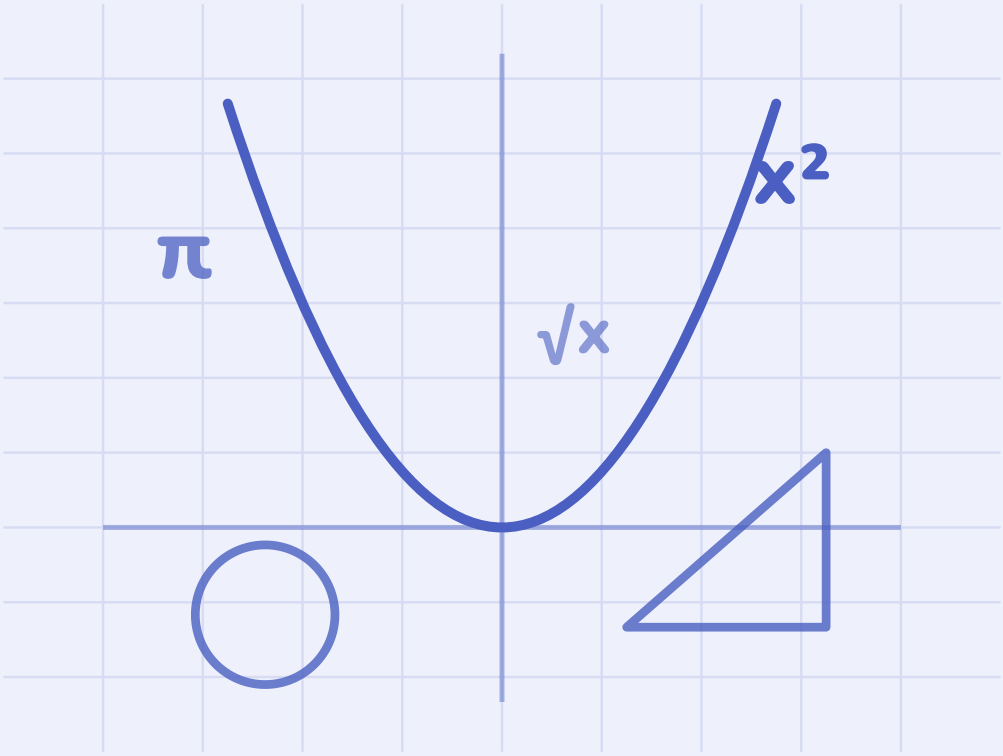
MATH PICNIC · WORKBOOK

Precalculus Advanced Workbook

10 lessons · Ages 16–18 · answer key at the back



Name _____ Date _____

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1-1

Radians and Degrees

WORKED EXAMPLE

30° in radians = $\frac{\pi}{6}$

1. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

1 225° in radians = _____

2 $\frac{5\pi}{4}$ in degrees = _____

3 $\frac{\pi}{4}$ in degrees = _____

4 315° in radians = _____

5 330° in radians = _____

6 $\frac{4\pi}{3}$ in degrees = _____

7 180° in radians = _____

8 270° in radians = _____

9 $\frac{11\pi}{6}$ in degrees = _____

10 135° in radians = _____

11 $\frac{3\pi}{2}$ in degrees = _____

12 $\frac{5\pi}{3}$ in degrees = _____

13 $\frac{\pi}{2}$ in degrees = _____

14 360° in radians = _____

15 π in degrees = _____

16 300° in radians = _____

Exact Values on the Unit Circle

WORKED EXAMPLE

sin 90° = 1

2. Exact values on the unit circle

Give each exact value. Leave surds and fractions exact — no calculator.

1 cos 30° = _____

2 tan 60° = _____

3 sin 60° = _____

4 tan 30° = _____

5 cos 45° = _____

6 tan 0° = _____

7 cos 60° = _____

8 tan 45° = _____

9 sin 45° = _____

10 cos 90° = _____

11 sin 30° = _____

12 cos 0° = _____

Simplifying with Trigonometric Identities

3. Trigonometric identities

Simplify each expression. Every answer is a whole number.

1 $5 - 5\sin^2A - 5\cos^2A =$ _____

2 $12(1 - \sin^2A) \div (1 - \sin^2A) =$ _____

3 $10 - \sin^2A - \cos^2A =$ _____

4 $10(1 - \cos^2A) \div \sin^2A =$ _____

5 $8(\sin^2A + \cos^2A)^3 =$ _____

6 $6(1 - \sin^2A) \div \cos^2A =$ _____

7 $2 + 3(\sin^2A + \cos^2A) =$ _____

8 $3\sin^2A \div (1 - \cos^2A) =$ _____

9 $(\sin^2A + \cos^2A) + (\sin^2B + \cos^2B) =$ _____

10 $\sin^2A + \cos^2A + 6 =$ _____

11 $\sin^2A + \cos^2A =$ _____

12 $4(\sin^2A + \cos^2A) =$ _____

The Equation of a Circle

WORKED EXAMPLE

$(x - 5)^2 + (y + 4)^2 = 16$

center (5, −4) r = 4

4. The equation of a circle

Write the center and radius of each circle.

1 $(x + 1)^2 + (y - 5)^2 = 25$

2 $(x + 2)^2 + (y - 6)^2 = 100$

3 $(x + 3)^2 + (y - 4)^2 = 49$

4 $(x + 5)^2 + (y + 5)^2 = 36$

5 $(x - 5)^2 + (y + 5)^2 = 49$

6 $(x + 5)^2 + (y - 6)^2 = 100$

7 $(x + 3)^2 + (y + 3)^2 = 64$

8 $(x + 6)^2 + (y - 1)^2 = 49$

9 $(x + 6)^2 + (y + 6)^2 = 4$

10 $(x + 1)^2 + (y + 4)^2 = 100$

11 $(x - 5)^2 + (y + 6)^2 = 81$

12 $(x + 1)^2 + (y + 5)^2 = 25$

5. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

13 360° in radians = _____

14 270° in radians = _____

15 $\frac{\pi}{3}$ in degrees = _____

16 330° in radians = _____

17 $\frac{7\pi}{6}$ in degrees = _____

18 225° in radians = _____

A Circle from Its General Form

WORKED EXAMPLE

$x^2 + y^2 - 10x + 12y + 57 = 0$

$(x^2 - 10x) + (y^2 + 12y) = -57$
$(x^2 - 10x + 25) + (y^2 + 12y + 36) = 4$
$(x - 5)^2 + (y + 6)^2 = 4$
center $(5, -6)$ $r = 2$

6. The equation of a circle

Complete the square to find the center and radius of each circle.

1 **$x^2 + y^2 + 2x - 10y + = 0$**

2 **$x^2 + y^2 + 10x + 6y - 47 = 0$**

3 **$x^2 + y^2 + 8x - 4y - 5 = 0$**

4 **$x^2 + y^2 + 8x - 12y - 48 = 0$**

5 **$x^2 + y^2 - 10x + 8y + 25 = 0$**

6 **$x^2 + y^2 + 6x + 4y - 23 = 0$**

7 **$x^2 + y^2 + 10x + 6y + 30 = 0$**

8 **$x^2 + y^2 - 2x + 8y - 8 = 0$**

9 **$x^2 + y^2 + 4x + 2y - 11 = 0$**

10 **$x^2 + y^2 + 2x + 12y - 27 = 0$**

7. Exact values on the unit circle

Give each exact value. Leave surds and fractions exact — no calculator.

11 $\tan 0° =$ _____

12 $\tan 60° =$ _____

13 $\sin 60° =$ _____

14 $\tan 30° =$ _____

15 $\cos 30° =$ _____

16 $\tan 45° =$ _____

The Equation of a Parabola

WORKED EXAMPLE

focus (−4, 4) **directrix y = 0**

vertex = (−4, 2)
p = 4 − 2 = 2
$(x + 4)^2 = 8(y - 2)$

8. The equation of a parabola

Work between a parabola's equation and its focus and directrix.

1 **$(x + 2)^2 = 4(y - 1)$**

2 **$x^2 = -8(y - 2)$**

3 **focus (3, −3)** **directrix y = 1**

4 **$(x - 1)^2 = -8y$**

5 **$(x - 4)^2 = 16y$**

6 **$(x - 2)^2 = -4(y + 4)$**

7 **$(x - 1)^2 = -16(y - 1)$**

8 **focus (1, 1)** **directrix y = 5**

9 **$(x - 4)^2 = -8(y + 2)$**

10 **$(x - 1)^2 = -12(y + 1)$**

9. Trigonometric identities

Simplify each expression. Every answer is a whole number.

11 $5 - 5\sin^2 A - 5\cos^2 A =$ ____

12 $12(1 - \sin^2 A) \div (1 - \sin^2 A) =$ ____

13 $6(1 - \sin^2 A) \div \cos^2 A =$ ____

14 $10(1 - \cos^2 A) \div \sin^2 A =$ ____

15 $10 - \sin^2 A - \cos^2 A =$ ____

16 $3\sin^2 A \div (1 - \cos^2 A) =$ ____

3-1

Limits by Substitution

WORKED EXAMPLE

$$\lim_{x\to -1} \frac{-x-3}{x+2}$$

$$\lim_{x\to -1} \frac{-x-3}{x+2} = \frac{-(-1)-3}{(-1)+2}$$

$$= -2$$

10. Limits

Find each limit. Check the bottom first: if it is not 0 there, substitute.

1 $\lim_{x\to 3} (x^2 + 2x - 2)$

2 $\lim_{x\to -3} \frac{-x-1}{x-2}$

3 $\lim_{x\to -2} (-x^2 + x - 2)$

4 $\lim_{x\to 3} \frac{x-2}{x+2}$

5 $\lim_{x\to -3} (x^2 - 3x - 1)$

6 $\lim_{x\to 2} \frac{-x-2}{x+3}$

7 $\lim_{x\to -2} \frac{-x-1}{x-3}$

8 $\lim_{x\to 2} (-x^2 + 2x - 2)$

9 $\lim_{x\to -3} \frac{x-2}{x-3}$

10 $\lim_{x\to -2} \frac{x}{x-2}$

11 $\lim_{x\to 3} \frac{-x-3}{x-1}$

12 $\lim_{x\to 1} \frac{-x}{x-3}$

11. The equation of a circle

Write the center and radius of each circle.

13 $x^2 + (y+3)^2 = 64$

14 $(x+6)^2 + (y+1)^2 = 25$

15 $(x-2)^2 + (y+1)^2 = 25$

16 $(x+2)^2 + (y+2)^2 = 36$

17 $(x-1)^2 + (y+4)^2 = 25$

18 $(x+4)^2 + (y+3)^2 = 4$

Limits by Factoring

WORKED EXAMPLE

$$\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6}$$

$x = 2$: $\frac{0}{0}$

$$\lim_{x\to 2}\frac{(x-2)(x-1)}{(x-2)(x-3)}$$

$$= \lim_{x\to 2}\frac{x-1}{x-3}$$

$$= \frac{1}{-1}$$

$$= -1$$

12. Limits

Substituting makes the top and bottom both 0: factor first, then cancel.

1 $\lim_{x\to 2}\frac{x^2-2x}{x^2-4}$

2 $\lim_{x\to 3}\frac{x^2-x-6}{x^2-2x-3}$

3 $\lim_{x\to -2}\frac{x^2+2x}{x^2+x-2}$

4 $\lim_{x\to -3}\frac{x^2+x-6}{x^2-9}$

5 $\lim_{x\to -3}\frac{x^2+3x}{x^2-9}$

6 $\lim_{x\to 3}\frac{x^2-2x-3}{x^2-3x}$

7 $\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$

8 $\lim_{x\to -1}\frac{x^2+3x+2}{x^2-2x-3}$

9 $\lim_{x\to -1}\frac{x^2+4x+3}{x^2-x-2}$

10 $\lim_{x\to -3}\frac{x^2+4x+3}{x^2-9}$

13. The equation of a circle

Complete the square to find the center and radius of each circle.

11 $x^2 + y^2 - 4x + 10y - 20 = 0$

12 $x^2 + y^2 + 2x + 4y - 31 = 0$

13 $x^2 + y^2 + 6x - 6y - 63 = 0$

14 $x^2 + y^2 + 8x - 6y - 75 = 0$

15 $x^2 + y^2 + 10x - 2y - 23 = 0$

Limits at Infinity

14. Limits

Find each limit. Divide every term by the highest power of x on the bottom.

1 $\lim_{x\to\infty} \frac{x^2 - 2x - 2}{3x^2 + 2}$

2 $\lim_{x\to\infty} \frac{-x^2 - 2x - 2}{3x^2 + 2}$

3 $\lim_{x\to\infty} \frac{-2x - 3}{x + 1}$

4 $\lim_{x\to\infty} \frac{3x + 3}{x^2 - 2}$

5 $\lim_{x\to\infty} \frac{-x}{2x^2 - 1}$

6 $\lim_{x\to\infty} \frac{-3x^2 + 2x - 2}{x^2 + 2}$

7 $\lim_{x\to\infty} \frac{-3x + 1}{2x + 1}$

8 $\lim_{x\to\infty} \frac{-3x^2 + x + 3}{3x^2 - 3}$

9 $\lim_{x\to\infty} \frac{x - 1}{x^2 + 1}$

10 $\lim_{x\to\infty} \frac{2x}{3x + 3}$

11 $\lim_{x\to\infty} \frac{2x^2 + x + 3}{3x^2 - 3}$

12 $\lim_{x\to\infty} \frac{-2x^2 + x + 3}{3x^2 - 3}$

15. The equation of a parabola

Work between a parabola's equation and its focus and directrix.

13 **focus $(-3, 2)$** directrix $y = 6$

14 $(x - 1)^2 = 16(y + 1)$

15 **focus $(4, -7)$** directrix $y = -1$

16 $(x + 4)^2 = 16(y + 2)$

17 **focus $(-2, -8)$** directrix $y = 0$

Choosing How to Find a Limit

WORKED EXAMPLE

$$\lim_{x\to 3}(x^2 + x + 3)$$
$$= 3^2 + 3 + 3$$
$$= 9 + 3 + 3$$
$$= 15$$

16. Limits

Find each limit. Substitute first; what you get decides the next move.

1 $\lim_{x\to 3} \dfrac{x^2 - 9}{x - 3}$

2 $\lim_{x\to \infty} \dfrac{-2x}{x^2 - 3}$

3 $\lim_{x\to 3} \dfrac{x^2 - x - 6}{x^2 - 3x}$

4 $\lim_{x\to \infty} \dfrac{-x + 3}{2x^2 - 3}$

5 $\lim_{x\to -2} \dfrac{x^2 - x - 6}{x^2 + 3x + 2}$

6 $\lim_{x\to \infty} \dfrac{-3x - 1}{3x^2 + 2}$

7 $\lim_{x\to \infty} \dfrac{-3x + 1}{3x - 1}$

8 $\lim_{x\to 2} \dfrac{x^2 - 3x + 2}{x^2 + x - 6}$

9 $\lim_{x\to \infty} \dfrac{-2x - 3}{2x + 3}$

10 $\lim_{x\to \infty} \dfrac{3x^2 + 2x - 2}{x^2 + 2}$

17. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

11 $\dfrac{3\pi}{2}$ in degrees = _____

12 $\dfrac{7\pi}{6}$ in degrees = _____

13 $\dfrac{\pi}{3}$ in degrees = _____

14 240° in radians = _____

15 330° in radians = _____

16 $\dfrac{5\pi}{3}$ in degrees = _____

Answer Key (1 of 3)

Radians and Degrees

Degrees and radians

1. 225° in radians = $5\pi/4$
2. $5\pi/4$ in degrees = 225°
3. $\pi/4$ in degrees = 45°
4. 315° in radians = $7\pi/4$
5. 330° in radians = $11\pi/6$
6. $4\pi/3$ in degrees = 240°
7. 180° in radians = π
8. 270° in radians = $3\pi/2$
9. $11\pi/6$ in degrees = 330°
10. 135° in radians = $3\pi/4$
11. $3\pi/2$ in degrees = 270°
12. $5\pi/3$ in degrees = 300°
13. $\pi/2$ in degrees = 90°
14. 360° in radians = 2π
15. π in degrees = 180°
16. 300° in radians = $5\pi/3$

Exact Values on the Unit Circle

Exact values on the unit circle

1. $\cos 30° = \frac{\sqrt{3}}{2}$
2. $\tan 60° = \sqrt{3}$
3. $\sin 60° = \frac{\sqrt{3}}{2}$
4. $\tan 30° = \frac{\sqrt{3}}{3}$
5. $\cos 45° = \frac{\sqrt{2}}{2}$
6. $\tan 0° = 0$
7. $\cos 60° = \frac{1}{2}$
8. $\tan 45° = 1$
9. $\sin 45° = \frac{\sqrt{2}}{2}$
10. $\cos 90° = 0$
11. $\sin 30° = \frac{1}{2}$
12. $\cos 0° = 1$

Simplifying with Trigonometric Identities

Trigonometric identities

1. $5 - 5\sin^2 A - 5\cos^2 A = 0$
2. $12(1 - \sin^2 A) \div (1 - \sin^2 A) = 12$
3. $10 - \sin^2 A - \cos^2 A = 9$
4. $10(1 - \cos^2 A) \div \sin^2 A = 10$
5. $8(\sin^2 A + \cos^2 A)^3 = 8$
6. $6(1 - \sin^2 A) \div \cos^2 A = 6$
7. $2 + 3(\sin^2 A + \cos^2 A) = 5$
8. $3\sin^2 A \div (1 - \cos^2 A) = 3$
9. $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) = 2$
10. $\sin^2 A + \cos^2 A + 6 = 7$
11. $\sin^2 A + \cos^2 A = 1$
12. $4(\sin^2 A + \cos^2 A) = 4$

The Equation of a Circle

The equation of a circle

1. $(x + 1)^2 + (y - 5)^2 = 25$ → center $(-1, 5)$, radius 5
2. $(x + 2)^2 + (y - 6)^2 = 100$ → center $(-2, 6)$, radius 10
3. $(x + 3)^2 + (y - 4)^2 = 49$ → center $(-3, 4)$, radius 7
4. $(x + 5)^2 + (y + 5)^2 = 36$ → center $(-5, -5)$, radius 6
5. $(x - 5)^2 + (y + 5)^2 = 49$ → center $(5, -5)$, radius 7
6. $(x + 5)^2 + (y - 6)^2 = 100$ → center $(-5, 6)$, radius 10
7. $(x + 3)^2 + (y + 3)^2 = 64$ → center $(-3, -3)$, radius 8
8. $(x + 6)^2 + (y - 1)^2 = 49$ → center $(-6, 1)$, radius 7
9. $(x + 6)^2 + (y + 6)^2 = 4$ → center $(-6, -6)$, radius 2
10. $(x + 1)^2 + (y + 4)^2 = 100$ → center $(-1, -4)$, radius 10
11. $(x - 5)^2 + (y + 6)^2 = 81$ → center $(5, -6)$, radius 9
12. $(x + 1)^2 + (y + 5)^2 = 25$ → center $(-1, -5)$, radius 5

Degrees and radians

13. 360° in radians = 2π
14. 270° in radians = $3\pi/2$
15. $\pi/3$ in degrees = 60°
16. 330° in radians = $11\pi/6$
17. $7\pi/6$ in degrees = 210°
18. 225° in radians = $5\pi/4$

A Circle from Its General Form

The equation of a circle

1. $x^2 + y^2 + 2x - 10y + = 0$ → center $(-1, 5)$, radius 5
2. $x^2 + y^2 + 10x + 6y - 47 = 0$ → center $(-5, -3)$, radius 9
3. $x^2 + y^2 + 8x - 4y - 5 = 0$ → center $(-4, 2)$, radius 5
4. $x^2 + y^2 + 8x - 12y - 48 = 0$ → center $(-4, 6)$, radius 10
5. $x^2 + y^2 - 10x + 8y + 25 = 0$ → center $(5, -4)$, radius 4
6. $x^2 + y^2 + 6x + 4y - 23 = 0$ → center $(-3, -2)$, radius 6
7. $x^2 + y^2 + 10x + 6y + 30 = 0$ → center $(-5, -3)$, radius 2
8. $x^2 + y^2 - 2x + 8y - 8 = 0$ → center $(1, -4)$, radius 5
9. $x^2 + y^2 + 4x + 2y - 11 = 0$ → center $(-2, -1)$, radius 4
10. $x^2 + y^2 + 2x + 12y - 27 = 0$ → center $(-1, -6)$, radius 8

Exact values on the unit circle

11. $\tan 0° = 0$
12. $\tan 60° = \sqrt{3}$
13. $\sin 60° = \frac{\sqrt{3}}{2}$
14. $\tan 30° = \frac{\sqrt{3}}{3}$
15. $\cos 30° = \frac{\sqrt{3}}{2}$
16. $\tan 45° = 1$

The Equation of a Parabola

The equation of a parabola

1. $(x + 2)^2 = 4(y - 1)$ → focus $(-2, 2)$, directrix $y = 0$
2. $x^2 = -8(y - 2)$ → focus $(0, 0)$, directrix $y = 4$
3. focus $(3, -3)$, $y = 1$ → $(x - 3)^2 = -8(y + 1)$
4. $(x - 1)^2 = -8y$ → focus $(1, -2)$, directrix $y = 2$
5. $(x - 4)^2 = 16y$ → focus $(4, 4)$, directrix $y = -4$

Answer Key (2 of 3)

The Equation of a Parabola (continued)

The equation of a parabola (continued)

6. $(x - 2)^2 = -4(y + 4)$ → focus (2, −5), directrix y = −3
7. $(x - 1)^2 = -16(y - 1)$ → focus (1, −3), directrix y = 5
8. focus (1, 1), y = 5 → $(x - 1)^2 = -8(y - 3)$
9. $(x - 4)^2 = -8(y + 2)$ → focus (4, −4), directrix y = 0
10. $(x - 1)^2 = -12(y + 1)$ → focus (1, −4), directrix y = 2

Trigonometric identities

11. $5 - 5\sin^2 A - 5\cos^2 A = 0$
12. $12(1 - \sin^2 A) \div (1 - \sin^2 A) = 12$
13. $6(1 - \sin^2 A) \div \cos^2 A = 6$
14. $10(1 - \cos^2 A) \div \sin^2 A = 10$
15. $10 - \sin^2 A - \cos^2 A = 9$
16. $3\sin^2 A \div (1 - \cos^2 A) = 3$

Limits by Substitution

Limits

1. $\lim_{x \to 3} (x^2 + 2x - 2) = 13$
2. $\lim_{x \to -3} \frac{-x - 1}{x - 2} = -\frac{2}{5}$
3. $\lim_{x \to -2} (-x^2 + x - 2) = -8$
4. $\lim_{x \to 3} \frac{x - 2}{x + 2} = \frac{1}{5}$
5. $\lim_{x \to -3} (x^2 - 3x - 1) = 17$
6. $\lim_{x \to 2} \frac{-x - 2}{x + 3} = -\frac{4}{5}$
7. $\lim_{x \to -2} \frac{-x - 1}{x - 3} = -\frac{1}{5}$
8. $\lim_{x \to 2} (-x^2 + 2x - 2) = -2$
9. $\lim_{x \to -3} \frac{x - 2}{x - 3} = \frac{5}{6}$
10. $\lim_{x \to -2} \frac{x}{x - 2} = \frac{1}{2}$
11. $\lim_{x \to 3} \frac{-x - 3}{x - 1} = -3$
12. $\lim_{x \to 1} \frac{-x}{x - 3} = \frac{1}{2}$

The equation of a circle

13. $x^2 + (y + 3)^2 = 64$ → center (0, −3), radius 8
14. $(x + 6)^2 + (y + 1)^2 = 25$ → center (−6, −1), radius 5
15. $(x - 2)^2 + (y + 1)^2 = 25$ → center (2, −1), radius 5
16. $(x + 2)^2 + (y + 2)^2 = 36$ → center (−2, −2), radius 6
17. $(x - 1)^2 + (y + 4)^2 = 25$ → center (1, −4), radius 5
18. $(x + 4)^2 + (y + 3)^2 = 4$ → center (−4, −3), radius 2

Limits by Factoring

Limits

1. $\lim_{x \to 2} \frac{x^2 - 2x}{x^2 - 4} = \frac{1}{2}$
2. $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 2x - 3} = \frac{5}{4}$
3. $\lim_{x \to -2} \frac{x^2 + 2x}{x^2 + x - 2} = \frac{2}{3}$
4. $\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 - 9} = \frac{5}{6}$
5. $\lim_{x \to -3} \frac{x^2 + 3x}{x^2 - 9} = \frac{1}{2}$
6. $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x^2 - 3x} = \frac{4}{3}$
7. $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4} = \frac{5}{4}$
8. $\lim_{x \to -1} \frac{x^2 + 3x + 2}{x^2 - 2x - 3} = -\frac{1}{4}$

Limits by Factoring (continued)

Limits (continued)

9. $\lim_{x \to -1} \frac{x^2 + 4x + 3}{x^2 - x - 2} = -\frac{2}{3}$
10. $\lim_{x \to -3} \frac{x^2 + 4x + 3}{x^2 - 9} = \frac{1}{3}$

The equation of a circle

11. $x^2 + y^2 - 4x + 10y - 20 = 0$ → center (2, −5), radius 7
12. $x^2 + y^2 + 2x + 4y - 31 = 0$ → center (−1, −2), radius 6
13. $x^2 + y^2 + 6x - 6y - 63 = 0$ → center (−3, 3), radius 9
14. $x^2 + y^2 + 8x - 6y - 75 = 0$ → center (−4, 3), radius 10
15. $x^2 + y^2 + 10x - 2y - 23 = 0$ → center (−5, 1), radius 7

Limits at Infinity

Limits

1. $\lim_{x \to \infty} \frac{x^2 - 2x - 2}{3x^2 + 2} = \frac{1}{3}$
2. $\lim_{x \to \infty} \frac{-x^2 - 2x - 2}{3x^2 + 2} = -\frac{1}{3}$
3. $\lim_{x \to \infty} \frac{-2x - 3}{x + 1} = -2$
4. $\lim_{x \to \infty} \frac{3x + 3}{x^2 - 2} = 0$
5. $\lim_{x \to \infty} \frac{-x}{2x^2 - 1} = 0$
6. $\lim_{x \to \infty} \frac{-3x^2 + 2x - 2}{x^2 + 2} = -3$
7. $\lim_{x \to \infty} \frac{-3x + 1}{2x + 1} = -\frac{3}{2}$
8. $\lim_{x \to \infty} \frac{-3x^2 + x + 3}{3x^2 - 3} = -1$
9. $\lim_{x \to \infty} \frac{x - 1}{x^2 + 1} = 0$
10. $\lim_{x \to \infty} \frac{2x}{3x + 3} = \frac{2}{3}$
11. $\lim_{x \to \infty} \frac{2x^2 + x + 3}{3x^2 - 3} = \frac{2}{3}$
12. $\lim_{x \to \infty} \frac{-2x^2 + x + 3}{3x^2 - 3} = -\frac{2}{3}$

The equation of a parabola

13. focus (−3, 2), y = 6 → $(x + 3)^2 = -8(y - 4)$
14. $(x - 1)^2 = 16(y + 1)$ → focus (1, 3), directrix y = −5
15. focus (4, −7), y = −1 → $(x - 4)^2 = -12(y + 4)$
16. $(x + 4)^2 = 16(y + 2)$ → focus (−4, 2), directrix y = −6
17. focus (−2, −8), y = 0 → $(x + 2)^2 = -16(y + 4)$

Choosing How to Find a Limit

Limits

1. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$
2. $\lim_{x \to \infty} \frac{-2x}{x^2 - 3} = 0$
3. $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 3x} = \frac{5}{3}$
4. $\lim_{x \to \infty} \frac{-x + 3}{2x^2 - 3} = 0$
5. $\lim_{x \to -2} \frac{x^2 - x - 6}{x^2 + 3x + 2} = 5$
6. $\lim_{x \to \infty} \frac{-3x - 1}{3x^2 + 2} = 0$
7. $\lim_{x \to \infty} \frac{-3x + 1}{3x - 1} = -1$
8. $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 + x - 6} = \frac{1}{5}$
9. $\lim_{x \to \infty} \frac{-2x - 3}{2x + 3} = -1$
10. $\lim_{x \to \infty} \frac{3x^2 + 2x - 2}{x^2 + 2} = 3$

Answer Key (3 of 3)

Choosing How to Find a Limit (continued)

Degrees and radians

11. $3\pi/2$ in degrees = 270°
12. $7\pi/6$ in degrees = 210°
13. $\pi/3$ in degrees = 60°
14. 240° in radians = $4\pi/3$
15. 330° in radians = $11\pi/6$
16. $5\pi/3$ in degrees = 300°

Solutions (1 of 3)

Radians and Degrees

Degrees and radians

1. 225° in radians = _____

$$225° \times \frac{\pi}{180}$$

$$225° \text{ in radians} = \frac{5\pi}{4}$$

A straight angle is 180° and also π radians, so multiply degrees by $\frac{\pi}{180}$.

$225 \times \frac{\pi}{180} = \frac{225\pi}{180}$. 45 goes into both 225 and 180, so it simplifies to $\frac{5\pi}{4}$.

2. $\frac{5\pi}{4}$ in degrees = _____

$$\frac{5\pi}{4} \times 180/\pi$$

$$\frac{5\pi}{4} \text{ in degrees} = 225°$$

π radians is 180°, so multiply radians by 180/π to get degrees.

The π cancels, leaving $180 \div 4 \times 5 = 45 \times 5 = 225°$.

Simplifying with Trigonometric Identities

Trigonometric identities

1. $5 - 5\sin^2A - 5\cos^2A =$ _____

$$\sin^2A + \cos^2A = 1$$

$$(\sin^2A + \cos^2A) \rightarrow 1$$

$$5 - 5\sin^2A - 5\cos^2A = 0$$

Every one of these collapses on the same fact: $\sin^2A + \cos^2A = 1$, for any angle at all. Find it in the expression and the rest is arithmetic.

Here it is in plain sight. Replace the whole $\sin^2A + \cos^2A$ with 1 and leave everything around it alone.

What is left is a number: 0.

2. $12(1 - \sin^2A) \div (1 - \sin^2A) =$ _____

$$\sin^2A + \cos^2A = 1$$

$$1 - \cos^2A = \sin^2A$$

$$12(1 - \sin^2A) \div (1 - \sin^2A) = 12$$

Every one of these collapses on the same fact: $\sin^2A + \cos^2A = 1$, for any angle at all. Find it in the expression and the rest is arithmetic.

This one hides it. $1 - \cos^2A$ IS $\sin^2A$, and $1 - \sin^2A$ IS $\cos^2A$ — the identity read backwards. Rewrite the bracket before doing anything else.

What is left is a number: 12.

Solutions (2 of 3)

The Equation of a Circle

The equation of a circle

1. $(x + 1)^2 + (y - 5)^2 = 25$

$$(x + 1)^2 + (y - 5)^2 = 25$$

center $(-1, 5)$ $\quad r = 5$

In $(x + 1)^2 + (y - 5)^2 = r^2$, the center is the numbers in the brackets with their signs REVERSED, and the right-hand side is r^2, not r.

So the center is $(-1, 5)$ and the radius is $\sqrt{25} = 5$.

2. $(x + 2)^2 + (y - 6)^2 = 100$

$$(x + 2)^2 + (y - 6)^2 = 100$$

center $(-2, 6)$ $\quad r = 10$

In $(x + 2)^2 + (y - 6)^2 = r^2$, the center is the numbers in the brackets with their signs REVERSED, and the right-hand side is r^2, not r.

So the center is $(-2, 6)$ and the radius is $\sqrt{100} = 10$.

The Equation of a Parabola

The equation of a parabola

1. $(x + 2)^2 = 4(y - 1)$

vertex $= (-2, 1)$

$4p = 4 \quad \rightarrow \quad p = 1$

focus $(-2, 2)$ $\quad \cdot \quad y = 0$

Read the vertex off the brackets, remembering that the signs flip: $(x + 2)^2 = 4(y - 1)$ has its vertex at $(-2, 1)$.

The number multiplying the bracket is 4p, so $p = 4 \div 4 = 1$. It is positive, so the parabola opens upwards.

The focus is p from the vertex, and the directrix is the same distance the other way.

2. $x^2 = -8(y - 2)$

vertex $= (0, 2)$

$4p = -8 \quad \rightarrow \quad p = -2$

focus $(0, 0)$ $\quad \cdot \quad y = 4$

Read the vertex off the brackets, remembering that the signs flip: $x^2 = -8(y - 2)$ has its vertex at $(0, 2)$.

The number multiplying the bracket is 4p, so $p = -8 \div 4 = -2$. It is negative, so the parabola opens downwards.

The focus is p from the vertex, and the directrix is the same distance the other way.

Solutions (3 of 3)

Limits by Substitution

Limits

1. $\lim_{x \to 3} (x^2 + 2x - 2)$

$$\lim_{x \to 3} (x^2 + 2x - 2) = 3^2 + 2(3) - 2$$

$$= 9 + 6 - 2$$

$$= \mathbf{13}$$

A polynomial has no breaks and no gaps, so its limit is just its value there. Put $x = 3$ into every term: $3^2 + 2(3) - 2$.

Work out each term on its own: $9 + 6 - 2$.

Add them up. The limit is 13.

2. $\lim_{x \to -3} \dfrac{-x - 1}{x - 2}$

$$\lim_{x \to -3} \frac{-x - 1}{x - 2} = \frac{-(-3) - 1}{(-3) - 2}$$

$$= \frac{2}{-5}$$

$$= \mathbf{\frac{-2}{5}}$$

Check the bottom first. At $x = -3$ it is -5, not 0, so the fraction has a value there and the limit is that value. Substitute into the top and the bottom.

The top is 2 and the bottom is -5.

In lowest terms that is $-\frac{2}{5}$.