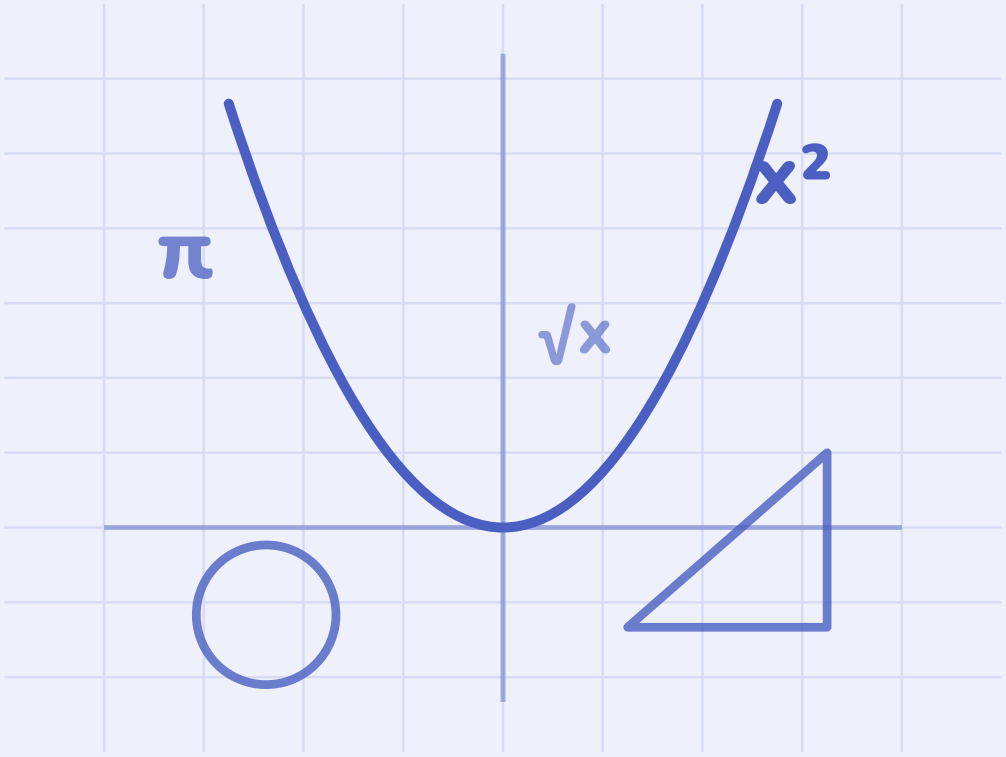
MATH PICNIC · WORKBOOK

Precalculus Limits Workbook

4 lessons · Ages 16–18 · answer key at the back



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1-1

Limits by Substitution

WORKED EXAMPLE

$$\lim_{x\to -1} \frac{-x-3}{x+2}$$

$$\lim_{x\to -1} \frac{-x-3}{x+2} = \frac{-(-1)-3}{(-1)+2}$$

$$= -2$$

1. Limits

Find each limit. Check the bottom first: if it is not 0 there, substitute.

1 $\lim_{x\to -1} (-x^2 + x + 3)$

2 $\lim_{x\to 1} (-x^2 + 2x - 2)$

3 $\lim_{x\to -3} (-x^2 - 3x - 1)$

4 $\lim_{x\to -1} (-x^2 - 2x + 2)$

5 $\lim_{x\to -2} \frac{-x+1}{x+3}$

6 $\lim_{x\to -2} (-x^2 + 2x + 1)$

7 $\lim_{x\to -1} (-x^2 - 3x - 3)$

8 $\lim_{x\to -1} \frac{-x+2}{x+2}$

9 $\lim_{x\to -2} \frac{x+3}{x-2}$

10 $\lim_{x\to -1} (x^2 - x - 3)$

11 $\lim_{x\to 3} \frac{-x-3}{x-1}$

12 $\lim_{x\to -1} (-x^2 - x + 2)$

2. The equation of a circle

Write the center and radius of each circle.

13 $(x-5)^2 + y^2 = 9$

14 $(x+2)^2 + (y-3)^2 = 4$

15 $(x-5)^2 + (y+5)^2 = 49$

16 $(x-1)^2 + (y-3)^2 = 16$

17 $(x+5)^2 + (y-6)^2 = 9$

18 $(x+1)^2 + (y-1)^2 = 36$

Limits by Factoring

WORKED EXAMPLE

$$\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6}$$

$x = 2$: $\frac{0}{0}$

$$\lim_{x\to 2}\frac{(x-2)(x-1)}{(x-2)(x-3)}$$

$$= \lim_{x\to 2}\frac{x-1}{x-3}$$

$$= \frac{1}{-1}$$

$$= -1$$

3. Limits

Substituting makes the top and bottom both 0: factor first, then cancel.

1 $\lim_{x\to -1}\frac{x^2-x-2}{x^2+x}$

2 $\lim_{x\to 1}\frac{x^2-x}{x^2-4x+3}$

3 $\lim_{x\to 3}\frac{x^2-2x-3}{x-3}$

4 $\lim_{x\to -2}\frac{x^2+2x}{x^2+3x+2}$

5 $\lim_{x\to -3}\frac{x^2+4x+3}{x^2-9}$

6 $\lim_{x\to -1}\frac{x^2+3x+2}{x^2-2x-3}$

7 $\lim_{x\to 2}\frac{x^2-5x+6}{x-2}$

8 $\lim_{x\to 1}\frac{x^2+2x-3}{x^2-x}$

9 $\lim_{x\to -1}\frac{x^2-x-2}{x^2-1}$

10 $\lim_{x\to -2}\frac{x^2-4}{x^2-x-6}$

4. The equation of a circle

Complete the square to find the center and radius of each circle.

11 $x^2 + y^2 + 2x - 12y + 28 = 0$

12 $x^2 + y^2 - 10x + 6y - 47 = 0$

13 $x^2 + y^2 - 8x + 7 = 0$

14 $x^2 + y^2 - 2x - 12y + 21 = 0$

15 $x^2 + y^2 + 8x + 6y + 21 = 0$

Limits at Infinity

5. Limits

Find each limit. Divide every term by the highest power of x on the bottom.

1 $\lim_{x\to\infty} \frac{-2x+1}{2x-3}$

2 $\lim_{x\to\infty} \frac{-2x-1}{2x^2+3}$

3 $\lim_{x\to\infty} \frac{-x-3}{x-3}$

4 $\lim_{x\to\infty} \frac{-2x^2-2x+2}{3x^2-2}$

5 $\lim_{x\to\infty} \frac{-2x}{x+2}$

6 $\lim_{x\to\infty} \frac{-2x+3}{x^2-2}$

7 $\lim_{x\to\infty} \frac{-3x}{2x+1}$

8 $\lim_{x\to\infty} \frac{x-1}{3x^2+2}$

9 $\lim_{x\to\infty} \frac{3x+1}{2x+1}$

10 $\lim_{x\to\infty} \frac{3x+3}{3x^2-1}$

11 $\lim_{x\to\infty} \frac{-3x+1}{2x+1}$

12 $\lim_{x\to\infty} \frac{3x^2-2x+3}{2x^2-3}$

6. The equation of a parabola

Work between a parabola's equation and its focus and directrix.

13 **focus (1, −2)** directrix $y = -6$

14 $(x+3)^2 = 4y$

15 **focus (3, −4)** directrix $y = -2$

16 $(x+4)^2 = -4(y-4)$

17 $(x-3)^2 = 12(y-1)$

Choosing How to Find a Limit

WORKED EXAMPLE

$$\lim_{x \to 3} (x^2 + x + 3)$$
$$= 3^2 + 3 + 3$$
$$= 9 + 3 + 3$$
$$= 15$$

7. Limits

Find each limit. Substitute first; what you get decides the next move.

1 $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

2 $\lim_{x \to -3} \dfrac{x + 3}{x - 1}$

3 $\lim_{x \to \infty} \dfrac{-2x}{x^2 - 3}$

4 $\lim_{x \to 3} \dfrac{x^2 - x - 6}{x^2 - 3x}$

5 $\lim_{x \to \infty} \dfrac{-x + 3}{2x^2 - 3}$

6 $\lim_{x \to -2} \dfrac{x^2 - x - 6}{x^2 + 3x + 2}$

7 $\lim_{x \to \infty} \dfrac{-3x - 1}{3x^2 + 2}$

8 $\lim_{x \to 3} (x^2 - 3x - 3)$

9 $\lim_{x \to \infty} \dfrac{-3x + 1}{3x - 1}$

10 $\lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x^2 + x - 6}$

8. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

11 $\frac{3\pi}{2}$ in degrees = _____

12 $\frac{\pi}{3}$ in degrees = _____

13 $\frac{7\pi}{6}$ in degrees = _____

14 180° in radians = _____

15 240° in radians = _____

16 330° in radians = _____

Answer Key

Limits by Substitution

Limits

1. $\lim_{x\to -1} (-x^2 + x + 3) = 1$
2. $\lim_{x\to 1} (-x^2 + 2x - 2) = -1$
3. $\lim_{x\to -3} (-x^2 - 3x - 1) = -1$
4. $\lim_{x\to -1} (-x^2 - 2x + 2) = 3$
5. $\lim_{x\to -2} \frac{-x+1}{x+3} = 3$
6. $\lim_{x\to -2} (-x^2 + 2x + 1) = -7$
7. $\lim_{x\to -1} (-x^2 - 3x - 3) = -1$
8. $\lim_{x\to -1} \frac{-x+2}{x+2} = 3$
9. $\lim_{x\to -2} \frac{x+3}{x-2} = -\frac{1}{4}$
10. $\lim_{x\to -1} (x^2 - x - 3) = -1$
11. $\lim_{x\to 3} \frac{-x-3}{x-1} = -3$
12. $\lim_{x\to -1} (-x^2 - x + 2) = 2$

The equation of a circle

13. $(x - 5)^2 + y^2 = 9$ → center (5, 0), radius 3
14. $(x + 2)^2 + (y - 3)^2 = 4$ → center (−2, 3), radius 2
15. $(x - 5)^2 + (y + 5)^2 = 49$ → center (5, −5), radius 7
16. $(x - 1)^2 + (y - 3)^2 = 16$ → center (1, 3), radius 4
17. $(x + 5)^2 + (y - 6)^2 = 9$ → center (−5, 6), radius 3
18. $(x + 1)^2 + (y - 1)^2 = 36$ → center (−1, 1), radius 6

Limits by Factoring

Limits

1. $\lim_{x\to -1} \frac{x^2 - x - 2}{x^2 + x} = 3$
2. $\lim_{x\to 1} \frac{x^2 - x}{x^2 - 4x + 3} = -\frac{1}{2}$
3. $\lim_{x\to 3} \frac{x^2 - 2x - 3}{x - 3} = 4$
4. $\lim_{x\to -2} \frac{x^2 + 2x}{x^2 + 3x + 2} = 2$
5. $\lim_{x\to -3} \frac{x^2 + 4x + 3}{x^2 - 9} = \frac{1}{3}$
6. $\lim_{x\to -1} \frac{x^2 + 3x + 2}{x^2 - 2x - 3} = -\frac{1}{4}$
7. $\lim_{x\to 2} \frac{x^2 - 5x + 6}{x - 2} = -1$
8. $\lim_{x\to 1} \frac{x^2 + 2x - 3}{x^2 - x} = 4$
9. $\lim_{x\to -1} \frac{x^2 - x - 2}{x^2 - 1} = \frac{3}{2}$
10. $\lim_{x\to -2} \frac{x^2 - 4}{x^2 - x - 6} = \frac{4}{5}$

The equation of a circle

11. $x^2 + y^2 + 2x - 12y + 28 = 0$ → center (−1, 6), radius 3
12. $x^2 + y^2 - 10x + 6y - 47 = 0$ → center (5, −3), radius 9
13. $x^2 + y^2 - 8x + 7 = 0$ → center (4, 0), radius 3
14. $x^2 + y^2 - 2x - 12y + 21 = 0$ → center (1, 6), radius 4
15. $x^2 + y^2 + 8x + 6y + 21 = 0$ → center (−4, −3), radius 2

Limits at Infinity

Limits

1. $\lim_{x\to \infty} \frac{-2x+1}{2x-3} = -1$
2. $\lim_{x\to \infty} \frac{-2x-1}{2x^2+3} = 0$

Limits at Infinity (continued)

Limits (continued)

3. $\lim_{x\to \infty} \frac{-x-3}{x-3} = -1$
4. $\lim_{x\to \infty} \frac{-2x^2 - 2x + 2}{3x^2 - 2} = -\frac{2}{3}$
5. $\lim_{x\to \infty} \frac{-2x}{x+2} = -2$
6. $\lim_{x\to \infty} \frac{-2x+3}{x^2-2} = 0$
7. $\lim_{x\to \infty} \frac{-3x}{2x+1} = -\frac{3}{2}$
8. $\lim_{x\to \infty} \frac{x-1}{3x^2+2} = 0$
9. $\lim_{x\to \infty} \frac{3x+1}{2x+1} = \frac{3}{2}$
10. $\lim_{x\to \infty} \frac{3x+3}{3x^2-1} = 0$
11. $\lim_{x\to \infty} \frac{-3x+1}{2x+1} = -\frac{3}{2}$
12. $\lim_{x\to \infty} \frac{3x^2 - 2x + 3}{2x^2 - 3} = \frac{3}{2}$

The equation of a parabola

13. focus (1, −2), $y = -6$ → $(x - 1)^2 = 8(y + 4)$
14. $(x + 3)^2 = 4y$ → focus (−3, 1), directrix $y = -1$
15. focus (3, −4), $y = -2$ → $(x - 3)^2 = -4(y + 3)$
16. $(x + 4)^2 = -4(y - 4)$ → focus (−4, 3), directrix $y = 5$
17. $(x - 3)^2 = 12(y - 1)$ → focus (3, 4), directrix $y = -2$

Choosing How to Find a Limit

Limits

1. $\lim_{x\to 3} \frac{x^2 - 9}{x - 3} = 6$
2. $\lim_{x\to -3} \frac{x+3}{x-1} = 0$
3. $\lim_{x\to \infty} \frac{-2x}{x^2-3} = 0$
4. $\lim_{x\to 3} \frac{x^2 - x - 6}{x^2 - 3x} = \frac{5}{3}$
5. $\lim_{x\to \infty} \frac{-x+3}{2x^2-3} = 0$
6. $\lim_{x\to -2} \frac{x^2 - x - 6}{x^2 + 3x + 2} = 5$
7. $\lim_{x\to \infty} \frac{-3x-1}{3x^2+2} = 0$
8. $\lim_{x\to 3} (x^2 - 3x - 3) = -3$
9. $\lim_{x\to \infty} \frac{-3x+1}{3x-1} = -1$
10. $\lim_{x\to 2} \frac{x^2 - 3x + 2}{x^2 + x - 6} = \frac{1}{5}$

Degrees and radians

11. $3\pi/2$ in degrees = 270°
12. $\pi/3$ in degrees = 60°
13. $7\pi/6$ in degrees = 210°
14. 180° in radians = π
15. 240° in radians = $4\pi/3$
16. 330° in radians = $11\pi/6$

Solutions (1 of 3)

Limits by Substitution

Limits

1. $\lim_{x \to -1} (-x^2 + x + 3)$

$$\lim_{x \to -1} (-x^2 + x + 3) = -(-1)^2 + (-1) + 3$$
$$= -1 - 1 + 3$$
$$= 1$$

A polynomial has no breaks and no gaps, so its limit is just its value there. Put $x = -1$ into every term: $-(-1)^2 + (-1) + 3$.

Work out each term on its own: $-1 - 1 + 3$.

Add them up. The limit is 1.

2. $\lim_{x \to 1} (-x^2 + 2x - 2)$

$$\lim_{x \to 1} (-x^2 + 2x - 2) = -1^2 + 2(1) - 2$$
$$= -1 + 2 - 2$$
$$= -1$$

A polynomial has no breaks and no gaps, so its limit is just its value there. Put $x = 1$ into every term: $-1^2 + 2(1) - 2$.

Work out each term on its own: $-1 + 2 - 2$.

Add them up. The limit is -1.

The equation of a circle

13. $(x - 5)^2 + y^2 = 9$

$(x - 5)^2 + y^2 = 9$

center $(5, 0)$ $r = 3$

In $(x - 5)^2 + y^2 = r^2$, the center is the numbers in the brackets with their signs REVERSED, and the right-hand side is r^2, not r.

So the center is $(5, 0)$ and the radius is $\sqrt{9} = 3$.

Solutions (2 of 3)

Limits by Substitution (continued)

The equation of a circle (continued)

14. $(x + 2)^2 + (y - 3)^2 = 4$

$(x + 2)^2 + (y - 3)^2 = 4$

center $(-2, 3)$ $r = 2$

In $(x + 2)^2 + (y - 3)^2 = r^2$, the center is the numbers in the brackets with their signs REVERSED, and the right-hand side is r^2, not r.

So the center is $(-2, 3)$ and the radius is $\sqrt{4} = 2$.

Limits at Infinity

The equation of a parabola

13. focus $(1, -2)$ directrix $y = -6$

vertex = $(1, -4)$

$p = -2 - (-4) = 2$

$(x - 1)^2 = 8(y + 4)$

Every point on a parabola is the same distance from the focus as from the directrix, so the vertex sits halfway between them: halfway from -2 to -6 is -4. The vertex is $(1, -4)$.

p is the gap from the vertex to the focus, and it is SIGNED: the focus is above the vertex, so $p = 2$ and the parabola opens upwards.

Put the vertex and p into $(x - h)^2 = 4p(y - k)$. With $4p = 8$, that is the equation.

14. $(x + 3)^2 = 4y$

vertex = $(-3, 0)$

$4p = 4 \rightarrow p = 1$

focus $(-3, 1)$ · $y = -1$

Read the vertex off the brackets, remembering that the signs flip: $(x + 3)^2 = 4y$ has its vertex at $(-3, 0)$.

The number multiplying the bracket is 4p, so $p = 4 \div 4 = 1$. It is positive, so the parabola opens upwards.

The focus is p from the vertex, and the directrix is the same distance the other way.

Solutions (3 of 3)

Choosing How to Find a Limit

Degrees and radians

11. $\frac{3\pi}{2}$ in degrees = _____

$$\frac{3\pi}{2} \times \mathbf{180/\pi}$$

$$\frac{3\pi}{2} \text{ in degrees} = \mathbf{270°}$$

π radians is 180°, so multiply radians by 180/π to get degrees.

The π cancels, leaving 180 ÷ 2 × 3 = 90 × 3 = 270°.

12. $\frac{\pi}{3}$ in degrees = _____

$$\frac{\pi}{3} \times \mathbf{180/\pi}$$

$$\frac{\pi}{3} \text{ in degrees} = \mathbf{60°}$$

π radians is 180°, so multiply radians by 180/π to get degrees.

The π cancels, leaving 180 ÷ 3 = 60°.