm
6. r = 8, 45°, area: 25.12 cm²

Right-triangle trigonometry

7. 30°, opp 20 → adj x = 34.6
8. 70°, opp 7 → adj x = 2.5
9. 60°, opp 6 → adj x = 3.5
10. 20°, opp 14 → hyp x = 40.9
11. 60°, opp 13 → adj x = 7.5
12. 20°, hyp 10 → opp x = 3.4

Inscribed Angles

Inscribed angle in a circle

1. center 100° → circle 50°
2. center 170° → circle 85°
3. center 90° → circle 45°
4. center 160° → circle 80°
5. center 110° → circle 55°
6. center 140° → circle 70°
7. center 150° → circle 75°
8. center 130° → circle 65°
9. center 120° → circle 60°
10. center 60° → circle 30°

Write a trigonometric ratio

11. tan B (legs 12, 5, hyp 13) = 12/5
12. tan A (legs 3, 4, hyp 5) = 3/4
13. sin B (legs 24, 7, hyp 25) = 24/25
14. tan A (legs 7, 24, hyp 25) = 7/24
15. tan A (legs 8, 15, hyp 17) = 8/15
16. tan A (legs 9, 40, hyp 41) = 9/40

Triangles and Circles

Circles of a right triangle

1. 10, 24, 26 — through the corners: 13 cm
2. 24, 45, 51 — through the corners: 25.5 cm
3. 21, 28, 35 — through the corners: 17.5 cm

Answer Key (4 of 5)

Triangles and Circles (continued)

Circles of a right triangle (continued)

4. 27, 36, 45 — through the corners: 22.5 cm
5. 12, 35, 37 — through the corners: 18.5 cm
6. 9, 40, 41 — through the corners: 20.5 cm
7. 24, 32, 40 — through the corners: 20 cm
8. 18, 24, 30 — through the corners: 15 cm
9. 20, 48, 52 — through the corners: 26 cm
10. 30, 40, 50 — through the corners: 25 cm

Laws of sines and cosines

11. In ΔABC, a = 6, ∠A = 60°, ∠B = 45°. Find b: 4.9
12. In ΔABC, a = 9, ∠A = 45°, ∠B = 55°. Find b: 10.4
13. In ΔABC, a = 7, ∠A = 40°, ∠B = 60°. Find b: 9.4
14. In ΔABC, a = 10, ∠A = 50°, ∠B = 70°. Find b: 12.3
15. In ΔABC, a = 12, ∠A = 40°, ∠B = 50°. Find b: 14.3
16. In ΔABC, a = 9, ∠A = 65°, ∠B = 40°. Find b: 6.4

Midpoints

Midpoint of a segment

1. (0, 1) and (4, 7): (2, 4)
2. (0, 9) and (10, 5): (5, 7)
3. (0, 3) and (10, 5): (5, 4)
4. (0, 10) and (6, 0): (3, 5)
5. (0, 2) and (10, 0): (5, 1)
6. (0, 0) and (8, 8): (4, 4)
7. (0, 2) and (10, 8): (5, 5)
8. (0, 7) and (8, 3): (4, 5)
9. (0, 0) and (8, 6): (4, 3)
10. (0, 6) and (10, 10): (5, 8)
11. (0, 6) and (6, 2): (3, 4)
12. (0, 2) and (10, 10): (5, 6)

Arc length and sector area

13. r = 3, 60°, arc: 3.14 cm
14. r = 12, 90°, area: 113.04 cm²
15. r = 9, 270°, arc: 42.39 cm
16. r = 6, 135°, area: 42.39 cm²
17. r = 6, 120°, arc: 12.56 cm
18. r = 9, 120°, area: 84.78 cm²

Parts of a Solid

Faces, edges and vertices

1. cuboid, edges: 12
2. triangular prism, faces (from the other two): 5
3. hexagonal prism, vertices: 12
4. pentagonal pyramid, faces (from the other two): 6
5. triangular pyramid, edges: 6
6. pentagonal prism, faces (from the other two): 7

Parts of a Solid (continued)

Inscribed angle in a circle

7. center 130° → circle 65°
8. center 160° → circle 80°
9. center 120° → circle 60°
10. center 140° → circle 70°
11. center 90° → circle 45°
12. center 150° → circle 75°

Missing Edges

Missing edge from the volume

1. V 132: x × 4 × 11 → x = 3
2. V 240: x × 10 × 3 → x = 8
3. V 200: 5 × x × 5 → x = 8
4. V 252: 9 × x × 4 → x = 7
5. V 144: 6 × x × 2 → x = 12
6. V 270: 5 × x × 6 → x = 9
7. V 280: 5 × 7 × x → x = 8
8. V 168: 2 × 12 × x → x = 7
9. V 396: x × 11 × 9 → x = 4
10. V 252: 6 × 7 × x → x = 6

Circles of a right triangle

11. 15, 36, 39 — through the corners: 19.5 cm
12. 24, 32, 40 — through the corners: 20 cm
13. 20, 21, 29 — through the corners: 14.5 cm
14. 24, 45, 51 — through the corners: 25.5 cm
15. 16, 30, 34 — through the corners: 17 cm
16. 9, 40, 41 — through the corners: 20.5 cm

Surface Area of a Sphere

Surface area of a sphere

1. r = 8: 803.84 cm²
2. d = 20: 1256 cm²
3. d = 16: 803.84 cm²
4. r = 9: 1017.36 cm²
5. r = 7: 615.44 cm²
6. d = 18: 1017.36 cm²
7. r = 10: 1256 cm²
8. r = 6: 452.16 cm²
9. d = 12: 452.16 cm²
10. d = 10: 314 cm²

Midpoint of a segment

11. (0, 3) and (8, 1): (4, 2)
12. (0, 1) and (8, 7): (4, 4)
13. (0, 7) and (6, 1): (3, 4)
14. (0, 10) and (4, 0): (2, 5)
15. (0, 2) and (10, 0): (5, 1)

Answer Key (5 of 5)

Surface Area of a Sphere (continued)

Midpoint of a segment (continued)

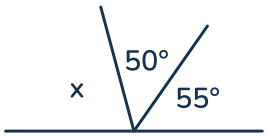
16. (0, 10) and (6, 2): (3, 6)

Solutions (1 of 24)

Angle Pairs

Angles on a line and around a point

1.



____ degrees

$$55° + 50° + x = \mathbf{180°}$$

$$55° + 50° = \mathbf{105°}$$

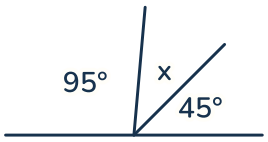
$$180° - 105° = \mathbf{75°}$$

The angles on a straight line add up to 180°, so all three together make 180°.

Add the two you are given first: 55 + 50 = 105.

Then take that from 180°: 180 – 105 = 75. So x = 75°.

2.



____ degrees

$$45° + 95° + x = \mathbf{180°}$$

$$45° + 95° = \mathbf{140°}$$

$$180° - 140° = \mathbf{40°}$$

The angles on a straight line add up to 180°, so all three together make 180°.

Add the two you are given first: 45 + 95 = 140.

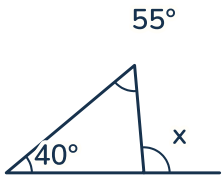
Then take that from 180°: 180 – 140 = 40. So x = 40°.

Solutions (2 of 24)

Exterior Angles

The exterior angle of a triangle

1.



____ degrees

$$x = 40° + 55°$$
$$40° + 55° = 95°$$
$$x = 95°$$

An exterior angle of a triangle equals the sum of the two remote interior angles — the two it does not sit against. Here those are 40° and 55°.

Add them: 40 + 55 = 95.

So x = 95°.

2.

____ degrees

$$x = 50° + 105°$$
$$50° + 105° = 155°$$
$$x = 155°$$

An exterior angle of a triangle equals the sum of the two remote interior angles — the two it does not sit against. Here those are 50° and 105°.

Add them: 50 + 105 = 155.

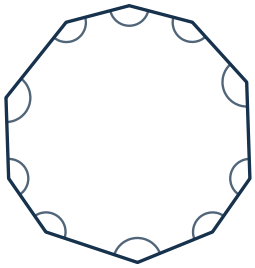
So x = 155°.

Solutions (3 of 24)

Angles in Polygons

Angles inside a polygon

1.



All the angles together

_____ degrees

1440°

There is no separate rule for each shape. Pick one corner of the 10-sided shape and draw a line from it to every corner it is not already joined to — the whole shape falls into triangles, and you already know what a triangle's angles come to.

Counting them: the corner you started from reaches every other corner except its own two neighbors, so a 10-sided shape splits into 10 − 2 = 8 triangles.

Every triangle's angles add to 180°, and between them the triangles use up every angle of the shape and nothing else. So the whole lot comes to 8 × 180 = 1440°.

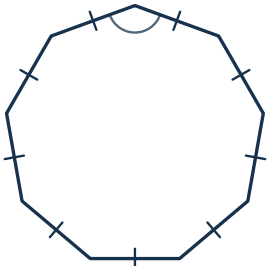
So the angles of a 10-sided shape add up to 1440°.

Solutions (4 of 24)

Angles in Polygons (continued)

Angles inside a polygon (continued)

2.



Each of its angles

____ degrees

140°

There is no separate rule for each shape. Pick one corner of the 9-sided shape and draw a line from it to every corner it is not already joined to — the whole shape falls into triangles, and you already know what a triangle's angles come to.

Counting them: the corner you started from reaches every other corner except its own two neighbors, so a 9-sided shape splits into 9 − 2 = 7 triangles.

Every triangle's angles add to 180°, and between them the triangles use up every angle of the shape and nothing else. So the whole lot comes to 7 × 180 = 1260°.

Now the question wants ONE angle, and it may be divided only because this shape is regular — every angle the same. 1260 shared between 9 is 1260 ÷ 9 = 140°.

So each angle of a regular 9-sided shape is 140°.

Solutions (5 of 24)

The Triangle Inequality

Can these be the sides of a triangle?

1.

— 4

——— 6

————— 12

Yes No

No

Three lengths make a triangle only when the two shorter sides add to more than the longest. The two shorter are 4 and 6; the longest is 12.

4 + 6 = 10, which is less than 12.

The two shorter sides do not reach past the longest, so they cannot close. No.

2.

— 3

————— 9

————— 11

Yes No

Yes

Three lengths make a triangle only when the two shorter sides add to more than the longest. The two shorter are 3 and 9; the longest is 11.

3 + 9 = 12, which is more than 11.

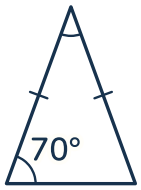
The two shorter sides reach past the longest, so they close into a triangle. Yes.

Solutions (6 of 24)

Isosceles Triangles

Angles in an isosceles triangle

1. x



___ degrees

base = base and

sum = 180°

180 − 140 = **40**

x = **40°**

The two ticked sides are equal, so the two base angles are equal — and the three angles still add to 180°.

Both base angles are 70°, so together they are 140°. The apex is what is left of 180°: 180 − 140 = 40.

So x = 40°.

2. x

___ degrees

base = base and

sum = 180°

180 − 150 = **30**

x = **30°**

The two ticked sides are equal, so the two base angles are equal — and the three angles still add to 180°.

Both base angles are 75°, so together they are 150°. The apex is what is left of 180°: 180 − 150 = 30.

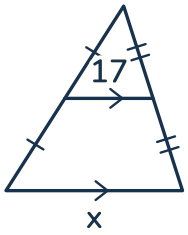
So x = 30°.

Solutions (7 of 24)

Midsegments

The midsegment of a triangle

1.



____ cm

midsegment = **half the base**

17 × 2 = **34**

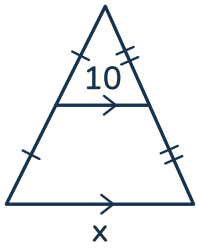
x = **34 cm**

A midsegment joins the midpoints of two sides, and it is half the length of the third side — so the base is always twice the midsegment.

The midsegment is 17 cm, so the base is twice it: 17 × 2 = 34.

So x = 34 cm.

2.



____ cm

midsegment = **half the base**

10 × 2 = **20**

x = **20 cm**

A midsegment joins the midpoints of two sides, and it is half the length of the third side — so the base is always twice the midsegment.

The midsegment is 10 cm, so the base is twice it: 10 × 2 = 20.

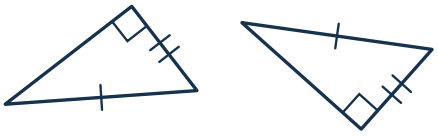
So x = 20 cm.

Solutions (8 of 24)

Congruence Criteria

Which congruence rule (SSS, SAS, ASA, AAS, HL)

1.



SSS SAS ASA AAS HL

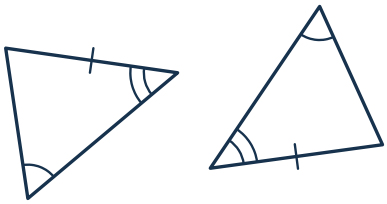
HL

Read the marks first — they are the givens. Here there are a right angle in each, and the hypotenuse with one leg marked equal.

In a right triangle the hypotenuse and one leg give the other leg by Pythagoras, so the two triangles have to match.

So the rule is HL.

2.



SSS SAS ASA AAS HL

AAS

Read the marks first — they are the givens. Here there are two pairs of equal angles and one pair of equal sides.

The marked side is NOT between the two marked angles — that is what makes this AAS and not ASA. It still works: two angles give the third for free (they sum to 180°), so the side is really between two known angles.

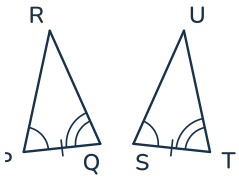
So the rule is AAS.

Solutions (9 of 24)

Proofs with Triangles

Complete the triangle congruence proof

1.



Given $\angle P \cong \angle S$; $PQ \cong ST$; $\angle Q \cong \angle T$

Prove $\Delta PQR \cong \Delta STU$

Statements	Reasons
1. $\angle P \cong \angle S$	Given
2. $PQ \cong ST$	Given
3. $\angle Q \cong \angle T$	Given
4. $\Delta PQR \cong \Delta STU$	?

Reason for step 4:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

Step 4: **$\Delta PQR \cong \Delta STU$**

$\Delta PQR \cong \Delta STU$ → **ASA**

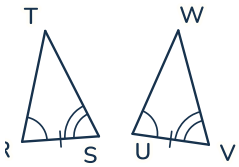
ASA

The proof is missing the reason for step 4: "$\Delta PQR \cong \Delta STU$". Read what that step claims.

Two pairs of angles and the side between them are equal: ASA.

So the missing reason is ASA.

2.



Given $\angle R \cong \angle U$; $RS \cong UV$; $\angle S \cong \angle V$

Prove $\Delta RST \cong \Delta UVW$

Statements	Reasons
1. $\angle R \cong \angle U$	Given
2. $RS \cong UV$	Given
3. $\angle S \cong \angle V$	Given
4. $\Delta RST \cong \Delta UVW$	?

Reason for step 4:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

Step 4: **$\Delta RST \cong \Delta UVW$**

$\Delta RST \cong \Delta UVW$ → **ASA**

ASA

The proof is missing the reason for step 4: "$\Delta RST \cong \Delta UVW$". Read what that step claims.

Two pairs of angles and the side between them are equal: ASA.

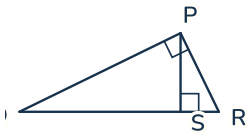
So the missing reason is ASA.

Solutions (10 of 24)

Proofs with Similar Triangles

Complete the similar-triangle proof

1.



Given ∠QPR is a right angle; PS ⊥ QR

Prove ΔSPR ~ ΔPQR

Statements	Reasons
1. ∠R ≅ ∠R	Reflexive Property
2. ∠PSR ≅ ∠QPR	Right angles are congruent
3. ΔSPR ~ ΔPQR	?

Reason for step 3:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

Step 3: **ΔSPR ~ ΔPQR**

ΔSPR ~ ΔPQR → **AA ~**

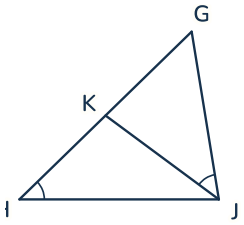
AA ~

The proof is missing the reason for step 3: "ΔSPR ~ ΔPQR". Read what that step claims.

Two pairs of angles are congruent, and two angles are enough to make triangles similar: AA.

So the missing reason is AA ~.

2.



Given ∠GJK ≅ ∠H

Prove ΔGJK ~ ΔGHJ

Statements	Reasons
1. ∠G ≅ ∠G	Reflexive Property
2. ∠GJK ≅ ∠H	Given
3. ΔGJK ~ ΔGHJ	?

Reason for step 3:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

Step 3: **ΔGJK ~ ΔGHJ**

ΔGJK ~ ΔGHJ → **AA ~**

AA ~

The proof is missing the reason for step 3: "ΔGJK ~ ΔGHJ". Read what that step claims.

Two pairs of angles are congruent, and two angles are enough to make triangles similar: AA.

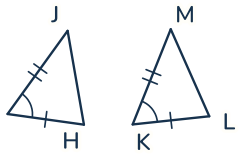
So the missing reason is AA ~.

Solutions (11 of 24)

Writing a Proof

Write a triangle proof

1.



Given GH ≅ KL; ∠G ≅ ∠K; GJ ≅ KM

Prove ΔGHJ ≅ ΔKLM

Statements	Reasons
1.	
2.	
3.	
4.	

Prove: **ΔGHJ ≅ ΔKLM**

1. GH ≅ KL → **Given**
2. ∠G ≅ ∠K → **Given**
3. GJ ≅ KM → **Given**
4. ΔGHJ ≅ ΔKLM → **SAS**

You are given GH ≅ KL; ∠G ≅ ∠K; GJ ≅ KM, and you have to prove ΔGHJ ≅ ΔKLM. Build it one line at a time — each line is a statement and the reason it is true.

1. GH ≅ KL — This fact is handed to you in the Given — nothing has to justify it.

2. ∠G ≅ ∠K — This fact is handed to you in the Given — nothing has to justify it.

3. GJ ≅ KM — This fact is handed to you in the Given — nothing has to justify it.

4. ΔGHJ ≅ ΔKLM — Two pairs of sides and the angle between them are equal: SAS.

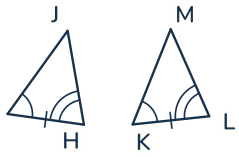
That is the whole proof: the last line, ΔGHJ ≅ ΔKLM, follows by SAS.

Solutions (12 of 24)

Writing a Proof (continued)

Write a triangle proof (continued)

2.



Given ∠G ≅ ∠K; GH ≅ KL; ∠H ≅ ∠L

Prove ΔGHJ ≅ ΔKLM

Statements	Reasons
1.	
2.	
3.	
4.	

Prove: **ΔGHJ ≅ ΔKLM**

1. ∠G ≅ ∠K → **Given**
2. GH ≅ KL → **Given**
3. ∠H ≅ ∠L → **Given**
4. ΔGHJ ≅ ΔKLM → **ASA**

You are given ∠G ≅ ∠K; GH ≅ KL; ∠H ≅ ∠L, and you have to prove ΔGHJ ≅ ΔKLM. Build it one line at a time — each line is a statement and the reason it is true.

1. ∠G ≅ ∠K — This fact is handed to you in the Given — nothing has to justify it.

2. GH ≅ KL — This fact is handed to you in the Given — nothing has to justify it.

3. ∠H ≅ ∠L — This fact is handed to you in the Given — nothing has to justify it.

4. ΔGHJ ≅ ΔKLM — Two pairs of angles and the side between them are equal: ASA.

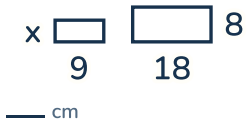
That is the whole proof: the last line, ΔGHJ ≅ ΔKLM, follows by ASA.

Solutions (13 of 24)

Similar Figures

Similar figures and scale

1.



___ cm

4 cm

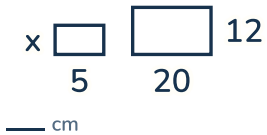
Two shapes, the same shape at two sizes. Every side of the big one is the matching side of the small one multiplied by the SAME number — so the first job is to pair the sides up: long with long, short with short.

Use the pair you can see both of. 9 became 18, and 18 ÷ 9 = 2 — so every side is 2 times bigger.

Now the missing side is on the SMALL shape, so it shrinks — divide instead of multiply: 8 ÷ 2 = 4.

Check it against the picture: the answer should be smaller than 8, and 4 is. So the missing side is 4 cm.

2.



___ cm

3 cm

Two shapes, the same shape at two sizes. Every side of the big one is the matching side of the small one multiplied by the SAME number — so the first job is to pair the sides up: long with long, short with short.

Use the pair you can see both of. 5 became 20, and 20 ÷ 5 = 4 — so every side is 4 times bigger.

Now the missing side is on the SMALL shape, so it shrinks — divide instead of multiply: 12 ÷ 4 = 3.

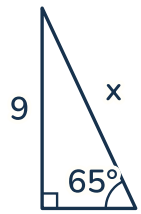
Check it against the picture: the answer should be smaller than 12, and 3 is. So the missing side is 3 cm.

Solutions (14 of 24)

Right Triangle Trigonometry

Right-triangle trigonometry

1.



____ cm

opposite ÷ hypotenuse = **sine**

sin 65° = 9 ÷ x

x = **9.9**

The angle is 65°. You know the opposite (9) and you want the hypotenuse (x). Those two sides are the opposite and the hypotenuse, so the ratio is the sine.

Write it with the numbers: sin 65° = 9 ÷ x.

Rearrange for x and work it out on a calculator: x = 9 ÷ sin 65° = 9.9.

2.

____ cm

adjacent ÷ hypotenuse = **cosine**

cos 70° = 5 ÷ x

x = **14.6**

The angle is 70°. You know the adjacent (5) and you want the hypotenuse (x). Those two sides are the adjacent and the hypotenuse, so the ratio is the cosine.

Write it with the numbers: cos 70° = 5 ÷ x.

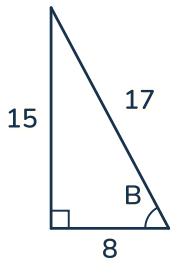
Rearrange for x and work it out on a calculator: x = 5 ÷ cos 70° = 14.6.

Solutions (15 of 24)

Trigonometric Ratios

Write a trigonometric ratio

1.



tan B = ____

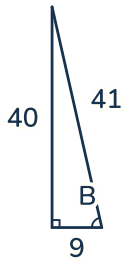
tan B = **15/8**

tan B is opposite over adjacent. Name the two sides you need, measured from angle B.

Read them off the triangle: the opposite side of angle B is 15, the adjacent is 8.

So tan B = 15/8.

2.



tan B = ____

tan B = **40/9**

tan B is opposite over adjacent. Name the two sides you need, measured from angle B.

Read them off the triangle: the opposite side of angle B is 40, the adjacent is 9.

So tan B = 40/9.

The Laws of Sines and Cosines

Laws of sines and cosines

1. In ΔABC, a = 6, ∠A = 60°, ∠B = 45°. Find b: ____

b = **a · sin B / sin A**

b = **6 · sin 45° / sin 60°**

b ≈ **4.9**

Two angles and the side opposite one — that is the law of sines: a/sin A = b/sin B.

Substitute a = 6, A = 60°, B = 45°.

Evaluate and round to one place: b ≈ 4.9.

Solutions (16 of 24)

The Laws of Sines and Cosines (continued)

Laws of sines and cosines (continued)

2. In ΔABC, a = 9, ∠A = 45°, ∠B = 55°. Find b: _____

$$b = a \cdot \sin B / \sin A$$

$$b = 9 \cdot \sin 55° / \sin 45°$$

$$b \approx 10.4$$

Two angles and the side opposite one — that is the law of sines: a/sin A = b/sin B.

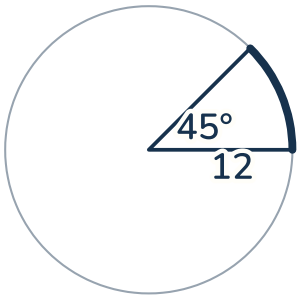
Substitute a = 9, A = 45°, B = 55°.

Evaluate and round to one place: b ≈ 10.4.

Parts of a Circle

Arc length and sector area

1.



Length of the curved edge

_____ cm

$$45/360 = 1/8$$

$$2 \times 3.14 \times 12 = 75.36$$

$$1/8 \text{ of } 75.36 = 9.42$$

9.42 cm

A slice is a FRACTION of the whole circle, and the angle says which fraction. A full turn is 360°, so 45° is 45 out of 360 — that is 1/8 of the circle. Write that down before touching a formula.

Now the whole circle, as if there were no slice. All the way round is 2 × π × r: 2 × 3.14 × 12 = 75.36 cm.

Then take the slice's share of it: 1/8 of 75.36 is 9.42.

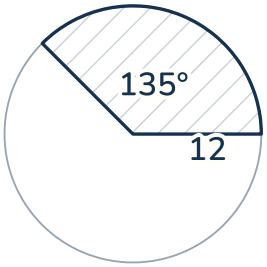
So the curved edge is 9.42 cm. Check the unit against what was asked — a curved edge is a length and a slice is an area, and getting cm where you expected the other one means you answered the other question.

Solutions (17 of 24)

Parts of a Circle (continued)

Arc length and sector area (continued)

2.



Area of the slice

_____ cm²

135/360 = **3/8**

3.14 × 12 × 12 = **452.16**

3/8 of 452.16 = **169.56**

169.56 cm²

A slice is a FRACTION of the whole circle, and the angle says which fraction. A full turn is 360°, so 135° is 135 out of 360 — that is 3/8 of the circle. Write that down before touching a formula.

Now the whole circle, as if there were no slice. Its area is π × r × r — only the r is squared: 3.14 × 12 × 12 = 452.16.

Then take the slice's share of it: 3/8 of 452.16 is 169.56.

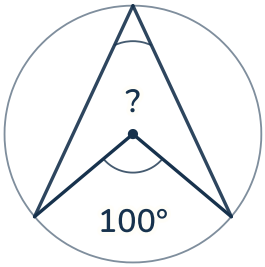
So the area of the slice is 169.56 cm². Check the unit against what was asked — a curved edge is a length and a slice is an area, and getting cm² where you expected the other one means you answered the other question.

Solutions (18 of 24)

Inscribed Angles

Inscribed angle in a circle

1.



___ degrees

same arc, two angles

center = 2 × circle so

circle = center ÷ 2

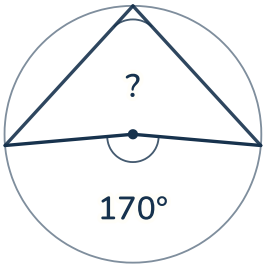
100 ÷ 2 = **50**

Both marked angles stand on the SAME arc — the two chords and the two radii end at the same two points on the circle. That is what makes them a pair; two angles in a circle that do not share an arc have nothing to say to each other.

The angle at the CENTER is always twice the one on the circle, wherever on the far arc that point sits. Here the center angle is the one given, so this is a halving.

Half of 100 is 50, so the angle on the circle is 50°.

2.



___ degrees

same arc, two angles

center = 2 × circle so

circle = center ÷ 2

170 ÷ 2 = **85**

Both marked angles stand on the SAME arc — the two chords and the two radii end at the same two points on the circle. That is what makes them a pair; two angles in a circle that do not share an arc have nothing to say to each other.

The angle at the CENTER is always twice the one on the circle, wherever on the far arc that point sits. Here the center angle is the one given, so this is a halving.

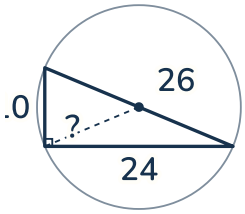
Half of 170 is 85, so the angle on the circle is 85°.

Solutions (19 of 24)

Triangles and Circles

Circles of a right triangle

1.



___ cm

right angle on the circle →

180° at the center

the 26 cm side =

the diameter

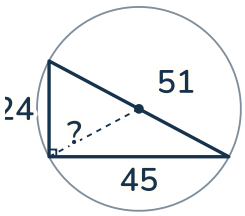
26 ÷ 2 = **13**

The corner between the two short sides is a RIGHT angle, and it is sitting on the circle looking straight across at the hypotenuse. An angle on a circle is half the one at the center, so the angle at the center here is 180° — a straight line.

A straight line through the center IS a diameter. So the hypotenuse is the diameter, and the center of the circle is its midpoint — not somewhere inside the triangle.

The radius is half the diameter: 26 ÷ 2 = 13. So the circle through all three corners has a radius of 13 cm, and the two short sides never come into it.

2.



_____ cm

right angle on the circle →

180° at the center

the 51 cm side =

the diameter

51 ÷ 2 = **25.5**

The corner between the two short sides is a RIGHT angle, and it is sitting on the circle looking straight across at the hypotenuse. An angle on a circle is half the one at the center, so the angle at the center here is 180° — a straight line.

A straight line through the center IS a diameter. So the hypotenuse is the diameter, and the center of the circle is its midpoint — not somewhere inside the triangle.

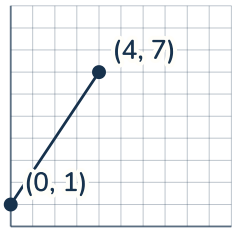
The radius is half the diameter: 51 ÷ 2 = 25.5. So the circle through all three corners has a radius of 25.5 cm, and the two short sides never come into it.

Solutions (20 of 24)

Midpoints

Midpoint of a segment

1.



(___, ___)

$(0 + 4) \div 2 = \mathbf{2}$

$(1 + 7) \div 2 = \mathbf{4}$

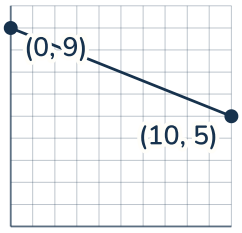
(2, 4)

The midpoint is halfway between, so average each coordinate on its own. The x's first: (0 + 4) ÷ 2 = 2.

Now the y's the same way: (1 + 7) ÷ 2 = 4.

So the midpoint is (2, 4).

2.



(___, ___)

$(0 + 10) \div 2 = \mathbf{5}$

$(9 + 5) \div 2 = \mathbf{7}$

(5, 7)

The midpoint is halfway between, so average each coordinate on its own. The x's first: (0 + 10) ÷ 2 = 5.

Now the y's the same way: (9 + 5) ÷ 2 = 7.

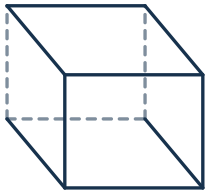
So the midpoint is (5, 7).

Solutions (21 of 24)

Parts of a Solid

Faces, edges and vertices

1.



How many edges?

—

cuboid

base: square **4 sides**

4 + 4 + 4 = **12**

edges: **12**

Half of a solid is round the back. The dashed lines in the drawing are the parts you cannot see, and they count exactly the same as the ones you can — counting only what faces you is where this goes wrong, every time.

Start by naming the base, because everything else is built on it. A cuboid has a square at the bottom, so the base has 4 sides.

Edges come in three sets: 4 round one end, 4 round the other, and 4 joining the two ends together. Nothing else has an edge, so it is 4 + 4 + 4.

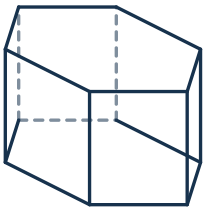
So a cuboid has 12 edges.

Solutions (22 of 24)

Parts of a Solid (continued)

Faces, edges and vertices (continued)

3.



How many vertices?

___

hexagonal prism

base: hexagon **6 sides**

6 + 6 = **12**

vertices: **12**

Half of a solid is round the back. The dashed lines in the drawing are the parts you cannot see, and they count exactly the same as the ones you can — counting only what faces you is where this goes wrong, every time.

Start by naming the base, because everything else is built on it. A hexagonal prism has a hexagon at the bottom, so the base has 6 sides.

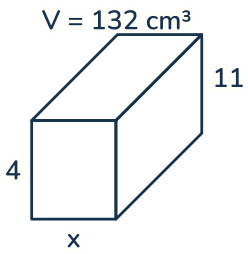
A vertex is a corner. There are 6 of them round one end and 6 round the other, and none anywhere in between: 6 + 6.

So a hexagonal prism has 12 vertices.

Missing Edges

Missing edge from the volume

1.



___ cm

$4 \times 11 \times x = 132$

$4 \times 11 = 44$

$132 \div 44 = 3$

The volume of a box is its three edges multiplied: length × width × height. You have the height (4), the depth (11), and the volume (132).

Multiply the two edges you know first: 4 × 11 = 44.

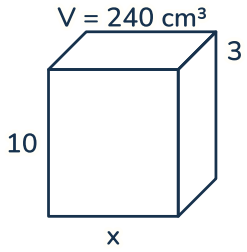
Then the missing edge is the volume divided by that: 132 ÷ 44 = 3.

Solutions (23 of 24)

Missing Edges (continued)

Missing edge from the volume (continued)

2.



___ cm

$$10 \times 3 \times x = 240$$

$$10 \times 3 = 30$$

$$240 \div 30 = 8$$

The volume of a box is its three edges multiplied: length × width × height. You have the height (10), the depth (3), and the volume (240).

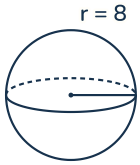
Multiply the two edges you know first: 10 × 3 = 30.

Then the missing edge is the volume divided by that: 240 ÷ 30 = 8.

Surface Area of a Sphere

Surface area of a sphere

1.



_____ cm²

radius: 8 cm

$$8 \times 8 = 64$$

$$12.56 \times 64 = 803.84$$

803.84 cm²

The 8 cm marked here runs from the center to the surface, so it is already the radius — which is what the formula wants.

Square the radius on a line of its own. r^2 means r × r, not r × 2: 8 × 8 = 64.

Now four times π, times that: 4 × 3.14 is 12.56, and 12.56 × 64 = 803.84. Nothing to round — 12.56 times a whole number always lands on a whole number of hundredths.

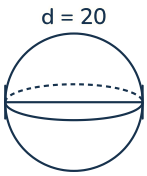
So the surface area is 803.84 square cm. Square, not cubic — a surface is skin rather than space, and the unit is the quickest way to notice you have answered the volume question instead.

Solutions (24 of 24)

Surface Area of a Sphere (continued)

Surface area of a sphere (continued)

2.



_____ cm²

diameter: 20 cm

20 ÷ 2 = **10**

10 × 10 = **100**

12.56 × 100 = **1256**

1256 cm²

The 20 cm marked here goes right ACROSS the sphere — that is the diameter. The formula wants the radius, and using the diameter instead makes the answer FOUR times too big, because it is squared.

Half of 20 is 10, so r = 10 cm.

Square the radius on a line of its own. r^2 means r × r, not r × 2: 10 × 10 = 100.

Now four times π, times that: 4 × 3.14 is 12.56, and 12.56 × 100 = 1256. Nothing to round — 12.56 times a whole number always lands on a whole number of hundredths.

So the surface area is 1256 square cm. Square, not cubic — a surface is skin rather than space, and the unit is the quickest way to notice you have answered the volume question instead.