Geometry Foundation Workbook

21 lessons · printable · answer key at the back

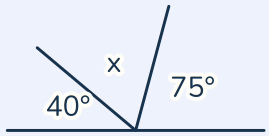
Name ________________________ Date ________________________

Contents (1 of 2)

Contents (2 of 2)

Angle Pairs

WORKED EXAMPLE

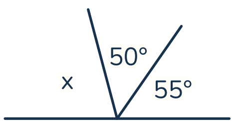


65 degrees

1. Angles on a line and around a point

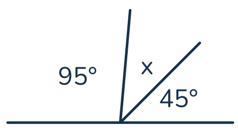
Find the angle marked x. The angles on each line add to 180°.

1



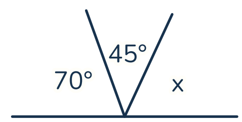
________________ degrees

2



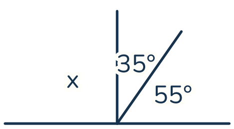
________________ degrees

3



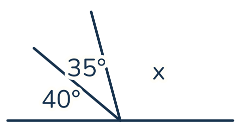
________________ degrees

4



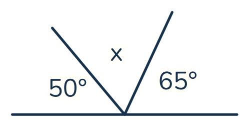
________________ degrees

5



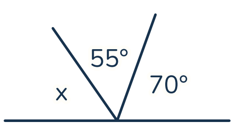
________________ degrees

6



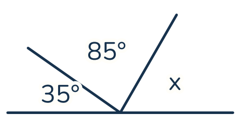
________________ degrees

7



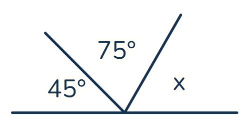
________________ degrees

8



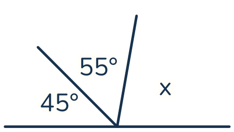
________________ degrees

9



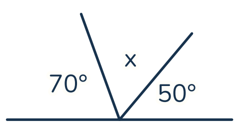
________________ degrees

10



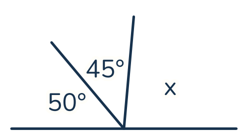
________________ degrees

11



________________ degrees

12

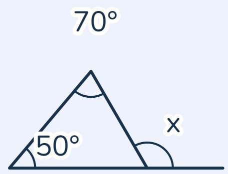


________________ degrees

1-2

Exterior Angles

WORKED EXAMPLE



120 degrees

2. The exterior angle of a triangle

Find the angle marked x. Give each answer in degrees.

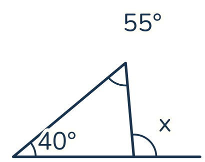
1

______________ degrees

2

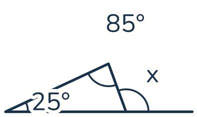
______________ degrees

3



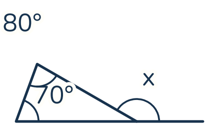
______________ degrees

4



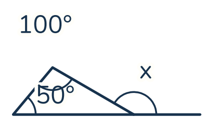
______________ degrees

5



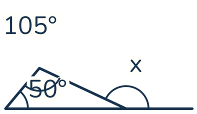
______________ degrees

6



______________ degrees

7



______________ degrees

8

______________ degrees

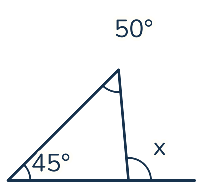
9

______________ degrees

10

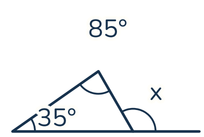
______________ degrees

11



______________ degrees

12

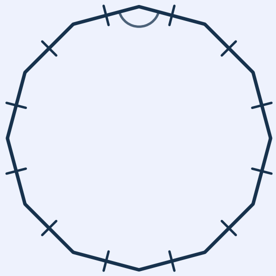


______________ degrees

1-3

Angles in Polygons

WORKED EXAMPLE



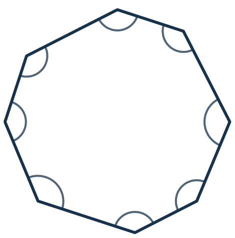
Each of its angles

150 degrees

3. Angles inside a polygon

Work out the angle each shape is asking for. Give every answer in degrees.

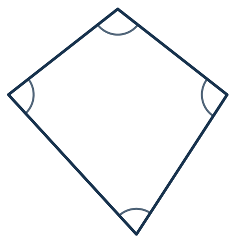
1



All the angles together

________________ degrees

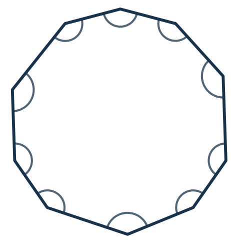
2



All the angles together

________________ degrees

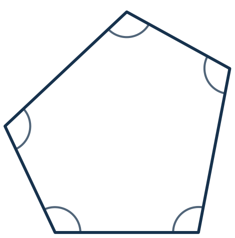
3



All the angles together

________________ degrees

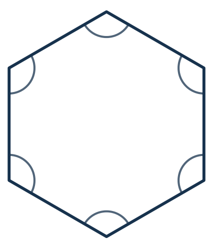
4



All the angles together

________________ degrees

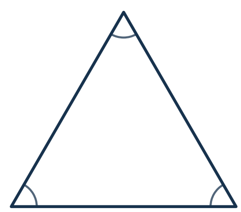
5



All the angles together

________________ degrees

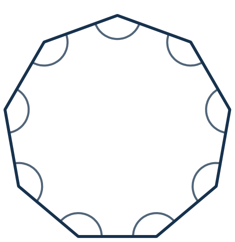
6



All the angles together

________________ degrees

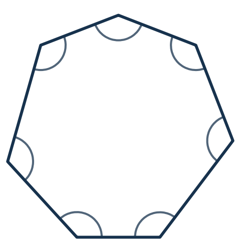
7



All the angles together

________________ degrees

8



All the angles together

________________ degrees

The Triangle Inequality

WORKED EXAMPLE

6
12
12

Yes No

4. Can these be the sides of a triangle?

Can these three lengths form a triangle? Answer Yes or No.

1
6
10
11
Yes No

2
7
8
9
Yes No

3
4
6
12
Yes No

4
11
16
16
Yes No

5
9
10
11
Yes No

6
14
14
14
Yes No

7
3
11
16
Yes No

8
10
11
13
Yes No

9
13
13
13
Yes No

10
8
13
16
Yes No

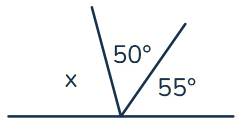
11
3
15
15
Yes No

12
10
12
13
Yes No

5. Angles on a line and around a point

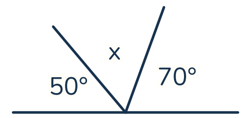
Find the angle marked x. The angles on each line add to 180°.

13



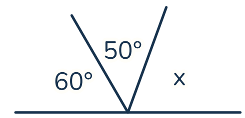
____________ degrees

14



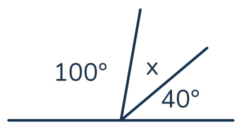
____________ degrees

15



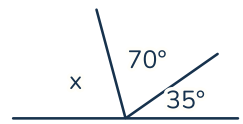
____________ degrees

16



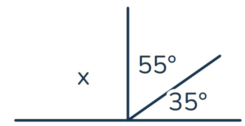
____________ degrees

17



____________ degrees

18



____________ degrees

Isosceles Triangles

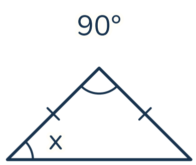
WORKED EXAMPLE

110 degrees

6. Angles in an isosceles triangle

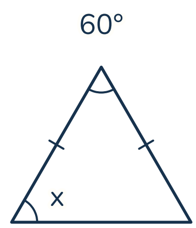
Find the angle marked x. The ticks mark the two equal sides.

1



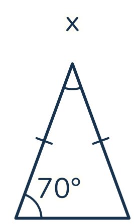
______ degrees

2



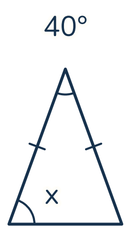
______ degrees

3



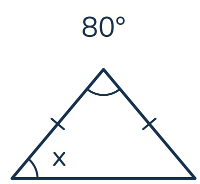
______ degrees

4



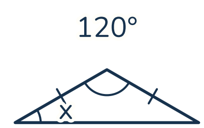
______ degrees

5



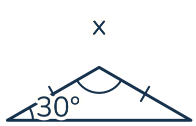
______ degrees

6



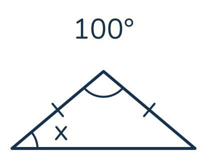
______ degrees

7



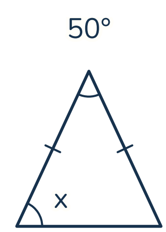
______ degrees

8



______ degrees

9



______ degrees

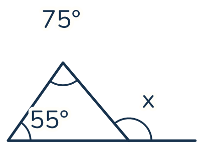
10

______ degrees

7. The exterior angle of a triangle

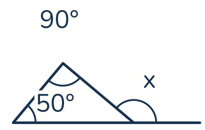
Find the angle marked x. Give each answer in degrees.

11



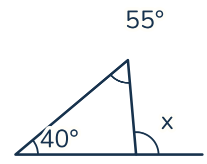
__________________ degrees

12



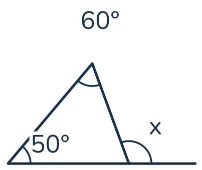
__________________ degrees

13



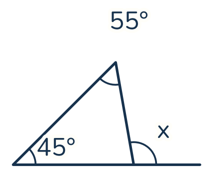
__________________ degrees

14



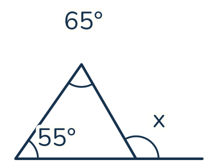
__________________ degrees

15



__________________ degrees

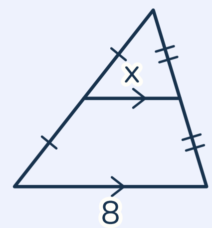
16



__________________ degrees

Midsegments

WORKED EXAMPLE

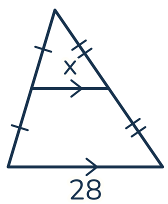


4 cm

8. The midsegment of a triangle

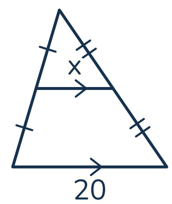
Find the length marked x. The dashes mark equal halves.

1



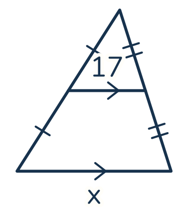
__________ cm

2



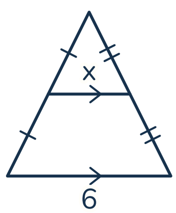
__________ cm

3



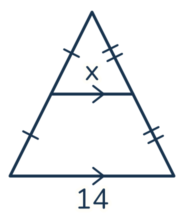
__________ cm

4



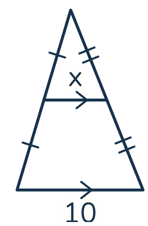
__________ cm

5



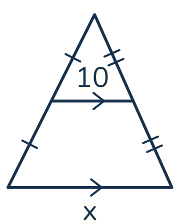
__________ cm

6



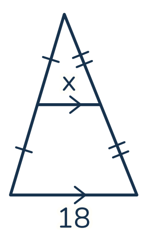
__________ cm

7



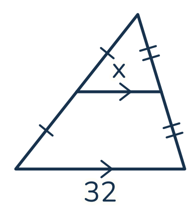
__________ cm

8



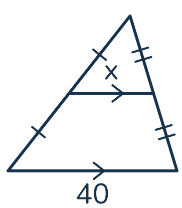
__________ cm

9



__________ cm

10

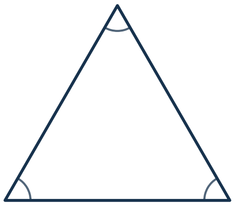


__________ cm

9. Angles inside a polygon

Work out the angle each shape is asking for. Give every answer in degrees.

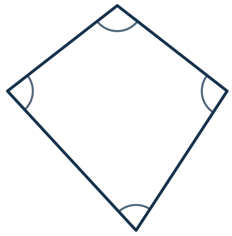
11



All the angles together

____________________ degrees

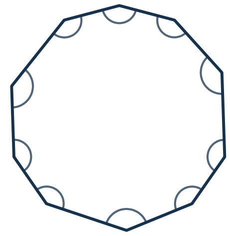
12



All the angles together

____________________ degrees

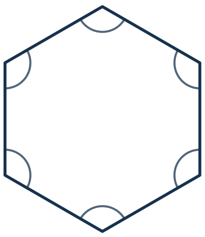
13



All the angles together

____________________ degrees

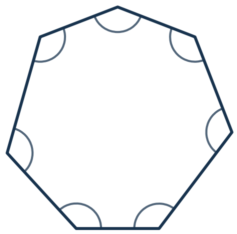
14



All the angles together

____________________ degrees

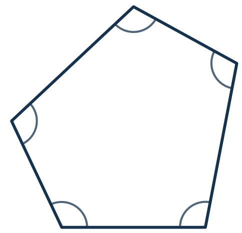
15



All the angles together

____________________ degrees

16

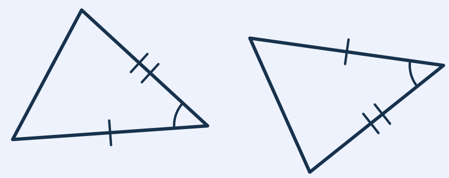


All the angles together

____________________ degrees

Congruence Criteria

WORKED EXAMPLE

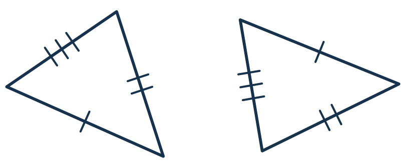


SSS **SAS** ASA AAS HL

10. Which congruence rule (SSS, SAS, ASA, AAS, HL)

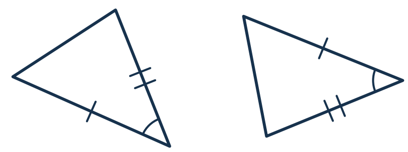
Which rule shows each pair of triangles is congruent?

1



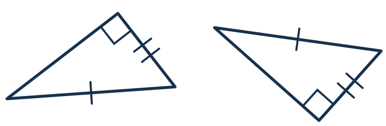
SSS SAS ASA AAS HL

2



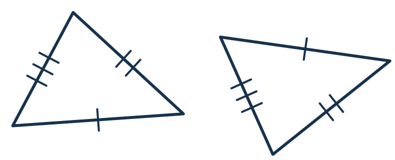
SSS SAS ASA AAS HL

3



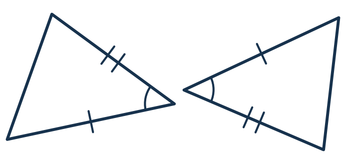
SSS SAS ASA AAS HL

4



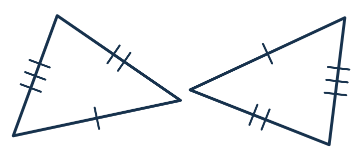
SSS SAS ASA AAS HL

5



SSS SAS ASA AAS HL

6

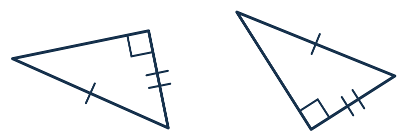


SSS SAS ASA AAS HL

10. Which congruence rule (SSS, SAS, ASA, AAS, HL) (continued)

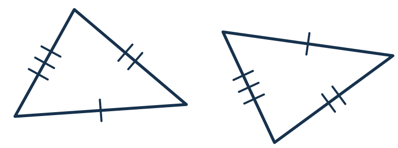
Which rule shows each pair of triangles is congruent?

7



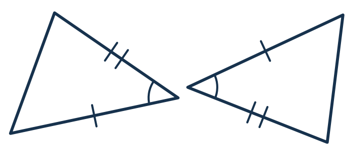
SSS SAS ASA AAS HL

8



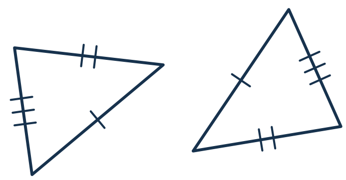
SSS SAS ASA AAS HL

9



SSS SAS ASA AAS HL

10



SSS SAS ASA AAS HL

11. Can these be the sides of a triangle?

Can these three lengths form a triangle? Answer Yes or No.

11

11
12
13

Yes No

12

10
11
13

Yes No

13

6
8
12

Yes No

14

9
12
12

Yes No

15

9
12
14

Yes No

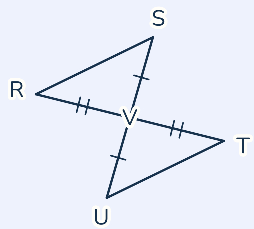
16

10
13
13

Yes No

Proofs with Triangles

WORKED EXAMPLE



Given V is the midpoint of SU; RV ≅ TV

Prove △RVS ≅ △TVU

Statements	Reasons
1. SV ≅ UV	Definition of Midpoint
2. ∠RVS ≅ ∠TVU	?
3. RV ≅ TV	Given
4. △RVS ≅ △TVU	SAS

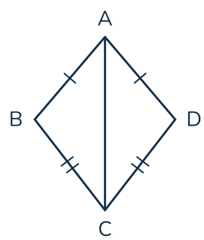
Reason for step 2:

Given | Reflexive Property | **Vertical Angles** | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

12. Complete the triangle congruence proof

Read each proof and choose the missing reason.

1



Given AB ≅ AD; CB ≅ CD

Prove △ABC ≅ △ADC

Statements	Reasons
1. AB ≅ AD	Given
2. CB ≅ CD	Given
3. AC ≅ AC	?
4. △ABC ≅ △ADC	SSS

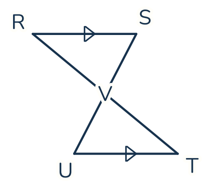
Reason for step 3:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

12. Complete the triangle congruence proof (continued)

Read each proof and choose the missing reason.

2



Given RS ∥ UT; RS ≅ UT

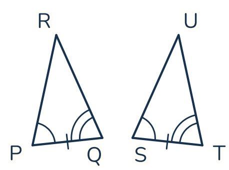
Prove ΔRSV ≅ ΔTUV

Statements	Reasons
1. ∠SRV ≅ ∠UTV	?
2. ∠RSV ≅ ∠TUV	Alternate Interior Angles
3. RS ≅ UT	Given
4. ΔRSV ≅ ΔTUV	ASA

Reason for step 1:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

3



Given ∠P ≅ ∠S; PQ ≅ ST; ∠Q ≅ ∠T

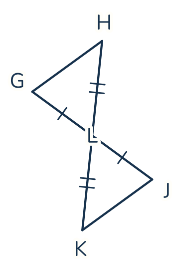
Prove ΔPQR ≅ ΔSTU

Statements	Reasons
1. ∠P ≅ ∠S	Given
2. PQ ≅ ST	Given
3. ∠Q ≅ ∠T	Given
4. ΔPQR ≅ ΔSTU	?

Reason for step 4:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

4



Given GL ≅ JL; HL ≅ KL

Prove ΔGLH ≅ ΔJLK

Statements	Reasons
1. GL ≅ JL	Given
2. ∠GLH ≅ ∠JLK	?
3. HL ≅ KL	Given
4. ΔGLH ≅ ΔJLK	SAS

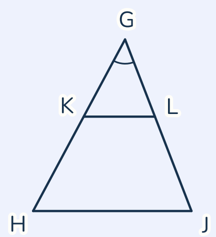
Reason for step 2:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

3-2

Proofs with Similar Triangles

WORKED EXAMPLE



Given GK/GH = GL/GJ

Prove ΔGKL ~ ΔGHJ

Statements	Reasons
1. ∠G ≅ ∠G	?
2. GK/GH = GL/GJ	Given
3. ΔGKL ~ ΔGHJ	SAS ~

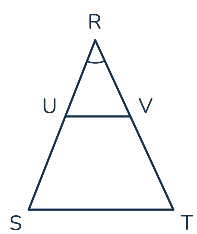
Reason for step 1:

Given | **Reflexive Property** | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

13. Complete the similar-triangle proof

Read each proof and choose the missing reason.

1



Given RU/RS = RV/RT

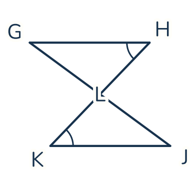
Prove ΔRUV ~ ΔRST

Statements	Reasons
1. ∠R ≅ ∠R	?
2. RU/RS = RV/RT	Given
3. ΔRUV ~ ΔRST	SAS ~

Reason for step 1:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

2



Given ∠GHL ≅ ∠JKL

Prove ΔGHL ~ ΔJKL

Statements	Reasons
1. ∠GHL ≅ ∠JKL	?
2. ∠GLH ≅ ∠JLK	Vertical Angles
3. ΔGHL ~ ΔJKL	AA ~

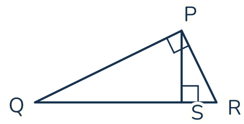
Reason for step 1:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

13. Complete the similar-triangle proof (continued)

Read each proof and choose the missing reason.

3



Given ∠QPR is a right angle; PS ⊥ QR

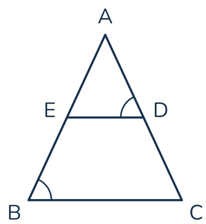
Prove △SPR ~ △PQR

Statements	Reasons
1. ∠R ≅ ∠R	Reflexive Property
2. ∠PSR ≅ ∠QPR	Right angles are congruent
3. △SPR ~ △PQR	?

Reason for step 3:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

4



Given ∠ADE ≅ ∠B

Prove △ADE ~ △ABC

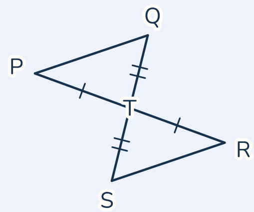
Statements	Reasons
1. ∠A ≅ ∠A	Reflexive Property
2. ∠ADE ≅ ∠B	?
3. △ADE ~ △ABC	AA ~

Reason for step 2:

Given | Reflexive Property | Vertical Angles | Corresponding Angles | Alternate Interior Angles | AA ~ | SAS ~ | SSS ~

Writing a Proof

WORKED EXAMPLE



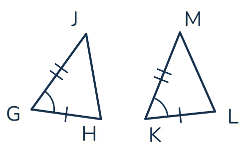
Given $\overline{PT} \cong \overline{RT}$; $\overline{QT} \cong \overline{ST}$
Prove $\triangle PTQ \cong \triangle RTS$

Statements	Reasons
1. $\overline{PT} \cong \overline{RT}$	Given
2. $\angle PTQ \cong \angle RTS$	Vertical Angles
3. $\overline{QT} \cong \overline{ST}$	Given
4. $\triangle PTQ \cong \triangle RTS$	SAS

14. Write a triangle proof

Write a full two-column proof for each. Reasons may be used more than once.

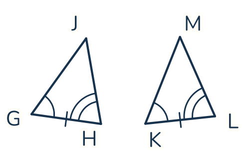
1



Given $GH \cong KL$; $\angle G \cong \angle K$; $GJ \cong KM$
Prove $\triangle GHJ \cong \triangle KLM$

Statements	Reasons
1.	
2.	
3.	
4.	

2



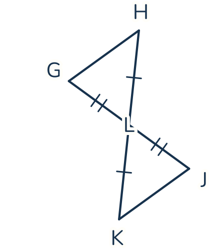
Given $\angle G \cong \angle K$; $GH \cong KL$; $\angle H \cong \angle L$
Prove $\triangle GHJ \cong \triangle KLM$

Statements	Reasons
1.	
2.	
3.	
4.	

14. Write a triangle proof (continued)

Write a full two-column proof for each. Reasons may be used more than once.

3

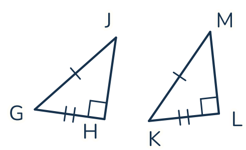


Given L is the midpoint of HK; GL ≅ JL

Prove △GLH ≅ △JLK

Statements	Reasons
1.	
2.	
3.	
4.	

4



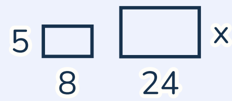
Given ∠H and ∠L are right angles; GJ ≅ KM; GH ≅ KL

Prove △GHJ ≅ △KLM

Statements	Reasons
1.	
2.	
3.	
4.	

Similar Figures

WORKED EXAMPLE

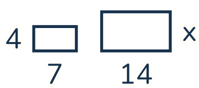


15 cm

15. Similar figures and scale

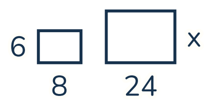
Each pair of shapes is the same shape at two sizes. Find the missing length.

1



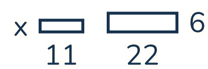
__________ cm

2



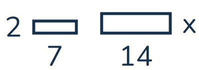
__________ cm

3



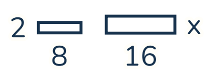
__________ cm

4



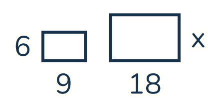
__________ cm

5



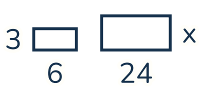
__________ cm

6



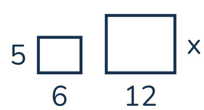
__________ cm

7



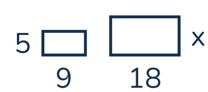
__________ cm

8



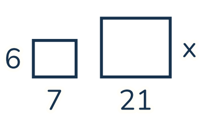
__________ cm

9



__________ cm

10

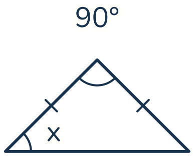


__________ cm

16. Angles in an isosceles triangle

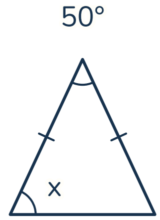
Find the angle marked x. The ticks mark the two equal sides.

11



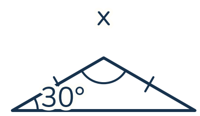
________________ degrees

12



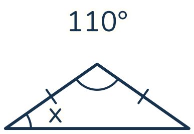
________________ degrees

13



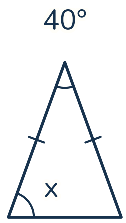
________________ degrees

14



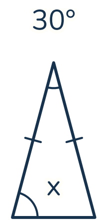
________________ degrees

15



________________ degrees

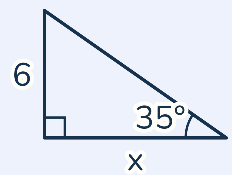
16



________________ degrees

Right Triangle Trigonometry

WORKED EXAMPLE

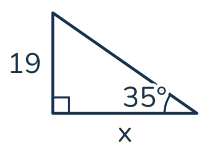


8.6 cm

17. Right-triangle trigonometry

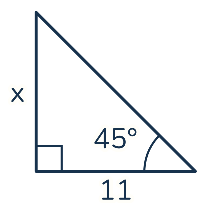
Find the side marked x, to one decimal place. Use a calculator.

1



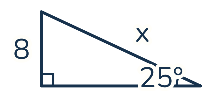
____________ cm

2



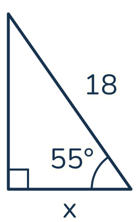
____________ cm

3



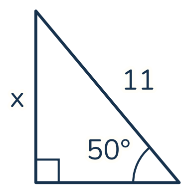
____________ cm

4



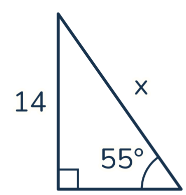
____________ cm

5



____________ cm

6

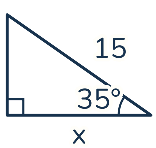


____________ cm

7

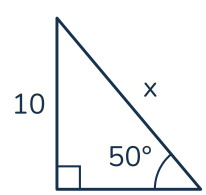
____________ cm

8



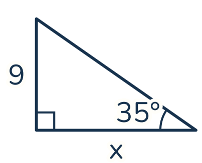
____________ cm

9



____________ cm

10

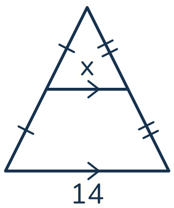


____________ cm

18. The midsegment of a triangle

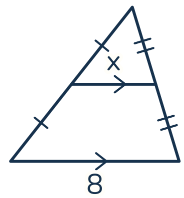
Find the length marked x. The dashes mark equal halves.

11



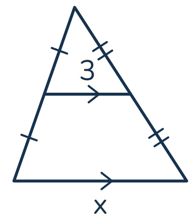
______________ cm

12



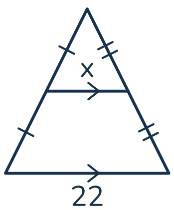
______________ cm

13



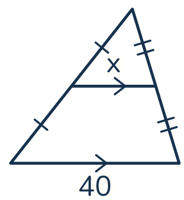
______________ cm

14



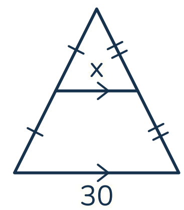
______________ cm

15



______________ cm

16

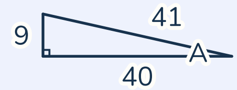


______________ cm

4-3

Trigonometric Ratios

WORKED EXAMPLE

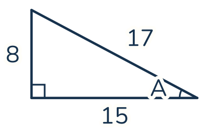


$\tan A = \frac{9}{40}$

19. Write a trigonometric ratio

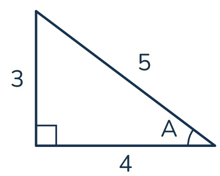
Write each ratio as a fraction. Read the sides straight off the triangle.

1



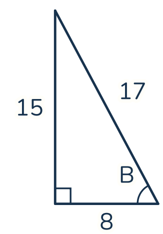
sin A = ____________

2



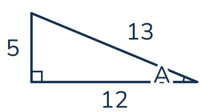
sin A = ____________

3



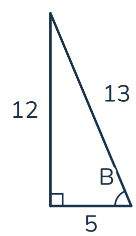
tan B = ____________

4



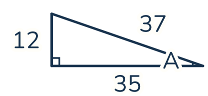
sin A = ____________

5



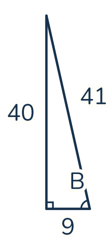
sin B = ____________

6



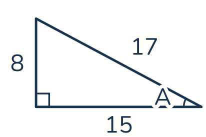
sin A = ____________

7



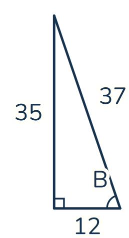
tan B = ____________

8



cos A = ____________

9

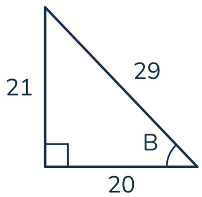


sin B = ____________

19. Write a trigonometric ratio (continued)

Write each ratio as a fraction. Read the sides straight off the triangle.

10

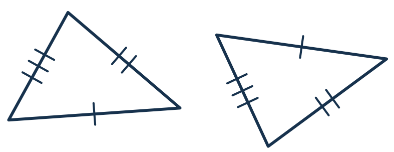


cos B = ________________

20. Which congruence rule (SSS, SAS, ASA, AAS, HL)

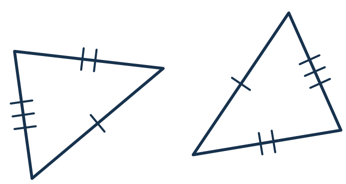
Which rule shows each pair of triangles is congruent?

11



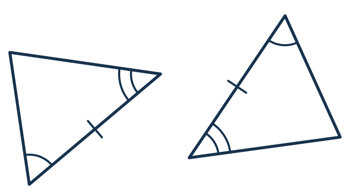
SSS SAS ASA AAS HL

12



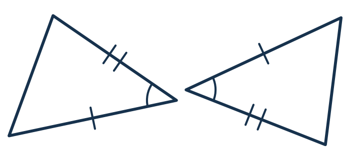
SSS SAS ASA AAS HL

13



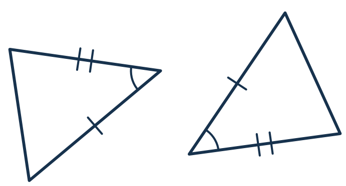
SSS SAS ASA AAS HL

14



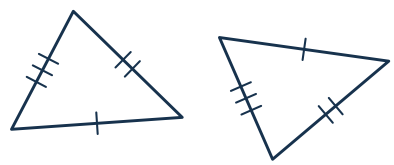
SSS SAS ASA AAS HL

15



SSS SAS ASA AAS HL

16



SSS SAS ASA AAS HL

The Laws of Sines and Cosines

WORKED EXAMPLE

In $\triangle ABC$, $a = 7$, $\angle A = 50^\circ$, $\angle B = 60^\circ$. Find b: **7.9**

21. Laws of sines and cosines

Use the law of sines. Round each length to one decimal place.

1 In $\triangle ABC$, $a = 6$, $\angle A = 60^\circ$, $\angle B = 45^\circ$. Find b:

______

2 In $\triangle ABC$, $a = 9$, $\angle A = 45^\circ$, $\angle B = 55^\circ$. Find b:

______

3 In $\triangle ABC$, $a = 7$, $\angle A = 40^\circ$, $\angle B = 60^\circ$. Find b:

______

4 In $\triangle ABC$, $a = 10$, $\angle A = 50^\circ$, $\angle B = 70^\circ$. Find b:

______

5 In $\triangle ABC$, $a = 12$, $\angle A = 40^\circ$, $\angle B = 50^\circ$. Find b:

______

6 In $\triangle ABC$, $a = 9$, $\angle A = 65^\circ$, $\angle B = 40^\circ$. Find b:

______

7 In $\triangle ABC$, $a = 5$, $\angle A = 55^\circ$, $\angle B = 65^\circ$. Find b:

______

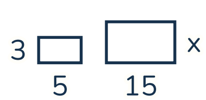
8 In $\triangle ABC$, $a = 11$, $\angle A = 30^\circ$, $\angle B = 75^\circ$. Find b:

______

22. Similar figures and scale

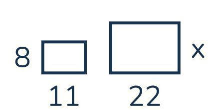
Each pair of shapes is the same shape at two sizes. Find the missing length.

9



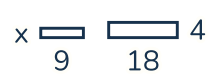
______ cm

10



______ cm

11

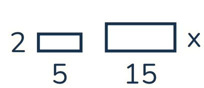


______ cm

12

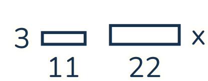
______ cm

13



______ cm

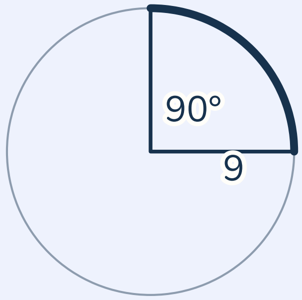
14



______ cm

Parts of a Circle

WORKED EXAMPLE

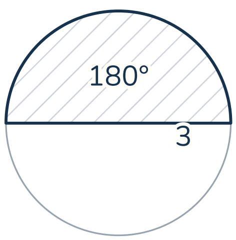


Length of the curved edge

14.13 cm

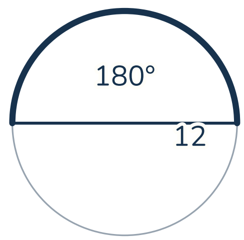
23. Arc length and sector area

Use π = 3.14. Each shape is part of a circle — find what the label asks for.

1


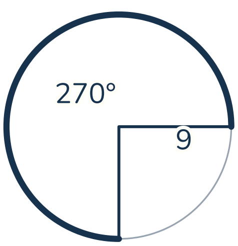
Area of the slice

__________ cm^2

2


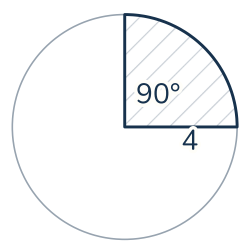
Length of the curved edge

__________ cm

3


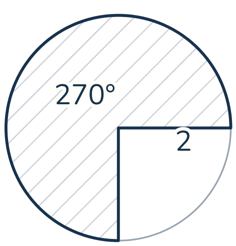
Length of the curved edge

__________ cm

4


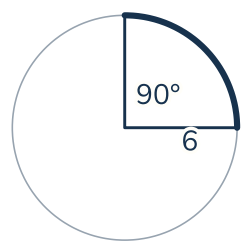
Area of the slice

__________ cm^2

5


Area of the slice

__________ cm^2

6


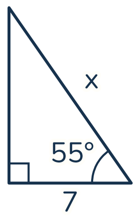
Length of the curved edge

__________ cm

24. Right-triangle trigonometry

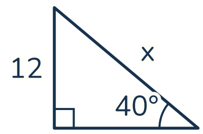
Find the side marked x, to one decimal place. Use a calculator.

7



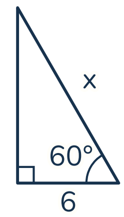
_______________ cm

8



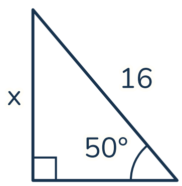
_______________ cm

9



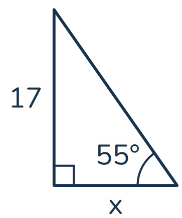
_______________ cm

10



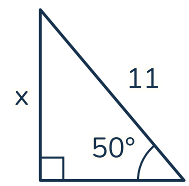
_______________ cm

11



_______________ cm

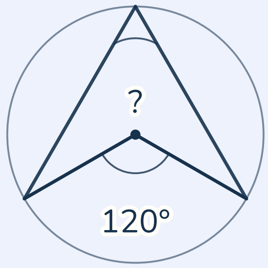
12



_______________ cm

Inscribed Angles

WORKED EXAMPLE

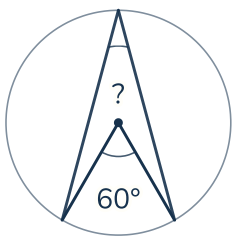


60 degrees

25. Inscribed angle in a circle

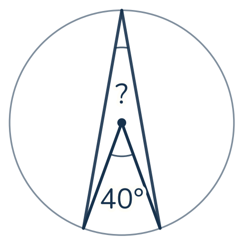
The angle at the center is given. Find the angle on the circle.

1



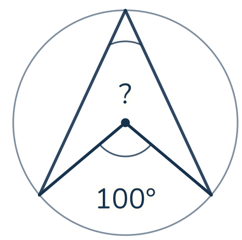
________ degrees

2



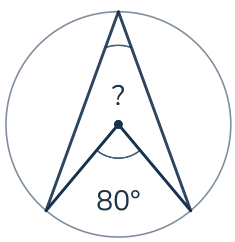
________ degrees

3



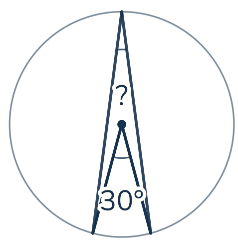
________ degrees

4



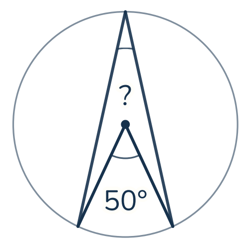
________ degrees

5



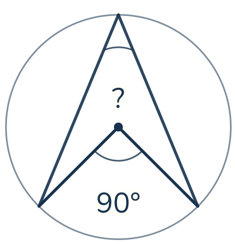
________ degrees

6



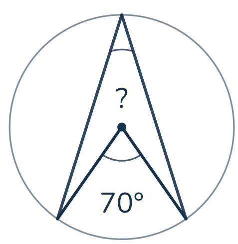
________ degrees

7



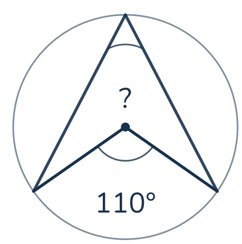
________ degrees

8



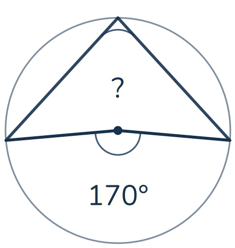
________ degrees

9



________ degrees

10

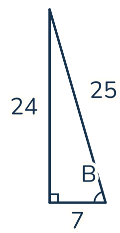


________ degrees

26. Write a trigonometric ratio

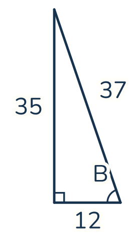
Write each ratio as a fraction. Read the sides straight off the triangle.

11



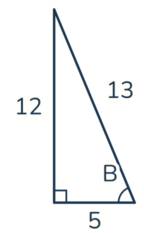
sin B = ________________

12



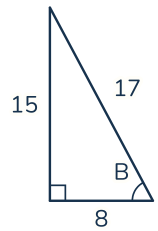
sin B = ________________

13



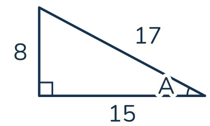
tan B = ________________

14



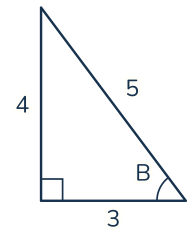
cos B = ________________

15



cos A = ________________

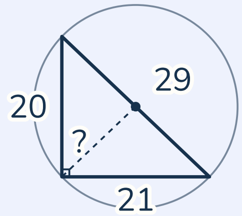
16



cos B = ________________

Triangles and Circles

WORKED EXAMPLE

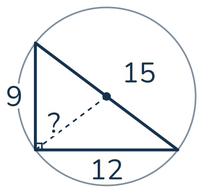


14.5 cm

27. Circles of a right triangle

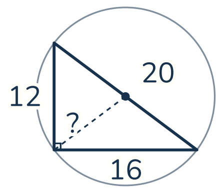
Each circle passes through all three corners. Find its radius.

1



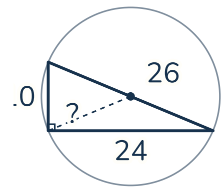
________ cm

2



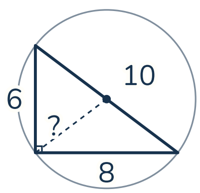
________ cm

3



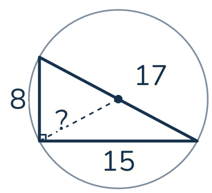
________ cm

4



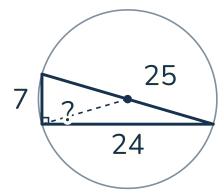
________ cm

5



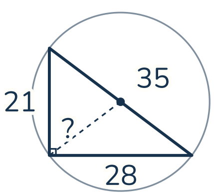
________ cm

6



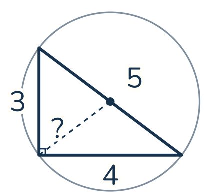
________ cm

7



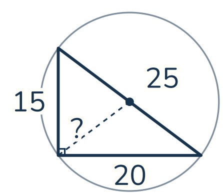
________ cm

8



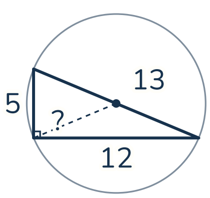
________ cm

9



________ cm

10



________ cm

28. Laws of sines and cosines

Use the law of sines. Round each length to one decimal place.

11 In $\triangle ABC$, $a = 6$, $\angle A = 60°$, $\angle B = 45°$. Find b:

____

12 In $\triangle ABC$, $a = 9$, $\angle A = 45°$, $\angle B = 55°$. Find b:

____

13 In $\triangle ABC$, $a = 7$, $\angle A = 40°$, $\angle B = 60°$. Find b:

____

14 In $\triangle ABC$, $a = 10$, $\angle A = 50°$, $\angle B = 70°$. Find b:

____

15 In $\triangle ABC$, $a = 12$, $\angle A = 40°$, $\angle B = 50°$. Find b:

____

16 In $\triangle ABC$, $a = 9$, $\angle A = 65°$, $\angle B = 40°$. Find b:

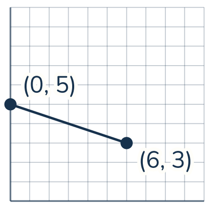
____

Midpoints

29. Midpoint of a segment

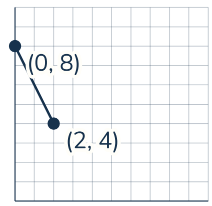
Find the midpoint of each segment.

1



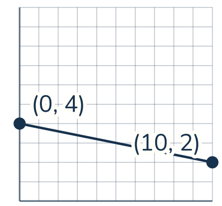
(______, ______)

2



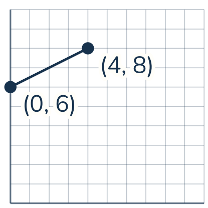
(______, ______)

3



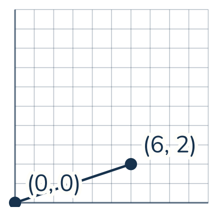
(______, ______)

4



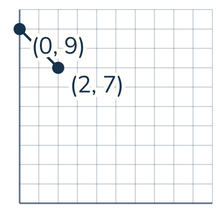
(______, ______)

5



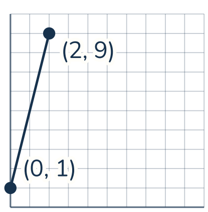
(______, ______)

6



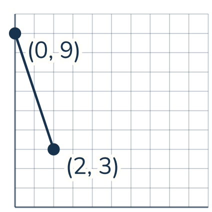
(______, ______)

7



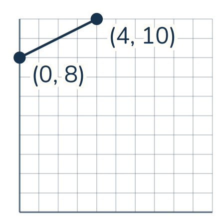
(______, ______)

8



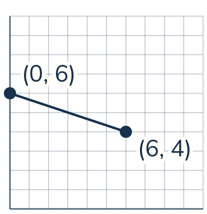
(______, ______)

9



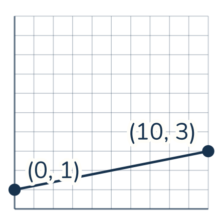
(______, ______)

10



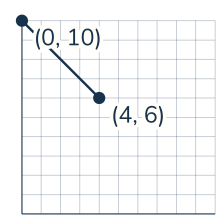
(______, ______)

11



(______, ______)

12

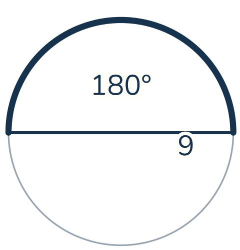


(______, ______)

30. Arc length and sector area

Use π = 3.14. Each shape is part of a circle — find what the label asks for.

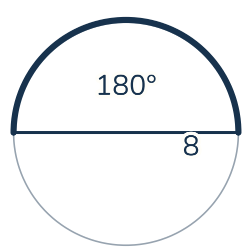
13



Length of the curved edge

______________________ cm

14



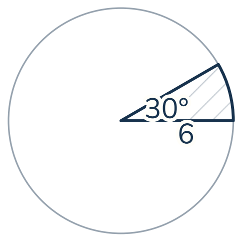
Length of the curved edge

______________________ cm

30. Arc length and sector area (continued)

Use π = 3.14. Each shape is part of a circle — find what the label asks for.

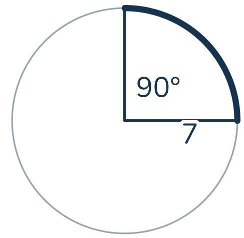
15



Area of the slice

________________ cm²

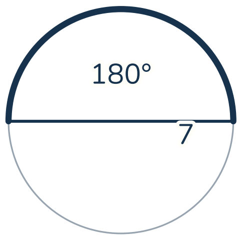
16



Length of the curved edge

________________ cm

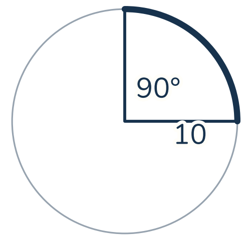
17



Length of the curved edge

________________ cm

18

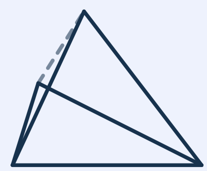


Length of the curved edge

________________ cm

Parts of a Solid

WORKED EXAMPLE



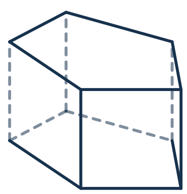
How many vertices?

4

31. Faces, edges and vertices

Hidden edges are dashed. Answer the question under each solid.

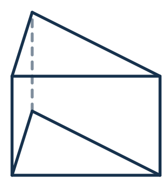
1



How many faces?

____

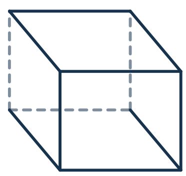
2



How many faces?

____

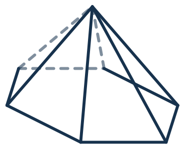
3



How many edges?

____

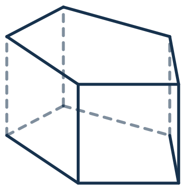
4



How many faces?

____

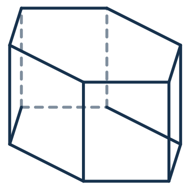
5



How many vertices?

____

6



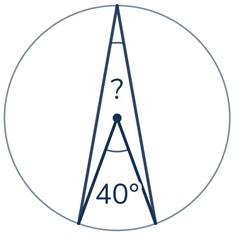
How many faces?

____

32. Inscribed angle in a circle

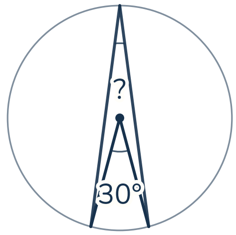
The angle at the center is given. Find the angle on the circle.

7



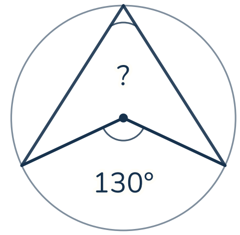
________ degrees

8



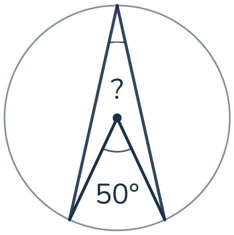
________ degrees

9



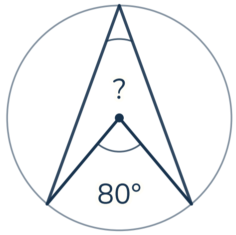
________ degrees

10



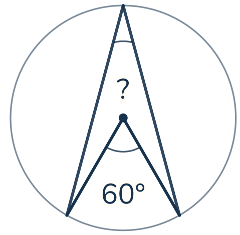
________ degrees

11



________ degrees

12

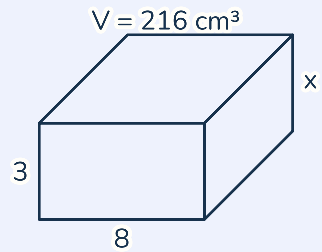


________ degrees

6-3

Missing Edges

WORKED EXAMPLE

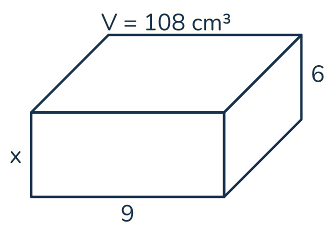


9 cm

33. Missing edge from the volume

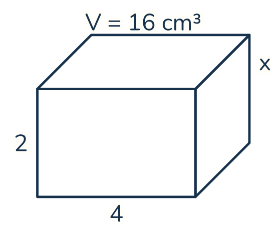
The volume of each box is shown. Find the missing edge, in centimeters.

1



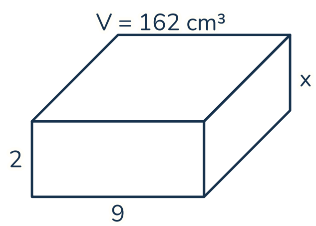
________________ cm

2



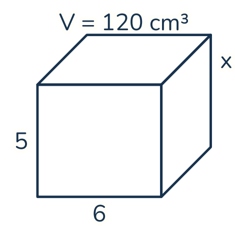
________________ cm

3



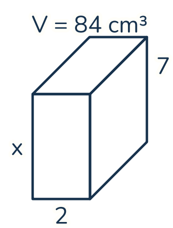
________________ cm

4



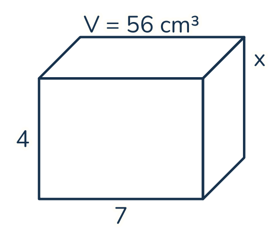
________________ cm

5



________________ cm

6

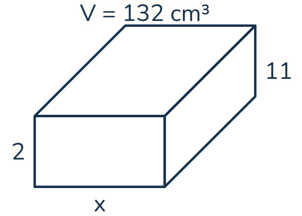


________________ cm

33. Missing edge from the volume (continued)

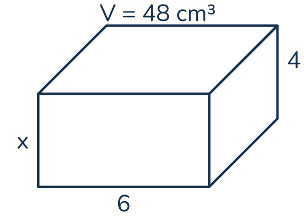
The volume of each box is shown. Find the missing edge, in centimeters.

7



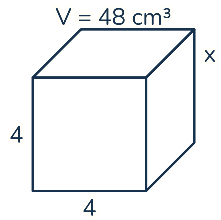
______________ cm

8



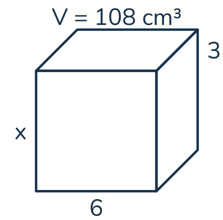
______________ cm

9



______________ cm

10

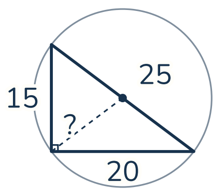


______________ cm

34. Circles of a right triangle

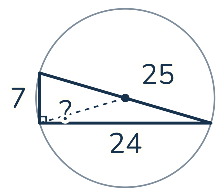
Each circle passes through all three corners. Find its radius.

11



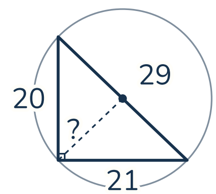
______________ cm

12



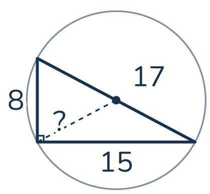
______________ cm

13



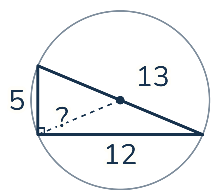
______________ cm

14



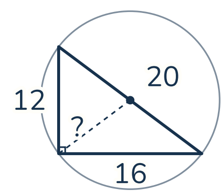
______________ cm

15



______________ cm

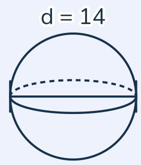
16



______________ cm

Surface Area of a Sphere

WORKED EXAMPLE

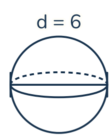


615.44 cm²

35. Surface area of a sphere

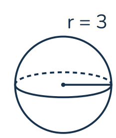
Find the surface area of each sphere. Use π = 3.14.

1



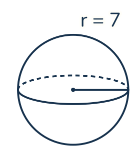
____________ cm^2

2



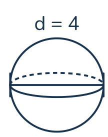
____________ cm^2

3



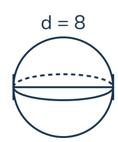
____________ cm^2

4



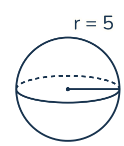
____________ cm^2

5



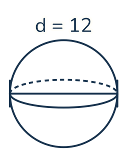
____________ cm^2

6



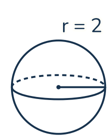
____________ cm^2

7



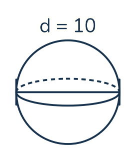
____________ cm^2

8



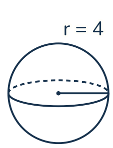
____________ cm^2

9



____________ cm^2

10

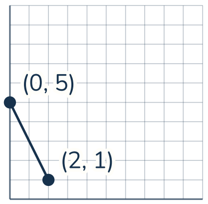


____________ cm^2

36. Midpoint of a segment

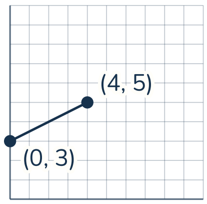
Find the midpoint of each segment.

11



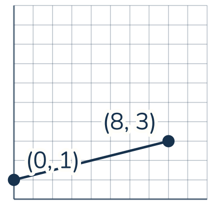
(______ , ______)

12



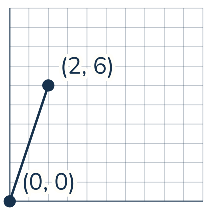
(______ , ______)

13



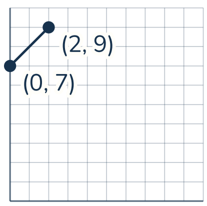
(______ , ______)

14



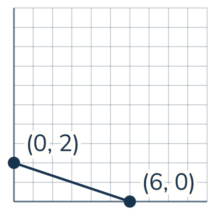
(______ , ______)

15



(______ , ______)

16



(______ , ______)

Answer Key (1 of 5)

Angle Pairs

Angles on a line and around a point

1. 180° (55°, 50°, x): 75°
2. 180° (45°, x, 95°): 40°
3. 180° (x, 45°, 70°): 65°
4. 180° (55°, 35°, x): 90°
5. 180° (x, 35°, 40°): 105°
6. 180° (65°, x, 50°): 65°
7. 180° (70°, 55°, x): 55°
8. 180° (x, 85°, 35°): 60°
9. 180° (x, 75°, 45°): 60°
10. 180° (x, 55°, 45°): 80°
11. 180° (50°, x, 70°): 60°
12. 180° (x, 45°, 50°): 85°

Exterior Angles

The exterior angle of a triangle

1. 25° + 115°: 140°
2. 55° + 85°: 140°
3. 40° + 55°: 95°
4. 25° + 85°: 110°
5. 70° + 80°: 150°
6. 50° + 100°: 150°
7. 50° + 105°: 155°
8. 45° + 105°: 150°
9. 75° + 75°: 150°
10. 30° + 110°: 140°
11. 45° + 50°: 95°
12. 35° + 85°: 120°

Angles in Polygons

Angles inside a polygon

1. 8-sided, all the angles: 1080°
2. 4-sided, all the angles: 360°
3. 10-sided, all the angles: 1440°
4. 5-sided, all the angles: 540°
5. 6-sided, all the angles: 720°
6. 3-sided, all the angles: 180°
7. 9-sided, all the angles: 1260°
8. 7-sided, all the angles: 900°

The Triangle Inequality

Can these be the sides of a triangle?

1. 6, 10, 11: Yes
2. 7, 8, 9: Yes
3. 4, 6, 12: No
4. 11, 16, 16: Yes

The Triangle Inequality (continued)

Can these be the sides of a triangle? (continued)

5. 9, 10, 11: Yes
6. 14, 14, 14: Yes
7. 3, 11, 16: No
8. 10, 11, 13: Yes
9. 13, 13, 13: Yes
10. 8, 13, 16: Yes
11. 3, 15, 15: Yes
12. 10, 12, 13: Yes

Angles on a line and around a point

13. 180° (55°, 50°, x): 75°
14. 180° (70°, x, 50°): 60°
15. 180° (x, 50°, 60°): 70°
16. 180° (40°, x, 100°): 40°
17. 180° (35°, 70°, x): 75°
18. 180° (35°, 55°, x): 90°

Isosceles Triangles

Angles in an isosceles triangle

1. apex 90°: 45°
2. apex 60°: 60°
3. base 70°: 40°
4. apex 40°: 70°
5. apex 80°: 50°
6. apex 120°: 30°
7. base 30°: 120°
8. apex 100°: 40°
9. apex 50°: 65°
10. apex 30°: 75°

The exterior angle of a triangle

11. 55° + 75°: 130°
12. 50° + 90°: 140°
13. 40° + 55°: 95°
14. 50° + 60°: 110°
15. 45° + 55°: 100°
16. 55° + 65°: 120°

Midsegments

The midsegment of a triangle

1. base 28: 14 cm
2. base 20: 10 cm
3. midsegment 17: 34 cm
4. base 6: 3 cm
5. base 14: 7 cm
6. base 10: 5 cm
7. midsegment 10: 20 cm

Answer Key (2 of 5)

Midsegments (continued)

The midsegment of a triangle (continued)

8. base 18: 9 cm
9. base 32: 16 cm
10. base 40: 20 cm

Angles inside a polygon

11. 3-sided, all the angles: 180°
12. 4-sided, all the angles: 360°
13. 10-sided, all the angles: 1440°
14. 6-sided, all the angles: 720°
15. 7-sided, all the angles: 900°
16. 5-sided, all the angles: 540°

Congruence Criteria

Which congruence rule (SSS, SAS, ASA, AAS, HL)

1. marked 3 sides: SSS
2. marked 2 sides, 1 angle: SAS
3. marked a right angle, 2 sides: HL
4. marked 3 sides: SSS
5. marked 2 sides, 1 angle: SAS
6. marked 3 sides: SSS
7. marked a right angle, 2 sides: HL
8. marked 3 sides: SSS
9. marked 2 sides, 1 angle: SAS
10. marked 3 sides: SSS

Can these be the sides of a triangle?

11. 11, 12, 13: Yes
12. 10, 11, 13: Yes
13. 6, 8, 12: Yes
14. 9, 12, 12: Yes
15. 9, 12, 14: Yes
16. 10, 13, 13: Yes

Proofs with Triangles

Complete the triangle congruence proof

1. ΔABC ≅ ΔADC (AB ≅ AD, CB ≅ CD), step 3: Reflexive Property
2. ΔRSV ≅ ΔTUV (RS ∥ UT, RS ≅ UT), step 1: Alternate Interior Angles
3. ΔPQR ≅ ΔSTU (∠P ≅ ∠S, PQ ≅ ST, ∠Q ≅ ∠T), step 4: ASA
4. ΔGLH ≅ ΔJLK (GL ≅ JL, HL ≅ KL), step 2: Vertical Angles

Proofs with Similar Triangles

Complete the similar-triangle proof

1. ΔRUV ~ ΔRST (RU/RS = RV/RT), step 1: Reflexive Property

Proofs with Similar Triangles (continued)

Complete the similar-triangle proof (continued)

2. ΔGHL ~ ΔJKL (∠GHL ≅ ∠JKL), step 1: Given
3. ΔSPR ~ ΔPQR (∠QPR is a right angle, PS ⊥ QR), step 3: AA ~
4. ΔADE ~ ΔABC (∠ADE ≅ ∠B), step 2: Given

Writing a Proof

Write a triangle proof

1. ΔGHJ ≅ ΔKLM — 1. GH ≅ KL — Given; 2. ∠G ≅ ∠K — Given; 3. GJ ≅ KM — Given; 4. ΔGHJ ≅ ΔKLM — SAS
2. ΔGHJ ≅ ΔKLM — 1. ∠G ≅ ∠K — Given; 2. GH ≅ KL — Given; 3. ∠H ≅ ∠L — Given; 4. ΔGHJ ≅ ΔKLM — ASA
3. ΔGLH ≅ ΔJLK — 1. HL ≅ KL — Definition of Midpoint; 2. ∠GLH ≅ ∠JLK — Vertical Angles; 3. GL ≅ JL — Given; 4. ΔGLH ≅ ΔJLK — SAS
4. ΔGHJ ≅ ΔKLM — 1. ∠H ≅ ∠L — Given; 2. GJ ≅ KM — Given; 3. GH ≅ KL — Given; 4. ΔGHJ ≅ ΔKLM — HL

Similar Figures

Similar figures and scale

1. 7 by 4 and 14 by x: 8
2. 8 by 6 and 24 by x: 18
3. 11 by x and 22 by 6: 3
4. 7 by 2 and 14 by x: 4
5. 8 by 2 and 16 by x: 4
6. 9 by 6 and 18 by x: 12
7. 6 by 3 and 24 by x: 12
8. 6 by 5 and 12 by x: 10
9. 9 by 5 and 18 by x: 10
10. 7 by 6 and 21 by x: 18

Angles in an isosceles triangle

11. apex 90°: 45°
12. apex 50°: 65°
13. base 30°: 120°
14. apex 110°: 35°
15. apex 40°: 70°
16. apex 30°: 75°

Right Triangle Trigonometry

Right-triangle trigonometry

1. 35°, opp 19 → adj x = 27.1
2. 45°, adj 11 → opp x = 11
3. 25°, opp 8 → hyp x = 18.9
4. 55°, hyp 18 → adj x = 10.3
5. 50°, hyp 11 → opp x = 8.4
6. 55°, opp 14 → hyp x = 17.1
7. 65°, hyp 16 → adj x = 6.8

Right Triangle Trigonometry (continued)

Right-triangle trigonometry (continued)

8. 35°, hyp 15 → adj x = 12.3
9. 50°, opp 10 → hyp x = 13.1
10. 35°, opp 9 → adj x = 12.9

The midsegment of a triangle

11. base 14: 7 cm
12. base 8: 4 cm
13. midsegment 3: 6 cm
14. base 22: 11 cm
15. base 40: 20 cm
16. base 30: 15 cm

Trigonometric Ratios

Write a trigonometric ratio

1. sin A (legs 8, 15, hyp 17) = 8/17
2. sin A (legs 3, 4, hyp 5) = 3/5
3. tan B (legs 15, 8, hyp 17) = 15/8
4. sin A (legs 5, 12, hyp 13) = 5/13
5. sin B (legs 12, 5, hyp 13) = 12/13
6. sin A (legs 12, 35, hyp 37) = 12/37
7. tan B (legs 40, 9, hyp 41) = 40/9
8. cos A (legs 8, 15, hyp 17) = 15/17
9. sin B (legs 35, 12, hyp 37) = 35/37
10. cos B (legs 21, 20, hyp 29) = 20/29

Which congruence rule (SSS, SAS, ASA, AAS, HL)

11. marked 3 sides: SSS
12. marked 3 sides: SSS
13. marked 1 side, 2 angles: ASA
14. marked 2 sides, 1 angle: SAS
15. marked 2 sides, 1 angle: SAS
16. marked 3 sides: SSS

The Laws of Sines and Cosines

Laws of sines and cosines

1. In $\triangle ABC$, $a = 6$, $\angle A = 60°$, $\angle B = 45°$. Find b: 4.9
2. In $\triangle ABC$, $a = 9$, $\angle A = 45°$, $\angle B = 55°$. Find b: 10.4
3. In $\triangle ABC$, $a = 7$, $\angle A = 40°$, $\angle B = 60°$. Find b: 9.4
4. In $\triangle ABC$, $a = 10$, $\angle A = 50°$, $\angle B = 70°$. Find b: 12.3
5. In $\triangle ABC$, $a = 12$, $\angle A = 40°$, $\angle B = 50°$. Find b: 14.3
6. In $\triangle ABC$, $a = 9$, $\angle A = 65°$, $\angle B = 40°$. Find b: 6.4
7. In $\triangle ABC$, $a = 5$, $\angle A = 55°$, $\angle B = 65°$. Find b: 5.5
8. In $\triangle ABC$, $a = 11$, $\angle A = 30°$, $\angle B = 75°$. Find b: 21.3

Similar figures and scale

9. 5 by 3 and 15 by x: 9
10. 11 by 8 and 22 by x: 16
11. 9 by x and 18 by 4: 2

The Laws of Sines and Cosines (continued)

Similar figures and scale (continued)

12. 8 by 2 and 24 by x: 6
13. 5 by 2 and 15 by x: 6
14. 11 by 3 and 22 by x: 6

Parts of a Circle

Arc length and sector area

1. r = 3, 180°, area: 14.13 cm²
2. r = 12, 180°, arc: 37.68 cm
3. r = 9, 270°, arc: 42.39 cm
4. r = 4, 90°, area: 12.56 cm²
5. r = 2, 270°, area: 9.42 cm²
6. r = 6, 90°, arc: 9.42 cm

Right-triangle trigonometry

7. 55°, adj 7 → hyp x = 12.2
8. 40°, opp 12 → hyp x = 18.7
9. 60°, adj 6 → hyp x = 12
10. 50°, hyp 16 → opp x = 12.3
11. 55°, opp 17 → adj x = 11.9
12. 50°, hyp 11 → opp x = 8.4

Inscribed Angles

Inscribed angle in a circle

1. center 60° → circle 30°
2. center 40° → circle 20°
3. center 100° → circle 50°
4. center 80° → circle 40°
5. center 30° → circle 15°
6. center 50° → circle 25°
7. center 90° → circle 45°
8. center 70° → circle 35°
9. center 110° → circle 55°
10. center 170° → circle 85°

Write a trigonometric ratio

11. sin B (legs 24, 7, hyp 25) = 24/25
12. sin B (legs 35, 12, hyp 37) = 35/37
13. tan B (legs 12, 5, hyp 13) = 12/5
14. cos B (legs 15, 8, hyp 17) = 8/17
15. cos A (legs 8, 15, hyp 17) = 15/17
16. cos B (legs 4, 3, hyp 5) = 3/5

Triangles and Circles

Circles of a right triangle

1. 9, 12, 15 — through the corners: 7.5 cm
2. 12, 16, 20 — through the corners: 10 cm
3. 10, 24, 26 — through the corners: 13 cm

Answer Key (4 of 5)

Triangles and Circles (continued)

Circles of a right triangle (continued)

4. 6, 8, 10 — through the corners: 5 cm
5. 8, 15, 17 — through the corners: 8.5 cm
6. 7, 24, 25 — through the corners: 12.5 cm
7. 21, 28, 35 — through the corners: 17.5 cm
8. 3, 4, 5 — through the corners: 2.5 cm
9. 15, 20, 25 — through the corners: 12.5 cm
10. 5, 12, 13 — through the corners: 6.5 cm

Laws of sines and cosines

11. In △ABC, a = 6, ∠A = 60°, ∠B = 45°. Find b: 4.9
12. In △ABC, a = 9, ∠A = 45°, ∠B = 55°. Find b: 10.4
13. In △ABC, a = 7, ∠A = 40°, ∠B = 60°. Find b: 9.4
14. In △ABC, a = 10, ∠A = 50°, ∠B = 70°. Find b: 12.3
15. In △ABC, a = 12, ∠A = 40°, ∠B = 50°. Find b: 14.3
16. In △ABC, a = 9, ∠A = 65°, ∠B = 40°. Find b: 6.4

Midpoints

Midpoint of a segment

1. (0, 5) and (6, 3): (3, 4)
2. (0, 8) and (2, 4): (1, 6)
3. (0, 4) and (10, 2): (5, 3)
4. (0, 6) and (4, 8): (2, 7)
5. (0, 0) and (6, 2): (3, 1)
6. (0, 9) and (2, 7): (1, 8)
7. (0, 1) and (2, 9): (1, 5)
8. (0, 9) and (2, 3): (1, 6)
9. (0, 8) and (4, 10): (2, 9)
10. (0, 6) and (6, 4): (3, 5)
11. (0, 1) and (10, 3): (5, 2)
12. (0, 10) and (4, 6): (2, 8)

Arc length and sector area

13. r = 9, 180°, arc: 28.26 cm
14. r = 8, 180°, arc: 25.12 cm
15. r = 6, 30°, area: 9.42 cm²
16. r = 7, 90°, arc: 10.99 cm
17. r = 7, 180°, arc: 21.98 cm
18. r = 10, 90°, arc: 15.7 cm

Parts of a Solid

Faces, edges and vertices

1. pentagonal prism, faces: 7
2. triangular prism, faces: 5
3. cuboid, edges: 12
4. hexagonal pyramid, faces: 7
5. pentagonal prism, vertices: 10
6. hexagonal prism, faces: 8

Parts of a Solid (continued)

Inscribed angle in a circle

7. center 40° → circle 20°
8. center 30° → circle 15°
9. center 130° → circle 65°
10. center 50° → circle 25°
11. center 80° → circle 40°
12. center 60° → circle 30°

Missing Edges

Missing edge from the volume

1. V 108: 9 × x × 6 → x = 2
2. V 16: 4 × 2 × x → x = 2
3. V 162: 9 × 2 × x → x = 9
4. V 120: 6 × 5 × x → x = 4
5. V 84: 2 × x × 7 → x = 6
6. V 56: 7 × 4 × x → x = 2
7. V 132: x × 2 × 11 → x = 6
8. V 48: 6 × x × 4 → x = 2
9. V 48: 4 × 4 × x → x = 3
10. V 108: 6 × x × 3 → x = 6

Circles of a right triangle

11. 15, 20, 25 — through the corners: 12.5 cm
12. 7, 24, 25 — through the corners: 12.5 cm
13. 20, 21, 29 — through the corners: 14.5 cm
14. 8, 15, 17 — through the corners: 8.5 cm
15. 5, 12, 13 — through the corners: 6.5 cm
16. 12, 16, 20 — through the corners: 10 cm

Surface Area of a Sphere

Surface area of a sphere

1. d = 6: 113.04 cm²
2. r = 3: 113.04 cm²
3. r = 7: 615.44 cm²
4. d = 4: 50.24 cm²
5. d = 8: 200.96 cm²
6. r = 5: 314 cm²
7. d = 12: 452.16 cm²
8. r = 2: 50.24 cm²
9. d = 10: 314 cm²
10. r = 4: 200.96 cm²

Midpoint of a segment

11. (0, 5) and (2, 1): (1, 3)
12. (0, 3) and (4, 5): (2, 4)
13. (0, 1) and (8, 3): (4, 2)
14. (0, 0) and (2, 6): (1, 3)
15. (0, 7) and (2, 9): (1, 8)

Answer Key (5 of 5)

Surface Area of a Sphere (continued)

Midpoint of a segment (continued)

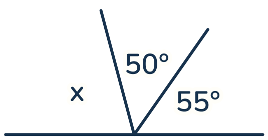
16. (0, 2) and (6, 0): (3, 1)

Solutions (1 of 25)

Angle Pairs

Angles on a line and around a point

1.



___ degrees

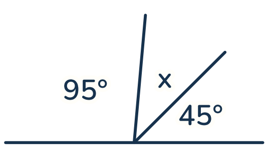
$55° + 50° + x$	=	**180°**
$55° + 50°$	=	**105°**
$180° - 105°$	=	**75°**

The angles on a straight line add up to 180°, so all three together make 180°.

Add the two you are given first: 55 + 50 = 105.

Then take that from 180°: 180 − 105 = 75. So x = 75°.

2.



___ degrees

$45° + 95° + x$	=	**180°**
$45° + 95°$	=	**140°**
$180° - 140°$	=	**40°**

The angles on a straight line add up to 180°, so all three together make 180°.

Add the two you are given first: 45 + 95 = 140.

Then take that from 180°: 180 − 140 = 40. So x = 40°.

Solutions (2 of 25)

Exterior Angles

The exterior angle of a triangle

1.

____ degrees

x	=	**25°**	+	**115°**
25°	+	115°	=	**140°**
	x	=	**140°**	

An exterior angle of a triangle equals the sum of the two remote interior angles — the two it does not sit against. Here those are 25° and 115°.

Add them: 25 + 115 = 140.

So x = 140°.

2.

____ degrees

x	=	**55°**	+	**85°**
55°	+	85°	=	**140°**
	x	=	**140°**	

An exterior angle of a triangle equals the sum of the two remote interior angles — the two it does not sit against. Here those are 55° and 85°.

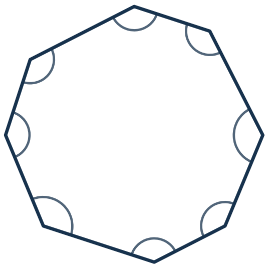
Add them: 55 + 85 = 140.

So x = 140°.

Angles in Polygons

Angles inside a polygon

1.



All the angles together

______ degrees

1080°

There is no separate rule for each shape. Pick one corner of the octagon and draw a line from it to every corner it is not already joined to — the whole shape falls into triangles, and you already know what a triangle's angles come to.

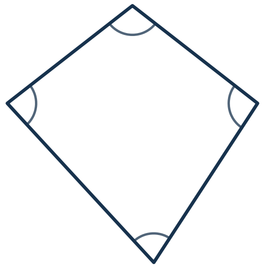
Counting them: the corner you started from reaches every other corner except its own two neighbors, so an 8-sided shape splits into 8 – 2 = 6 triangles.

Every triangle's angles add to 180°, and between them the triangles use up every angle of the shape and nothing else. So the whole lot comes to 6 × 180 = 1080°.

So the angles of an octagon add up to 1080°.

Angles in Polygons (continued)

Angles inside a polygon (continued)

2. 

All the angles together

____ degrees

360°

There is no separate rule for each shape. Pick one corner of the quadrilateral and draw a line from it to every corner it is not already joined to — the whole shape falls into triangles, and you already know what a triangle's angles come to.

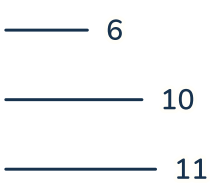
Counting them: the corner you started from reaches every other corner except its own two neighbors, so a 4-sided shape splits into 4 – 2 = 2 triangles.

Every triangle's angles add to 180°, and between them the triangles use up every angle of the shape and nothing else. So the whole lot comes to 2 × 180 = 360°.

So the angles of a quadrilateral add up to 360°.

The Triangle Inequality

Can these be the sides of a triangle?

1. 

Yes No

Yes

Three lengths make a triangle only when the two shorter sides add to more than the longest. The two shorter are 6 and 10; the longest is 11.

6 + 10 = 16, which is more than 11.

The two shorter sides reach past the longest, so they close into a triangle. Yes.

The Triangle Inequality (continued)

Can these be the sides of a triangle? (continued)

2.

7

8

9

Yes No

Yes

Three lengths make a triangle only when the two shorter sides add to more than the longest. The two shorter are 7 and 8; the longest is 9.

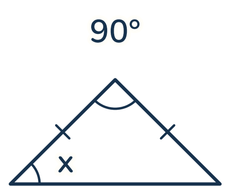
7 + 8 = 15, which is more than 9.

The two shorter sides reach past the longest, so they close into a triangle. Yes.

Isosceles Triangles

Angles in an isosceles triangle

1.



___ degrees

base	=	**base**	and
sum	=	180°	
90	÷ 2	**=**	45
x	=	**45°**	

The two ticked sides are equal, so the two base angles are equal — and the three angles still add to 180°.

The apex is 90°, so the two base angles share the other 90° equally: (180 – 90) ÷ 2 = 45.

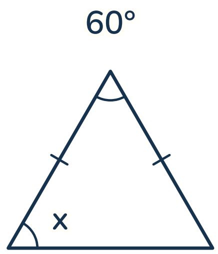
So x = 45°.

Solutions (6 of 25)

Isosceles Triangles (continued)

Angles in an isosceles triangle (continued)

2.



___ degrees

base = base and

sum = 180°

120 ÷ 2 = 60

x = **60°**

The two ticked sides are equal, so the two base angles are equal — and the three angles still add to 180°.

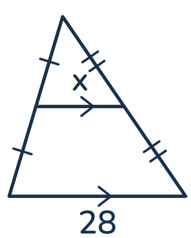
The apex is 60°, so the two base angles share the other 120° equally: (180 – 60) ÷ 2 = 60.

So x = 60°.

Midsegments

The midsegment of a triangle

1.



___ cm

midsegment =

half the base

28 ÷ 2 = **14**

x = **14 cm**

A midsegment joins the midpoints of two sides, and it is half the length of the third side — so the base is always twice the midsegment.

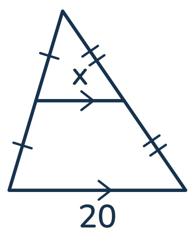
The base is 28 cm, so the midsegment is half of it: 28 ÷ 2 = 14.

So x = 14 cm.

Midsegments (continued)

The midsegment of a triangle (continued)

2.



___ cm

midsegment =
half the base
20 ÷ 2 = **10**
x = **10 cm**

A midsegment joins the midpoints of two sides, and it is half the length of the third side — so the base is always twice the midsegment.

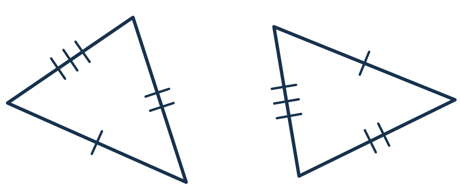
The base is 20 cm, so the midsegment is half of it: 20 ÷ 2 = 10.

So x = 10 cm.

Congruence Criteria

Which congruence rule (SSS, SAS, ASA, AAS, HL)

1.



SSS SAS ASA AAS HL

SSS

Read the marks first — they are the givens. Here there are three pairs of equal sides, and no angles.

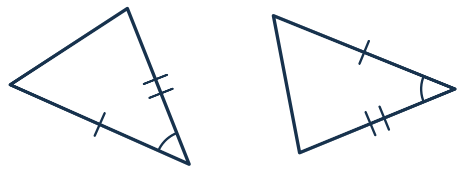
Three pairs of equal sides pin a triangle down completely — there is only one triangle with those three lengths.

So the rule is SSS.

Congruence Criteria (continued)

Which congruence rule (SSS, SAS, ASA, AAS, HL) (continued)

2.



SSS | SAS | ASA | AAS | HL

SAS

Read the marks first — they are the givens. Here there are two pairs of equal sides and one pair of equal angles.

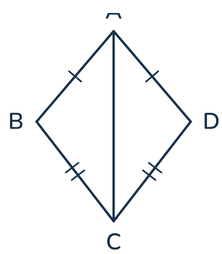
The marked angle sits BETWEEN the two marked sides — an included angle — and two sides with the angle between them fix the third side.

So the rule is SAS.

Proofs with Triangles

Complete the triangle congruence proof

1.



Given AB ≅ AD; CB ≅ CD

Prove ΔABC ≅ ΔADC

Statements	Reasons
1. AB ≅ AD	Given
2. CB ≅ CD	Given
3. AC ≅ AC	?
4. ΔABC ≅ ΔADC	SSS

Reason for step 3:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

Step 3: **AC ≅ AC**

AC ≅ AC →

Reflexive Property

Reflexive Property

The proof is missing the reason for step 3: "AC ≅ AC". Read what that step claims.

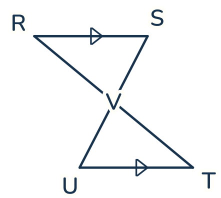
A segment (or angle) is congruent to itself. The two triangles share this side, so it is equal in both by the reflexive property.

So the missing reason is Reflexive Property.

Proofs with Triangles (continued)

Complete the triangle congruence proof (continued)

2.



Given RS ∥ UT; RS ≅ UT

Prove ΔRSV ≅ ΔTUV

Statements	Reasons
1. ∠SRV ≅ ∠UTV	?
2. ∠RSV ≅ ∠TUV	Alternate Interior Angles
3. RS ≅ UT	Given
4. ΔRSV ≅ ΔTUV	ASA

Reason for step 1:

Given | Reflexive Property | Vertical Angles | Alternate Interior Angles | Definition of Midpoint | SSS | SAS | ASA | AAS | HL | Definition of Perpendicular

Step 1: ∠SRV ≅ ∠UTV

∠SRV ≅ ∠UTV →

Alternate Interior Angles

Alternate Interior Angles

The proof is missing the reason for step 1: "∠SRV ≅ ∠UTV". Read what that step claims.

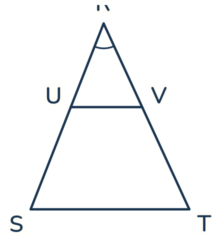
The two lines are parallel, so a transversal makes alternate interior angles congruent.

So the missing reason is Alternate Interior Angles.

Proofs with Similar Triangles

Complete the similar-triangle proof

1.



Given RU/RS = RV/RT

Prove ΔRUV ~ ΔRST

Statements	Reasons
1. ∠R ≅ ∠R	?
2. RU/RS = RV/RT	Given
3. ΔRUV ~ ΔRST	SAS ~

Reason for step 1:

Given · Reflexive Property · Vertical Angles · Corresponding Angles · Alternate Interior Angles · AA ~ · SAS ~ · SSS ~

Step 1: **∠R ≅ ∠R**

∠R ≅ ∠R →

Reflexive Property

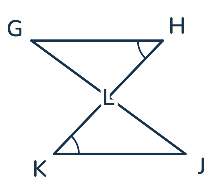
Reflexive Property

The proof is missing the reason for step 1: "∠R ≅ ∠R". Read what that step claims.

A segment (or angle) is congruent to itself. The two triangles share this side, so it is equal in both by the reflexive property.

So the missing reason is Reflexive Property.

2.



Given ∠GHL ≅ ∠JKL

Prove ΔGHL ~ ΔJKL

Statements	Reasons
1. ∠GHL ≅ ∠JKL	?
2. ∠GLH ≅ ∠JLK	Vertical Angles
3. ΔGHL ~ ΔJKL	AA ~

Reason for step 1:

Given · Reflexive Property · Vertical Angles · Corresponding Angles · Alternate Interior Angles · AA ~ · SAS ~ · SSS ~

Step 1: **∠GHL ≅ ∠JKL**

∠GHL ≅ ∠JKL → **Given**

Given

The proof is missing the reason for step 1: "∠GHL ≅ ∠JKL". Read what that step claims.

This fact is handed to you in the Given — nothing has to justify it.

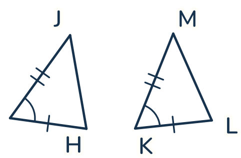
So the missing reason is Given.

Solutions (11 of 25)

Writing a Proof

Write a triangle proof

1.



Given GH ≅ KL; ∠G ≅ ∠K; GJ ≅ KM

Prove ΔGHJ ≅ ΔKLM

Statements	Reasons
1.	
2.	
3.	
4.	

Prove: **ΔGHJ ≅ ΔKLM**

1. GH ≅ KL → **Given**
2. ∠G ≅ ∠K → **Given**
3. GJ ≅ KM → **Given**
4. ΔGHJ ≅ ΔKLM → **SAS**

You are given GH ≅ KL; ∠G ≅ ∠K; GJ ≅ KM, and you have to prove ΔGHJ ≅ ΔKLM. Build it one line at a time — each line is a statement and the reason it is true.

1. GH ≅ KL — This fact is handed to you in the Given — nothing has to justify it.

2. ∠G ≅ ∠K — This fact is handed to you in the Given — nothing has to justify it.

3. GJ ≅ KM — This fact is handed to you in the Given — nothing has to justify it.

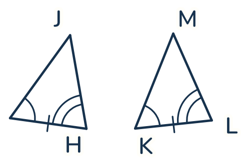
4. ΔGHJ ≅ ΔKLM — Two pairs of sides and the angle between them are equal: SAS.

That is the whole proof: the last line, ΔGHJ ≅ ΔKLM, follows by SAS.

Writing a Proof (continued)

Write a triangle proof (continued)

2.



Given ∠G ≅ ∠K; GH ≅ KL; ∠H ≅ ∠L

Prove ΔGHJ ≅ ΔKLM

Statements	Reasons
1.	
2.	
3.	
4.	

Prove: ΔGHJ ≅ ΔKLM

1. ∠G ≅ ∠K → Given
2. GH ≅ KL → Given
3. ∠H ≅ ∠L → Given
4. ΔGHJ ≅ ΔKLM → ASA

You are given ∠G ≅ ∠K; GH ≅ KL; ∠H ≅ ∠L, and you have to prove ΔGHJ ≅ ΔKLM. Build it one line at a time — each line is a statement and the reason it is true.

1. ∠G ≅ ∠K — This fact is handed to you in the Given — nothing has to justify it.

2. GH ≅ KL — This fact is handed to you in the Given — nothing has to justify it.

3. ∠H ≅ ∠L — This fact is handed to you in the Given — nothing has to justify it.

4. ΔGHJ ≅ ΔKLM — Two pairs of angles and the side between them are equal: ASA.

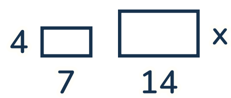
That is the whole proof: the last line, ΔGHJ ≅ ΔKLM, follows by ASA.

Solutions (13 of 25)

Similar Figures

Similar figures and scale

1.



___ cm

8 cm

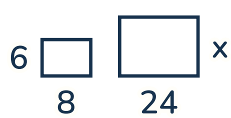
Two shapes, the same shape at two sizes. Every side of the big one is the matching side of the small one multiplied by the SAME number — so the first job is to pair the sides up: long with long, short with short.

Use the pair you can see both of. 7 became 14, and $14 \div 7 = 2$ — so every side is 2 times bigger.

Now the missing side is on the BIG shape, so it grows: $4 \times 2 = 8$.

Check it against the picture: the answer should be bigger than 4, and 8 is. So the missing side is 8 cm.

2.



___ cm

18 cm

Two shapes, the same shape at two sizes. Every side of the big one is the matching side of the small one multiplied by the SAME number — so the first job is to pair the sides up: long with long, short with short.

Use the pair you can see both of. 8 became 24, and $24 \div 8 = 3$ — so every side is 3 times bigger.

Now the missing side is on the BIG shape, so it grows: $6 \times 3 = 18$.

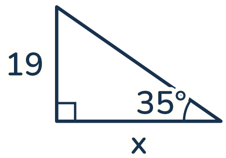
Check it against the picture: the answer should be bigger than 6, and 18 is. So the missing side is 18 cm.

Solutions (14 of 25)

Right Triangle Trigonometry

Right-triangle trigonometry

1.



____ cm

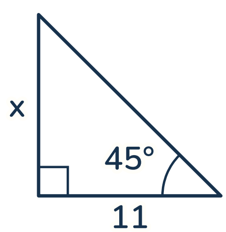
opposite ÷ adjacent =
tangent
tan 35° = 19 ÷ x
x = **27.1**

The angle is 35°. You know the opposite (19) and you want the adjacent (x). Those two sides are the opposite and the adjacent, so the ratio is the tangent.

Write it with the numbers: tan 35° = 19 ÷ x.

Rearrange for x and work it out on a calculator: x = 19 ÷ tan 35° = 27.1.

2.



____ cm

opposite ÷ adjacent =
tangent
tan 45° = x ÷ 11
x = **11**

The angle is 45°. You know the adjacent (11) and you want the opposite (x). Those two sides are the opposite and the adjacent, so the ratio is the tangent.

Write it with the numbers: tan 45° = x ÷ 11.

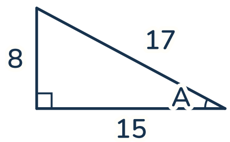
Rearrange for x and work it out on a calculator: x = 11 × tan 45° = 11.

Solutions (15 of 25)

Trigonometric Ratios

Write a trigonometric ratio

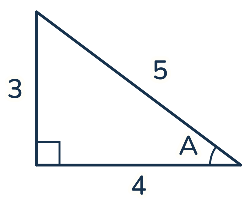
1.



sin A = ____

sin A = **8/17**	sin A is opposite over hypotenuse. Name the two sides you need, measured from angle A. Read them off the triangle: the opposite side of angle A is 8, the hypotenuse is 17. So sin A = 8/17.

2.



sin A = ____

sin A = **3/5**	sin A is opposite over hypotenuse. Name the two sides you need, measured from angle A. Read them off the triangle: the opposite side of angle A is 3, the hypotenuse is 5. So sin A = 3/5.

The Laws of Sines and Cosines

Laws of sines and cosines

1. In $\triangle ABC$, $a = 6$, $\angle A = 60°$, $\angle B = 45°$. Find b: ____

b = **a · sin B / sin A** b = **6 · sin 45° / sin 60°** b ≈ **4.9**	Two angles and the side opposite one — that is the law of sines: a/sin A = b/sin B. Substitute a = 6, A = 60°, B = 45°. Evaluate and round to one place: b ≈ 4.9.

The Laws of Sines and Cosines (continued)

Laws of sines and cosines (continued)

2. In ΔABC, $a = 9$, $\angle A = 45°$, $\angle B = 55°$. Find b: ______

$$b = a \cdot \sin B / \sin A$$
$$b = 9 \cdot \sin 55° / \sin 45°$$
$$b \approx 10.4$$

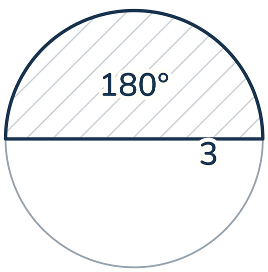
Two angles and the side opposite one — that is the law of sines: a/sin A = b/sin B.

Substitute a = 9, A = 45°, B = 55°.

Evaluate and round to one place: b ≈ 10.4.

Parts of a Circle

Arc length and sector area

1. 

Area of the slice

______ cm²

$$180/360 = 1/2$$
$$3.14 \times 3 \times 3 = 28.26$$
$$1/2 \text{ of } 28.26 = 14.13$$
$$14.13 \text{ cm}^2$$

A slice is a FRACTION of the whole circle, and the angle says which fraction. A full turn is 360°, so 180° is 180 out of 360 — that is 1/2 of the circle. Write that down before touching a formula.

Now the whole circle, as if there were no slice. Its area is π × r × r — only the r is squared: 3.14 × 3 × 3 = 28.26.

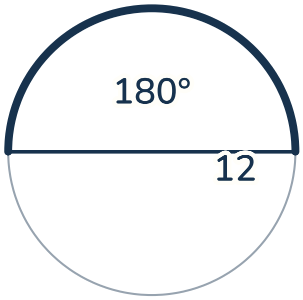
Then take the slice's share of it: 1/2 of 28.26 is 14.13.

So the area of the slice is 14.13 cm². Check the unit against what was asked — a curved edge is a length and a slice is an area, and getting cm² where you expected the other one means you answered the other question.

Parts of a Circle (continued)

Arc length and sector area (continued)

2.



Length of the curved edge

_______ cm

180/360		=	**1/2**
2 × 3.14 × 12		=	**75.36**
1/2	of 75.36	=	**37.68**

37.68 cm

A slice is a FRACTION of the whole circle, and the angle says which fraction. A full turn is 360°, so 180° is 180 out of 360 — that is 1/2 of the circle. Write that down before touching a formula.

Now the whole circle, as if there were no slice. All the way round is 2 × π × r: 2 × 3.14 × 12 = 75.36 cm.

Then take the slice's share of it: 1/2 of 75.36 is 37.68.

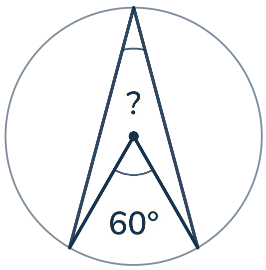
So the curved edge is 37.68 cm. Check the unit against what was asked — a curved edge is a length and a slice is an area, and getting cm where you expected the other one means you answered the other question.

Solutions (18 of 25)

Inscribed Angles

Inscribed angle in a circle

1.



___ degrees

same arc, two angles

center = 2 × circle so

circle = center ÷ 2

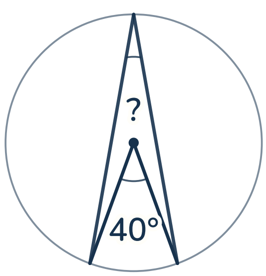
60 ÷ 2 = 30

Both marked angles stand on the SAME arc — the two chords and the two radii end at the same two points on the circle. That is what makes them a pair; two angles in a circle that do not share an arc have nothing to say to each other.

The angle at the CENTER is always twice the one on the circle, wherever on the far arc that point sits. Here the center angle is the one given, so this is a halving.

Half of 60 is 30, so the angle on the circle is 30°.

2.



___ degrees

same arc, two angles

center = 2 × circle so

circle = center ÷ 2

40 ÷ 2 = 20

Both marked angles stand on the SAME arc — the two chords and the two radii end at the same two points on the circle. That is what makes them a pair; two angles in a circle that do not share an arc have nothing to say to each other.

The angle at the CENTER is always twice the one on the circle, wherever on the far arc that point sits. Here the center angle is the one given, so this is a halving.

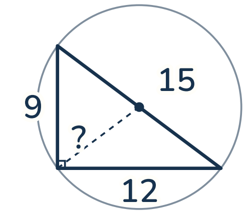
Half of 40 is 20, so the angle on the circle is 20°.

Solutions (19 of 25)

Triangles and Circles

Circles of a right triangle

1.



____ cm

right angle on the circle → **180° at the center**

the 15 cm side = **the diameter**

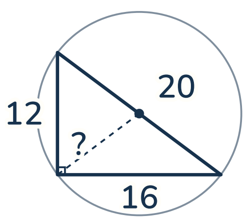
15 ÷ 2 = **7.5**

The corner between the two short sides is a RIGHT angle, and it is sitting on the circle looking straight across at the hypotenuse. An angle on a circle is half the one at the center, so the angle at the center here is 180° — a straight line.

A straight line through the center IS a diameter. So the hypotenuse is the diameter, and the center of the circle is its midpoint — not somewhere inside the triangle.

The radius is half the diameter: 15 ÷ 2 = 7.5. So the circle through all three corners has a radius of 7.5 cm, and the two short sides never come into it.

2.



____ cm

right angle on the circle → **180° at the center**

the 20 cm side = **the diameter**

20 ÷ 2 = **10**

The corner between the two short sides is a RIGHT angle, and it is sitting on the circle looking straight across at the hypotenuse. An angle on a circle is half the one at the center, so the angle at the center here is 180° — a straight line.

A straight line through the center IS a diameter. So the hypotenuse is the diameter, and the center of the circle is its midpoint — not somewhere inside the triangle.

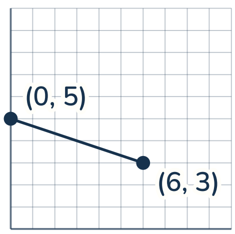
The radius is half the diameter: 20 ÷ 2 = 10. So the circle through all three corners has a radius of 10 cm, and the two short sides never come into it.

Solutions (20 of 25)

Midpoints

Midpoint of a segment

1.



(___, ___)

$(0 + 6) \div 2 = \boxed{3}$

$(5 + 3) \div 2 = \boxed{4}$

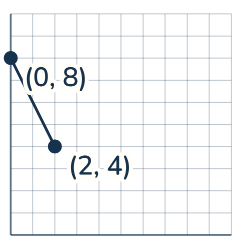
(3, 4)

The midpoint is halfway between, so average each coordinate on its own. The x's first: (0 + 6) ÷ 2 = 3.

Now the y's the same way: (5 + 3) ÷ 2 = 4.

So the midpoint is (3, 4).

2.



(___, ___)

$(0 + 2) \div 2 = \boxed{1}$

$(8 + 4) \div 2 = \boxed{6}$

(1, 6)

The midpoint is halfway between, so average each coordinate on its own. The x's first: (0 + 2) ÷ 2 = 1.

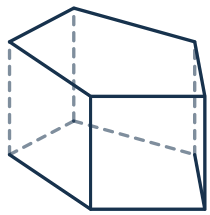
Now the y's the same way: (8 + 4) ÷ 2 = 6.

So the midpoint is (1, 6).

Parts of a Solid

Faces, edges and vertices

1.



How many faces?

___

pentagonal prism

base: pentagon **5 sides**

5 + 2 = **7**

faces: **7**

Half of a solid is round the back. The dashed lines in the drawing are the parts you cannot see, and they count exactly the same as the ones you can — counting only what faces you is where this goes wrong, every time.

Start by naming the base, because everything else is built on it. A pentagonal prism has a pentagon at the bottom, so the base has 5 sides.

A prism has the same shape at both ends and a flat rectangle down each side of it. That is 5 rectangles round the outside, plus the two ends: 5 + 2.

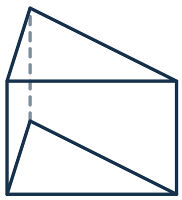
So a pentagonal prism has 7 faces.

Solutions (22 of 25)

Parts of a Solid (continued)

Faces, edges and vertices (continued)

2.



How many faces?

___

triangular prism

base: triangle **3 sides**

3 + 2 = **5**

faces: **5**

Half of a solid is round the back. The dashed lines in the drawing are the parts you cannot see, and they count exactly the same as the ones you can — counting only what faces you is where this goes wrong, every time.

Start by naming the base, because everything else is built on it. A triangular prism has a triangle at the bottom, so the base has 3 sides.

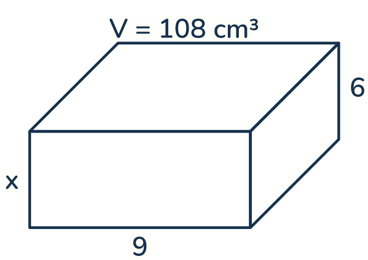
A prism has the same shape at both ends and a flat rectangle down each side of it. That is 3 rectangles round the outside, plus the two ends: 3 + 2.

So a triangular prism has 5 faces.

Missing Edges

Missing edge from the volume

1.



___ cm

$9 \times 6 \times x$ = **108**

9×6 = **54**

$108 \div 54$ = **2**

The volume of a box is its three edges multiplied: length × width × height. You have the width (9), the depth (6), and the volume (108).

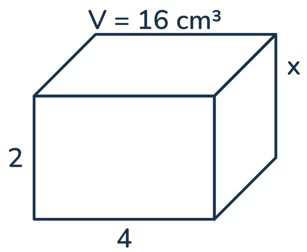
Multiply the two edges you know first: 9 × 6 = 54.

Then the missing edge is the volume divided by that: 108 ÷ 54 = 2.

Missing Edges (continued)

Missing edge from the volume (continued)

2.



___ cm

$4 \times 2 \times x$	=	**16**
4×2	=	**8**
$16 \div 8$	=	**2**

The volume of a box is its three edges multiplied: length × width × height. You have the width (4), the height (2), and the volume (16).

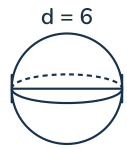
Multiply the two edges you know first: $4 \times 2 = 8$.

Then the missing edge is the volume divided by that: $16 \div 8 = 2$.

Surface Area of a Sphere

Surface area of a sphere

1.



______ cm^2

diameter: 6 cm

$6 \div 2 = \mathbf{3}$

$3 \times 3 = \mathbf{9}$

$12.56 \times 9 = \mathbf{113.04}$

113.04 cm^2

The 6 cm marked here goes right ACROSS the sphere — that is the diameter. The formula wants the radius, and using the diameter instead makes the answer FOUR times too big, because it is squared.

Half of 6 is 3, so $r = 3$ cm.

Square the radius on a line of its own. r^2 means $r \times r$, not $r \times 2$: $3 \times 3 = 9$.

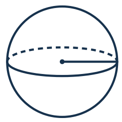
Now four times π, times that: 4×3.14 is 12.56, and $12.56 \times 9 = 113.04$. Nothing to round — 12.56 times a whole number always lands on a whole number of hundredths.

So the surface area is 113.04 square cm. Square, not cubic — a surface is skin rather than space, and the unit is the quickest way to notice you have answered the volume question instead.

Surface Area of a Sphere (continued)

Surface area of a sphere (continued)

2.



______ cm^2

radius: 3 cm

$3 \times 3 = 9$

$12.56 \times 9 = 113.04$

113.04 cm^2

The 3 cm marked here runs from the center to the surface, so it is already the radius — which is what the formula wants.

Square the radius on a line of its own. r^2 means $r \times r$, not $r \times 2$: $3 \times 3 = 9$.

Now four times π, times that: 4×3.14 is 12.56, and $12.56 \times 9 = 113.04$. Nothing to round — 12.56 times a whole number always lands on a whole number of hundredths.

So the surface area is 113.04 square cm. Square, not cubic — a surface is skin rather than space, and the unit is the quickest way to notice you have answered the volume question instead.