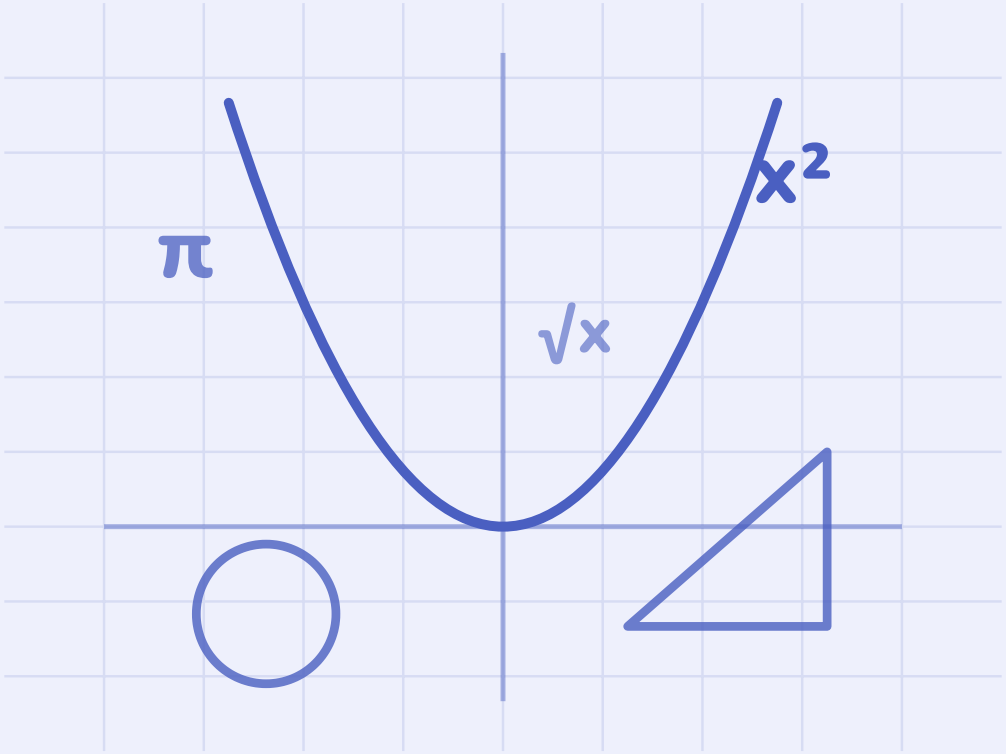
MATH PICNIC · WORKBOOK

# Precalculus Math Review Workbook

3 lessons · Ages 16–18 · answer key at the back




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# Simplifying with Trigonometric Identities

## 1. Trigonometric identities

Simplify each expression. Every answer is a whole number.

1 $2 + 3(\sin^2 A + \cos^2 A) =$ ____

2 $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) =$ ____

3 $5 - 5\sin^2 A - 5\cos^2 A =$ ____

4 $\sin^2 A + \cos^2 A + 6 =$ ____

5 $\sin^2 A + \cos^2 A =$ ____

6 $10 - \sin^2 A - \cos^2 A =$ ____

7 $4(\sin^2 A + \cos^2 A) =$ ____

8 $12(1 - \sin^2 A) \div (1 - \sin^2 A) =$ ____

9 $8(\sin^2 A + \cos^2 A)^3 =$ ____

10 $6(1 - \sin^2 A) \div \cos^2 A =$ ____

11 $10(1 - \cos^2 A) \div \sin^2 A =$ ____

12 $3\sin^2 A \div (1 - \cos^2 A) =$ ____

# The Equation of a Parabola

WORKED EXAMPLE

**focus (−4, 4)** **directrix y = 0**

vertex = (−4, 2)
p = 4 − 2 = 2
$(x + 4)^2 = 8(y - 2)$

## 2. The equation of a parabola

Work between a parabola's equation and its focus and directrix.

1 **focus (3, 3)** **directrix y = 5**

2 $(x + 1)^2 = 8(y + 3)$

3 **focus (1, 3)** **directrix y = 1**

4 $(x - 1)^2 = -4(y - 1)$

5 **focus (−1, 0)** **directrix y = 2**

6 $(x - 1)^2 = 8y$

7 **focus (4, 7)** **directrix y = −1**

8 $x^2 = -4(y + 4)$

9 **focus (−4, −2)** **directrix y = 4**

10 **focus (0, −2)** **directrix y = 0**

## 3. Trigonometric identities

Simplify each expression. Every answer is a whole number.

11 $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) =$ ____

12 $3\sin^2 A \div (1 - \cos^2 A) =$ ____

13 $2 + 3(\sin^2 A + \cos^2 A) =$ ____

14 $12(1 - \sin^2 A) \div (1 - \sin^2 A) =$ ____

15 $4(\sin^2 A + \cos^2 A) =$ ____

16 $10 - \sin^2 A - \cos^2 A =$ ____

# Choosing How to Find a Limit

WORKED EXAMPLE

$$\lim_{x \to 3} (x^2 + x + 3)$$
$$= 3^2 + 3 + 3$$
$$= 9 + 3 + 3$$
$$= 15$$

## 4. Limits

Find each limit. Substitute first; what you get decides the next move.

1 $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

2 $\lim_{x \to -3} \dfrac{x + 3}{x - 1}$

3 $\lim_{x \to \infty} \dfrac{-2x}{x^2 - 3}$

4 $\lim_{x \to 3} \dfrac{x^2 - x - 6}{x^2 - 3x}$

5 $\lim_{x \to \infty} \dfrac{-x + 3}{2x^2 - 3}$

6 $\lim_{x \to -2} \dfrac{x^2 - x - 6}{x^2 + 3x + 2}$

7 $\lim_{x \to \infty} \dfrac{-3x - 1}{3x^2 + 2}$

8 $\lim_{x \to 3} (x^2 - 3x - 3)$

9 $\lim_{x \to \infty} \dfrac{-3x + 1}{3x - 1}$

10 $\lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x^2 + x - 6}$

## 5. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

11 $\frac{3\pi}{2}$ in degrees = _____

12 $\frac{\pi}{3}$ in degrees = _____

13 $\frac{7\pi}{6}$ in degrees = _____

14 180° in radians = _____

15 240° in radians = _____

16 330° in radians = _____

# Answer Key

## Simplifying with Trigonometric Identities

Trigonometric identities

1. $2 + 3(\sin^2 A + \cos^2 A) = 5$
2. $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) = 2$
3. $5 - 5\sin^2 A - 5\cos^2 A = 0$
4. $\sin^2 A + \cos^2 A + 6 = 7$
5. $\sin^2 A + \cos^2 A = 1$
6. $10 - \sin^2 A - \cos^2 A = 9$
7. $4(\sin^2 A + \cos^2 A) = 4$
8. $12(1 - \sin^2 A) \div (1 - \sin^2 A) = 12$
9. $8(\sin^2 A + \cos^2 A)^3 = 8$
10. $6(1 - \sin^2 A) \div \cos^2 A = 6$
11. $10(1 - \cos^2 A) \div \sin^2 A = 10$
12. $3\sin^2 A \div (1 - \cos^2 A) = 3$

## The Equation of a Parabola

The equation of a parabola

1. focus $(3, 3)$, $y = 5 \rightarrow (x - 3)^2 = -4(y - 4)$
2. $(x + 1)^2 = 8(y + 3) \rightarrow$ focus $(-1, -1)$, directrix $y = -5$
3. focus $(1, 3)$, $y = 1 \rightarrow (x - 1)^2 = 4(y - 2)$
4. $(x - 1)^2 = -4(y - 1) \rightarrow$ focus $(1, 0)$, directrix $y = 2$
5. focus $(-1, 0)$, $y = 2 \rightarrow (x + 1)^2 = -4(y - 1)$
6. $(x - 1)^2 = 8y \rightarrow$ focus $(1, 2)$, directrix $y = -2$
7. focus $(4, 7)$, $y = -1 \rightarrow (x - 4)^2 = 16(y - 3)$
8. $x^2 = -4(y + 4) \rightarrow$ focus $(0, -5)$, directrix $y = -3$
9. focus $(-4, -2)$, $y = 4 \rightarrow (x + 4)^2 = -12(y - 1)$
10. focus $(0, -2)$, $y = 0 \rightarrow x^2 = -4(y + 1)$

Trigonometric identities

11. $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) = 2$
12. $3\sin^2 A \div (1 - \cos^2 A) = 3$
13. $2 + 3(\sin^2 A + \cos^2 A) = 5$
14. $12(1 - \sin^2 A) \div (1 - \sin^2 A) = 12$
15. $4(\sin^2 A + \cos^2 A) = 4$
16. $10 - \sin^2 A - \cos^2 A = 9$

## Choosing How to Find a Limit

Limits

1. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$
2. $\lim_{x \to -3} \frac{x + 3}{x - 1} = 0$
3. $\lim_{x \to \infty} \frac{-2x}{x^2 - 3} = 0$
4. $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 3x} = \frac{5}{3}$
5. $\lim_{x \to \infty} \frac{-x + 3}{2x^2 - 3} = 0$
6. $\lim_{x \to -2} \frac{x^2 - x - 6}{x^2 + 3x + 2} = 5$
7. $\lim_{x \to \infty} \frac{-3x - 1}{3x^2 + 2} = 0$
8. $\lim_{x \to 3} (x^2 - 3x - 3) = -3$
9. $\lim_{x \to \infty} \frac{-3x + 1}{3x - 1} = -1$
10. $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 + x - 6} = \frac{1}{5}$

## Choosing How to Find a Limit (continued)

Degrees and radians

11. $3\pi/2$ in degrees $= 270°$
12. $\pi/3$ in degrees $= 60°$
13. $7\pi/6$ in degrees $= 210°$
14. $180°$ in radians $= \pi$
15. $240°$ in radians $= 4\pi/3$
16. $330°$ in radians $= 11\pi/6$

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# Solutions (1 of 3)

## Simplifying with Trigonometric Identities

Trigonometric identities

1. $2 + 3(\sin^2 A + \cos^2 A) =$ ____

$$\sin^2 A + \cos^2 A = 1$$
$$(\sin^2 A + \cos^2 A) \rightarrow 1$$
$$2 + 3(\sin^2 A + \cos^2 A) = 5$$

Every one of these collapses on the same fact: $\sin^2 A + \cos^2 A = 1$, for any angle at all. Find it in the expression and the rest is arithmetic.

Here it is in plain sight. Replace the whole $\sin^2 A + \cos^2 A$ with 1 and leave everything around it alone.

What is left is a number: 5.

2. $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) =$ ____

$$\sin^2 A + \cos^2 A = 1$$
$$(\sin^2 A + \cos^2 A) \rightarrow 1$$
$$(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) = 2$$

Every one of these collapses on the same fact: $\sin^2 A + \cos^2 A = 1$, for any angle at all. Find it in the expression and the rest is arithmetic.

Here it is in plain sight. Replace the whole $\sin^2 A + \cos^2 A$ with 1 and leave everything around it alone.

What is left is a number: 2.

## The Equation of a Parabola

The equation of a parabola

1. **focus (3, 3)** directrix $y = 5$

$$\text{vertex} = (3, 4)$$
$$p = 3 - 4 = -1$$
$$(x - 3)^2 = -4(y - 4)$$

Every point on a parabola is the same distance from the focus as from the directrix, so the vertex sits halfway between them: halfway from 3 to 5 is 4. The vertex is (3, 4).

p is the gap from the vertex to the focus, and it is SIGNED: the focus is below the vertex, so $p = -1$ and the parabola opens downwards.

Put the vertex and p into $(x - h)^2 = 4p(y - k)$. With $4p = -4$, that is the equation.

# Solutions (2 of 3)

**The Equation of a Parabola** (continued)

The equation of a parabola (continued)

2. $(x + 1)^2 = 8(y + 3)$

**vertex = (–1, –3)**

$4p = 8 \rightarrow$ **p = 2**

**focus (–1, –1)** · **y = –5**

Read the vertex off the brackets, remembering that the signs flip: $(x + 1)^2 = 8(y + 3)$ has its vertex at (–1, –3).

The number multiplying the bracket is 4p, so $p = 8 \div 4 = 2$. It is positive, so the parabola opens upwards.

The focus is p from the vertex, and the directrix is the same distance the other way.

**Choosing How to Find a Limit**

Limits

1. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

$x = 3: \quad \frac{0}{0}$

$$\lim_{x \to 3} \frac{(x - 3)(x + 3)}{x - 3} = \lim_{x \to 3} x + 3$$

**= 6**

Try x = 3 first. The top is 0 and the bottom is 0, and $\frac{0}{0}$ is not an answer. It says the top and the bottom share a factor that is 0 at x = 3: (x – 3).

The top is a difference of squares: it is (x – 3)(x + 3), and the bottom is x – 3 already.

(x – 3) cancels. That is allowed because a limit asks what happens NEAR x = 3, never AT it, and near it (x – 3) is not 0.

Now nothing on the bottom is 0 at x = 3, so substitute.

2. $\lim_{x \to -3} \frac{x + 3}{x - 1}$

$$\lim_{x \to -3} \frac{x + 3}{x - 1} = \frac{(-3) + 3}{(-3) - 1} = \frac{0}{-4}$$

**= 0**

Check the bottom first. At x = –3 it is –4, not 0, so the fraction has a value there and the limit is that value. Substitute into the top and the bottom.

The top is 0 and the bottom is –4.

In lowest terms that is 0.

# Solutions (3 of 3)

**Choosing How to Find a Limit** (continued)

Degrees and radians

11. $\frac{3\pi}{2}$ in degrees = _____

$\frac{3\pi}{2} \times$ **180/π**

$\frac{3\pi}{2}$ in degrees = **270°**

π radians is 180°, so multiply radians by 180/π to get degrees.

The π cancels, leaving 180 ÷ 2 × 3 = 90 × 3 = 270°.

12. $\frac{\pi}{3}$ in degrees = _____

$\frac{\pi}{3} \times$ **180/π**

$\frac{\pi}{3}$ in degrees = **60°**

π radians is 180°, so multiply radians by 180/π to get degrees.

The π cancels, leaving 180 ÷ 3 = 60°.