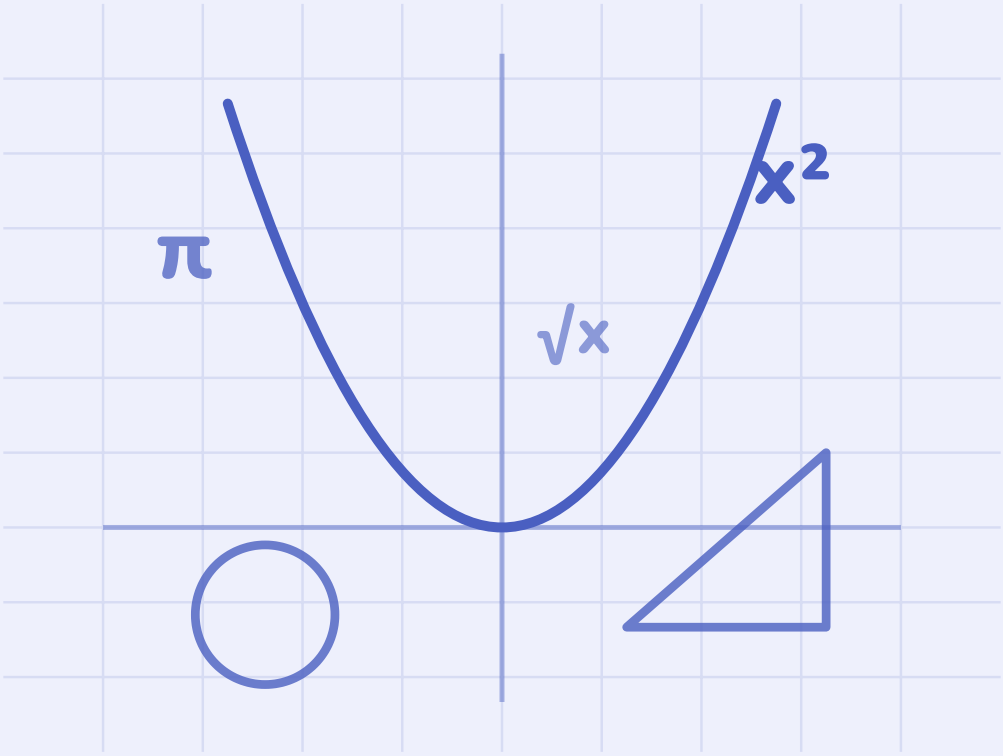
MATH PICNIC · WORKBOOK

Precalculus Foundation Workbook

10 lessons · Ages 16–18 · answer key at the back



Name _____ Date _____

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1-1

Radians and Degrees

WORKED EXAMPLE

$\frac{5\pi}{3}$ in degrees = **300°**

1. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

1 $\frac{\pi}{4}$ in degrees = _____

2 180° in radians = _____

3 225° in radians = _____

4 135° in radians = _____

5 $\frac{\pi}{2}$ in degrees = _____

6 π in degrees = _____

7 $\frac{5\pi}{4}$ in degrees = _____

8 150° in radians = _____

9 120° in radians = _____

10 $\frac{2\pi}{3}$ in degrees = _____

11 315° in radians = _____

12 $\frac{5\pi}{6}$ in degrees = _____

13 30° in radians = _____

14 330° in radians = _____

15 $\frac{3\pi}{4}$ in degrees = _____

16 60° in radians = _____

Exact Values on the Unit Circle

WORKED EXAMPLE

$$\sin 45° = \frac{\sqrt{2}}{2}$$

2. Exact values on the unit circle

Give each exact value. Leave surds and fractions exact — no calculator.

1 $\cos 60° =$ _____

2 $\cos 90° =$ _____

3 $\cos 30° =$ _____

4 $\sin 30° =$ _____

5 $\cos 0° =$ _____

6 $\sin 0° =$ _____

7 $\sin 60° =$ _____

8 $\sin 90° =$ _____

9 $\cos 45° =$ _____

10 $\tan 60° =$ _____

11 $\tan 0° =$ _____

12 $\tan 30° =$ _____

Simplifying with Trigonometric Identities

3. Trigonometric identities

Simplify each expression. Every answer is a whole number.

1 $2 + 3(\sin^2 A + \cos^2 A) =$ ____

2 $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) =$ ____

3 $5 - 5\sin^2 A - 5\cos^2 A =$ ____

4 $\sin^2 A + \cos^2 A + 6 =$ ____

5 $\sin^2 A + \cos^2 A =$ ____

6 $4(\sin^2 A + \cos^2 A) =$ ____

7 $10 - \sin^2 A - \cos^2 A =$ ____

8 $12(1 - \sin^2 A) \div (1 - \sin^2 A) =$ ____

9 $8(\sin^2 A + \cos^2 A)^3 =$ ____

10 $6(1 - \sin^2 A) \div \cos^2 A =$ ____

11 $10(1 - \cos^2 A) \div \sin^2 A =$ ____

12 $3\sin^2 A \div (1 - \cos^2 A) =$ ____

The Equation of a Circle

WORKED EXAMPLE

$(x - 5)^2 + (y + 4)^2 = 16$

center (5, −4) r = 4

4. The equation of a circle

Write the center and radius of each circle.

1 $(x + 2)^2 + y^2 = 4$

2 $(x - 6)^2 + (y + 3)^2 = 4$

3 $(x - 2)^2 + (y + 5)^2 = 49$

4 $x^2 + (y - 1)^2 = 64$

5 $(x + 2)^2 + (y - 6)^2 = 9$

6 $(x - 5)^2 + (y - 3)^2 = 100$

7 $(x + 5)^2 + y^2 = 81$

8 $(x - 6)^2 + (y - 4)^2 = 64$

9 $x^2 + (y - 2)^2 = 36$

10 $x^2 + (y - 6)^2 = 25$

11 $(x + 4)^2 + (y - 1)^2 = 49$

12 $(x - 1)^2 + (y - 2)^2 = 36$

5. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

13 90° in radians = _____

14 120° in radians = _____

15 300° in radians = _____

16 $\frac{2\pi}{3}$ in degrees = _____

17 $\frac{\pi}{4}$ in degrees = _____

18 $\frac{5\pi}{6}$ in degrees = _____

A Circle from Its General Form

WORKED EXAMPLE

$x^2 + y^2 - 10x + 12y + 57 = 0$

$(x^2 - 10x) + (y^2 + 12y) = -57$
$(x^2 - 10x + 25) + (y^2 + 12y + 36) = 4$
$(x - 5)^2 + (y + 6)^2 = 4$
center $(5, -6)$ $r = 2$

6. The equation of a circle

Complete the square to find the center and radius of each circle.

1 **$x^2 + y^2 - 12x - 10y + 25 = 0$**

2 **$x^2 + y^2 - 4x - 8y - 61 = 0$**

3 **$x^2 + y^2 + 2x - 10y + = 0$**

4 **$x^2 + y^2 + 4x = 0$**

5 **$x^2 + y^2 - 10x - 6y - 66 = 0$**

6 **$x^2 + y^2 - 8x + 6y + 21 = 0$**

7 **$x^2 + y^2 + 8x - 4y - 5 = 0$**

8 **$x^2 + y^2 - 8x - 6y + 16 = 0$**

9 **$x^2 + y^2 + 12x + 32 = 0$**

10 **$x^2 + y^2 - 6x = 0$**

7. Exact values on the unit circle

Give each exact value. Leave surds and fractions exact — no calculator.

11 $\sin 90° =$ _____

12 $\cos 90° =$ _____

13 $\tan 0° =$ _____

14 $\sin 0° =$ _____

15 $\cos 60° =$ _____

16 $\sin 30° =$ _____

The Equation of a Parabola

WORKED EXAMPLE

focus (−2, 8) **directrix y = 0**

vertex = (−2, 4)
p = 8 − 4 = 4
$(x + 2)^2 = 16(y - 4)$

8. The equation of a parabola

Work between a parabola's equation and its focus and directrix.

1 **focus (1, 3)** **directrix y = 1**

2 **focus (−4, −1)** **directrix y = 1**

3 **$(x - 3)^2 = 12(y - 1)$**

4 **focus (−1, 4)** **directrix y = −2**

5 **focus (−2, 1)** **directrix y = −7**

6 **focus (−2, 4)** **directrix y = 0**

7 **focus (−2, −6)** **directrix y = 0**

8 **focus (0, 8)** **directrix y = 0**

9 **focus (−4, 4)** **directrix y = 0**

10 **focus (0, −6)** **directrix y = 0**

9. Trigonometric identities

Simplify each expression. Every answer is a whole number.

11 $\sin^2 A + \cos^2 A =$ ____

12 $4(\sin^2 A + \cos^2 A) =$ ____

13 $5 - 5\sin^2 A - 5\cos^2 A =$ ____

14 $2 + 3(\sin^2 A + \cos^2 A) =$ ____

15 $\sin^2 A + \cos^2 A + 6 =$ ____

16 $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) =$ ____

Limits by Substitution

WORKED EXAMPLE

$$\lim_{x\to -1} \frac{-x-3}{x+2}$$

$$\lim_{x\to -1} \frac{-x-3}{x+2} = \frac{-(-1)-3}{(-1)+2}$$

$$= -2$$

10. Limits

Find each limit. Check the bottom first: if it is not 0 there, substitute.

1 $\lim_{x\to -1} (-x^2 + x + 3)$

2 $\lim_{x\to 1} (-x^2 + 2x - 2)$

3 $\lim_{x\to -3} (-x^2 - 3x - 1)$

4 $\lim_{x\to -1} (-x^2 - 2x + 2)$

5 $\lim_{x\to -2} \frac{-x+1}{x+3}$

6 $\lim_{x\to -1} (-x^2 - 3x - 3)$

7 $\lim_{x\to -2} (-x^2 + 2x + 1)$

8 $\lim_{x\to -1} \frac{-x+2}{x+2}$

9 $\lim_{x\to -1} (x^2 - x - 3)$

10 $\lim_{x\to -1} (-x^2 - x + 2)$

11 $\lim_{x\to 3} \frac{-x-3}{x-1}$

12 $\lim_{x\to 2} \frac{x-2}{x-3}$

11. The equation of a circle

Write the center and radius of each circle.

13 $(x-5)^2 + y^2 = 9$

14 $(x+2)^2 + (y-3)^2 = 4$

15 $(x-5)^2 + (y+5)^2 = 49$

16 $(x-1)^2 + (y-3)^2 = 16$

17 $(x+5)^2 + (y-6)^2 = 9$

18 $(x+6)^2 + y^2 = 4$

Limits by Factoring

WORKED EXAMPLE

$$\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6}$$

$x = 2$: $\frac{0}{0}$

$$\lim_{x\to 2}\frac{(x-2)(x-1)}{(x-2)(x-3)}$$

$$= \lim_{x\to 2}\frac{x-1}{x-3}$$

$$= \frac{1}{-1}$$

$$= -1$$

12. Limits

Substituting makes the top and bottom both 0: factor first, then cancel.

1 $\lim_{x\to 1}\frac{x^2-3x+2}{x-1}$

2 $\lim_{x\to 2}\frac{x^2-2x}{x^2-3x+2}$

3 $\lim_{x\to -2}\frac{x^2-x-6}{x^2+5x+6}$

4 $\lim_{x\to -2}\frac{x^2+x-2}{x+2}$

5 $\lim_{x\to -2}\frac{x^2+2x}{x+2}$

6 $\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}$

7 $\lim_{x\to -1}\frac{x^2-x-2}{x^2-1}$

8 $\lim_{x\to -1}\frac{x^2+3x+2}{x+1}$

9 $\lim_{x\to -3}\frac{x^2+2x-3}{x^2+5x+6}$

10 $\lim_{x\to -3}\frac{x^2+4x+3}{x^2+5x+6}$

13. The equation of a circle

Complete the square to find the center and radius of each circle.

11 $x^2 + y^2 - 2x - 10y - 23 = 0$

12 $x^2 + y^2 - 10x - 75 = 0$

13 $x^2 + y^2 - 12x + 8y + 36 = 0$

14 $x^2 + y^2 - 6x - 6y + 2 = 0$

15 $x^2 + y^2 - 10x - 10y + 14 = 0$

Limits at Infinity

14. Limits

Find each limit. Divide every term by the highest power of x on the bottom.

1 $\lim_{x\to\infty} \frac{-2x+1}{2x+1}$

2 $\lim_{x\to\infty} \frac{-2x-3}{3x+2}$

3 $\lim_{x\to\infty} \frac{2x+3}{x^2+2}$

4 $\lim_{x\to\infty} \frac{-x-3}{2x+3}$

5 $\lim_{x\to\infty} \frac{3x+1}{2x-3}$

6 $\lim_{x\to\infty} \frac{2x-3}{x-3}$

7 $\lim_{x\to\infty} \frac{-2x}{x^2-3}$

8 $\lim_{x\to\infty} \frac{-2x-3}{2x+3}$

9 $\lim_{x\to\infty} \frac{3x-2}{x-3}$

10 $\lim_{x\to\infty} \frac{2x+1}{3x-1}$

11 $\lim_{x\to\infty} \frac{-x^2-x-3}{x^2+3}$

12 $\lim_{x\to\infty} \frac{-2x-3}{x-3}$

15. The equation of a parabola

Work between a parabola's equation and its focus and directrix.

13 **focus (1, −4)** directrix y = −2

14 **focus (−2, 5)** directrix y = −3

15 **focus (3, −5)** directrix y = 1

16 **focus (−3, 7)** directrix y = −1

17 **focus (−3, −2)** directrix y = 2

Choosing How to Find a Limit

WORKED EXAMPLE

$$\lim_{x\to 3}(x^2 + x + 3)$$
$$= 3^2 + 3 + 3$$
$$= 9 + 3 + 3$$
$$= 15$$

16. Limits

Find each limit. Substitute first; what you get decides the next move.

1 $\lim_{x\to 1} \dfrac{-x + 2}{x + 2}$

2 $\lim_{x\to 1} \dfrac{-x + 2}{x - 3}$

3 $\lim_{x\to -2} \dfrac{x^2 + x - 2}{x^2 + 3x + 2}$

4 $\lim_{x\to 1}(x^2 - 2x - 3)$

5 $\lim_{x\to 3} \dfrac{-x - 1}{x - 1}$

6 $\lim_{x\to 3} \dfrac{x - 2}{x + 2}$

7 $\lim_{x\to \infty} \dfrac{-3x}{x + 2}$

8 $\lim_{x\to 2} \dfrac{x - 2}{x + 2}$

9 $\lim_{x\to 2} \dfrac{x - 1}{x - 3}$

10 $\lim_{x\to -3}(x^2 + x + 3)$

17. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

11 120° in radians = _____

12 60° in radians = _____

13 300° in radians = _____

14 $\dfrac{\pi}{2}$ in degrees = _____

15 $\dfrac{5\pi}{6}$ in degrees = _____

16 π in degrees = _____

Answer Key (1 of 3)

Radians and Degrees

Degrees and radians

1. $\pi/4$ in degrees = 45°
2. 180° in radians = π
3. 225° in radians = $5\pi/4$
4. 135° in radians = $3\pi/4$
5. $\pi/2$ in degrees = 90°
6. π in degrees = 180°
7. $5\pi/4$ in degrees = 225°
8. 150° in radians = $5\pi/6$
9. 120° in radians = $2\pi/3$
10. $2\pi/3$ in degrees = 120°
11. 315° in radians = $7\pi/4$
12. $5\pi/6$ in degrees = 150°
13. 30° in radians = $\pi/6$
14. 330° in radians = $11\pi/6$
15. $3\pi/4$ in degrees = 135°
16. 60° in radians = $\pi/3$

Exact Values on the Unit Circle

Exact values on the unit circle

1. $\cos 60° = \frac{1}{2}$
2. $\cos 90° = 0$
3. $\cos 30° = \frac{\sqrt{3}}{2}$
4. $\sin 30° = \frac{1}{2}$
5. $\cos 0° = 1$
6. $\sin 0° = 0$
7. $\sin 60° = \frac{\sqrt{3}}{2}$
8. $\sin 90° = 1$
9. $\cos 45° = \frac{\sqrt{2}}{2}$
10. $\tan 60° = \sqrt{3}$
11. $\tan 0° = 0$
12. $\tan 30° = \frac{\sqrt{3}}{3}$

Simplifying with Trigonometric Identities

Trigonometric identities

1. $2 + 3(\sin^2 A + \cos^2 A) = 5$
2. $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) = 2$
3. $5 - 5\sin^2 A - 5\cos^2 A = 0$
4. $\sin^2 A + \cos^2 A + 6 = 7$
5. $\sin^2 A + \cos^2 A = 1$
6. $4(\sin^2 A + \cos^2 A) = 4$
7. $10 - \sin^2 A - \cos^2 A = 9$
8. $12(1 - \sin^2 A) \div (1 - \sin^2 A) = 12$
9. $8(\sin^2 A + \cos^2 A)^3 = 8$
10. $6(1 - \sin^2 A) \div \cos^2 A = 6$
11. $10(1 - \cos^2 A) \div \sin^2 A = 10$
12. $3\sin^2 A \div (1 - \cos^2 A) = 3$

The Equation of a Circle

The equation of a circle

1. $(x + 2)^2 + y^2 = 4$ → center (−2, 0), radius 2
2. $(x - 6)^2 + (y + 3)^2 = 4$ → center (6, −3), radius 2
3. $(x - 2)^2 + (y + 5)^2 = 49$ → center (2, −5), radius 7
4. $x^2 + (y - 1)^2 = 64$ → center (0, 1), radius 8
5. $(x + 2)^2 + (y - 6)^2 = 9$ → center (−2, 6), radius 3
6. $(x - 5)^2 + (y - 3)^2 = 100$ → center (5, 3), radius 10
7. $(x + 5)^2 + y^2 = 81$ → center (−5, 0), radius 9
8. $(x - 6)^2 + (y - 4)^2 = 64$ → center (6, 4), radius 8
9. $x^2 + (y - 2)^2 = 36$ → center (0, 2), radius 6
10. $x^2 + (y - 6)^2 = 25$ → center (0, 6), radius 5
11. $(x + 4)^2 + (y - 1)^2 = 49$ → center (−4, 1), radius 7
12. $(x - 1)^2 + (y - 2)^2 = 36$ → center (1, 2), radius 6

Degrees and radians

13. 90° in radians = $\pi/2$
14. 120° in radians = $2\pi/3$
15. 300° in radians = $5\pi/3$
16. $2\pi/3$ in degrees = 120°
17. $\pi/4$ in degrees = 45°
18. $5\pi/6$ in degrees = 150°

A Circle from Its General Form

The equation of a circle

1. $x^2 + y^2 - 12x - 10y + 25 = 0$ → center (6, 5), radius 6
2. $x^2 + y^2 - 4x - 8y - 61 = 0$ → center (2, 4), radius 9
3. $x^2 + y^2 + 2x - 10y + = 0$ → center (−1, 5), radius 5
4. $x^2 + y^2 + 4x = 0$ → center (−2, 0), radius 2
5. $x^2 + y^2 - 10x - 6y - 66 = 0$ → center (5, 3), radius 10
6. $x^2 + y^2 - 8x + 6y + 21 = 0$ → center (4, −3), radius 2
7. $x^2 + y^2 + 8x - 4y - 5 = 0$ → center (−4, 2), radius 5
8. $x^2 + y^2 - 8x - 6y + 16 = 0$ → center (4, 3), radius 3
9. $x^2 + y^2 + 12x + 32 = 0$ → center (−6, 0), radius 2
10. $x^2 + y^2 - 6x = 0$ → center (3, 0), radius 3

Exact values on the unit circle

11. $\sin 90° = 1$
12. $\cos 90° = 0$
13. $\tan 0° = 0$
14. $\sin 0° = 0$
15. $\cos 60° = \frac{1}{2}$
16. $\sin 30° = \frac{1}{2}$

The Equation of a Parabola

The equation of a parabola

1. focus (1, 3), $y = 1$ → $(x - 1)^2 = 4(y - 2)$
2. focus (−4, −1), $y = 1$ → $(x + 4)^2 = -4y$
3. $(x - 3)^2 = 12(y - 1)$ → focus (3, 4), directrix $y = -2$
4. focus (−1, 4), $y = -2$ → $(x + 1)^2 = 12(y - 1)$
5. focus (−2, 1), $y = -7$ → $(x + 2)^2 = 16(y + 3)$

Answer Key (2 of 3)

The Equation of a Parabola (continued)

The equation of a parabola (continued)

6. focus $(-2, 4)$, $y = 0 \rightarrow (x + 2)^2 = 8(y - 2)$
7. focus $(-2, -6)$, $y = 0 \rightarrow (x + 2)^2 = -12(y + 3)$
8. focus $(0, 8)$, $y = 0 \rightarrow x^2 = 16(y - 4)$
9. focus $(-4, 4)$, $y = 0 \rightarrow (x + 4)^2 = 8(y - 2)$
10. focus $(0, -6)$, $y = 0 \rightarrow x^2 = -12(y + 3)$

Trigonometric identities

11. $\sin^2 A + \cos^2 A = 1$
12. $4(\sin^2 A + \cos^2 A) = 4$
13. $5 - 5\sin^2 A - 5\cos^2 A = 0$
14. $2 + 3(\sin^2 A + \cos^2 A) = 5$
15. $\sin^2 A + \cos^2 A + 6 = 7$
16. $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) = 2$

Limits by Substitution

Limits

1. $\lim_{x \to -1} (-x^2 + x + 3) = 1$
2. $\lim_{x \to 1} (-x^2 + 2x - 2) = -1$
3. $\lim_{x \to -3} (-x^2 - 3x - 1) = -1$
4. $\lim_{x \to -1} (-x^2 - 2x + 2) = 3$
5. $\lim_{x \to -2} \frac{-x + 1}{x + 3} = 3$
6. $\lim_{x \to -1} (-x^2 - 3x - 3) = -1$
7. $\lim_{x \to -2} (-x^2 + 2x + 1) = -7$
8. $\lim_{x \to -1} \frac{-x + 2}{x + 2} = 3$
9. $\lim_{x \to -1} (x^2 - x - 3) = -1$
10. $\lim_{x \to -1} (-x^2 - x + 2) = 2$
11. $\lim_{x \to 3} \frac{-x - 3}{x - 1} = -3$
12. $\lim_{x \to 2} \frac{x - 2}{x - 3} = 0$

The equation of a circle

13. $(x - 5)^2 + y^2 = 9 \rightarrow$ center $(5, 0)$, radius 3
14. $(x + 2)^2 + (y - 3)^2 = 4 \rightarrow$ center $(-2, 3)$, radius 2
15. $(x - 5)^2 + (y + 5)^2 = 49 \rightarrow$ center $(5, -5)$, radius 7
16. $(x - 1)^2 + (y - 3)^2 = 16 \rightarrow$ center $(1, 3)$, radius 4
17. $(x + 5)^2 + (y - 6)^2 = 9 \rightarrow$ center $(-5, 6)$, radius 3
18. $(x + 6)^2 + y^2 = 4 \rightarrow$ center $(-6, 0)$, radius 2

Limits by Factoring

Limits

1. $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x - 1} = -1$
2. $\lim_{x \to 2} \frac{x^2 - 2x}{x^2 - 3x + 2} = 2$
3. $\lim_{x \to -2} \frac{x^2 - x - 6}{x^2 + 5x + 6} = -5$
4. $\lim_{x \to -2} \frac{x^2 + x - 2}{x + 2} = -3$
5. $\lim_{x \to -2} \frac{x^2 + 2x}{x + 2} = -2$
6. $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x} = 3$
7. $\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 1} = \frac{3}{2}$
8. $\lim_{x \to -1} \frac{x^2 + 3x + 2}{x + 1} = 1$

Limits by Factoring (continued)

Limits (continued)

9. $\lim_{x \to -3} \frac{x^2 + 2x - 3}{x^2 + 5x + 6} = 4$
10. $\lim_{x \to -3} \frac{x^2 + 4x + 3}{x^2 + 5x + 6} = 2$

The equation of a circle

11. $x^2 + y^2 - 2x - 10y - 23 = 0 \rightarrow$ center $(1, 5)$, radius 7
12. $x^2 + y^2 - 10x - 75 = 0 \rightarrow$ center $(5, 0)$, radius 10
13. $x^2 + y^2 - 12x + 8y + 36 = 0 \rightarrow$ center $(6, -4)$, radius 4
14. $x^2 + y^2 - 6x - 6y + 2 = 0 \rightarrow$ center $(3, 3)$, radius 4
15. $x^2 + y^2 - 10x - 10y + 14 = 0 \rightarrow$ center $(5, 5)$, radius 6

Limits at Infinity

Limits

1. $\lim_{x \to \infty} \frac{-2x + 1}{2x + 1} = -1$
2. $\lim_{x \to \infty} \frac{-2x - 3}{3x + 2} = -\frac{2}{3}$
3. $\lim_{x \to \infty} \frac{2x + 3}{x^2 + 2} = 0$
4. $\lim_{x \to \infty} \frac{-x - 3}{2x + 3} = -\frac{1}{2}$
5. $\lim_{x \to \infty} \frac{3x + 1}{2x - 3} = \frac{3}{2}$
6. $\lim_{x \to \infty} \frac{2x - 3}{x - 3} = 2$
7. $\lim_{x \to \infty} \frac{-2x}{x^2 - 3} = 0$
8. $\lim_{x \to \infty} \frac{-2x - 3}{2x + 3} = -1$
9. $\lim_{x \to \infty} \frac{3x - 2}{x - 3} = 3$
10. $\lim_{x \to \infty} \frac{2x + 1}{3x - 1} = \frac{2}{3}$
11. $\lim_{x \to \infty} \frac{-x^2 - x - 3}{x^2 + 3} = -1$
12. $\lim_{x \to \infty} \frac{-2x - 3}{x - 3} = -2$

The equation of a parabola

13. focus $(1, -4)$, $y = -2 \rightarrow (x - 1)^2 = -4(y + 3)$
14. focus $(-2, 5)$, $y = -3 \rightarrow (x + 2)^2 = 16(y - 1)$
15. focus $(3, -5)$, $y = 1 \rightarrow (x - 3)^2 = -12(y + 2)$
16. focus $(-3, 7)$, $y = -1 \rightarrow (x + 3)^2 = 16(y - 3)$
17. focus $(-3, -2)$, $y = 2 \rightarrow (x + 3)^2 = -8y$

Choosing How to Find a Limit

Limits

1. $\lim_{x \to 1} \frac{-x + 2}{x + 2} = \frac{1}{3}$
2. $\lim_{x \to 1} \frac{-x + 2}{x - 3} = -\frac{1}{2}$
3. $\lim_{x \to -2} \frac{x^2 + x - 2}{x^2 + 3x + 2} = 3$
4. $\lim_{x \to 1} (x^2 - 2x - 3) = -4$
5. $\lim_{x \to 3} \frac{-x - 1}{x - 1} = -2$
6. $\lim_{x \to 3} \frac{x - 2}{x + 2} = \frac{1}{5}$
7. $\lim_{x \to \infty} \frac{-3x}{x + 2} = -3$
8. $\lim_{x \to 2} \frac{x - 2}{x + 2} = 0$
9. $\lim_{x \to 2} \frac{x - 1}{x - 3} = -1$
10. $\lim_{x \to -3} (x^2 + x + 3) = 9$

Answer Key (3 of 3)

Choosing How to Find a Limit (continued)

Degrees and radians

11. 120° in radians = $2\pi/3$
12. 60° in radians = $\pi/3$
13. 300° in radians = $5\pi/3$
14. $\pi/2$ in degrees = 90°
15. $5\pi/6$ in degrees = 150°
16. π in degrees = 180°

Solutions (1 of 4)

Radians and Degrees

Degrees and radians

1. $\frac{\pi}{4}$ in degrees = ____

$$\frac{\pi}{4} \times \boxed{180/\pi}$$

$$\frac{\pi}{4} \text{ in degrees} = \boxed{45°}$$

π radians is 180°, so multiply radians by 180/π to get degrees.

The π cancels, leaving 180 ÷ 4 = 45°.

2. 180° in radians = ____

$$180° \times \boxed{\frac{\pi}{180}}$$

$$180° \text{ in radians} = \boxed{\pi}$$

A straight angle is 180° and also π radians, so multiply degrees by $\frac{\pi}{180}$.

$180 \times \frac{\pi}{180} = \frac{180\pi}{180}$, which is π.

Simplifying with Trigonometric Identities

Trigonometric identities

1. $2 + 3(\sin^2 A + \cos^2 A)$ = ____

$$\sin^2 A + \cos^2 A = \boxed{1}$$

$$(\sin^2 A + \cos^2 A) \rightarrow \boxed{1}$$

$$2 + 3(\sin^2 A + \cos^2 A) = \boxed{5}$$

Every one of these collapses on the same fact: $\sin^2 A + \cos^2 A = 1$, for any angle at all. Find it in the expression and the rest is arithmetic.

Here it is in plain sight. Replace the whole $\sin^2 A + \cos^2 A$ with 1 and leave everything around it alone.

What is left is a number: 5.

2. $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B)$ = ____

$$\sin^2 A + \cos^2 A = \boxed{1}$$

$$(\sin^2 A + \cos^2 A) \rightarrow \boxed{1}$$

$$(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) = \boxed{2}$$

Every one of these collapses on the same fact: $\sin^2 A + \cos^2 A = 1$, for any angle at all. Find it in the expression and the rest is arithmetic.

Here it is in plain sight. Replace the whole $\sin^2 A + \cos^2 A$ with 1 and leave everything around it alone.

What is left is a number: 2.

Solutions (2 of 4)

The Equation of a Circle

The equation of a circle

1. $(x + 2)^2 + y^2 = 4$

$$(x + 2)^2 + y^2 = 4$$

center $(-2, 0)$ $r = 2$

In $(x + 2)^2 + y^2 = r^2$, the center is the numbers in the brackets with their signs REVERSED, and the right-hand side is r^2, not r.

So the center is $(-2, 0)$ and the radius is $\sqrt{4} = 2$.

2. $(x - 6)^2 + (y + 3)^2 = 4$

$$(x - 6)^2 + (y + 3)^2 = 4$$

center $(6, -3)$ $r = 2$

In $(x - 6)^2 + (y + 3)^2 = r^2$, the center is the numbers in the brackets with their signs REVERSED, and the right-hand side is r^2, not r.

So the center is $(6, -3)$ and the radius is $\sqrt{4} = 2$.

The Equation of a Parabola

The equation of a parabola

1. focus $(1, 3)$ directrix $y = 1$

vertex $= (1, 2)$

$p = 3 - 2 = 1$

$(x - 1)^2 = 4(y - 2)$

Every point on a parabola is the same distance from the focus as from the directrix, so the vertex sits halfway between them: halfway from 3 to 1 is 2. The vertex is $(1, 2)$.

p is the gap from the vertex to the focus, and it is SIGNED: the focus is above the vertex, so $p = 1$ and the parabola opens upwards.

Put the vertex and p into $(x - h)^2 = 4p(y - k)$. With $4p = 4$, that is the equation.

Solutions (3 of 4)

The Equation of a Parabola (continued)

The equation of a parabola (continued)

2. **focus (−4, −1)** directrix y = 1

vertex = (−4, 0)

p = −1 − 0 = −1

$(x + 4)^2 = -4y$

Every point on a parabola is the same distance from the focus as from the directrix, so the vertex sits halfway between them: halfway from −1 to 1 is 0. The vertex is (−4, 0).

p is the gap from the vertex to the focus, and it is SIGNED: the focus is below the vertex, so p = −1 and the parabola opens downwards.

Put the vertex and p into $(x - h)^2 = 4p(y - k)$. With $4p = -4$, that is the equation.

Limits by Substitution

Limits

1. $\lim_{x \to -1} (-x^2 + x + 3)$

$$\lim_{x \to -1} (-x^2 + x + 3) =$$

$$-(-1)^2 + (-1) + 3$$

$$= -1 - 1 + 3$$

$$= 1$$

A polynomial has no breaks and no gaps, so its limit is just its value there. Put x = −1 into every term: $-(-1)^2 + (-1) + 3$.

Work out each term on its own: −1 − 1 + 3.

Add them up. The limit is 1.

Solutions (4 of 4)

Limits by Substitution (continued)

Limits (continued)

2. $\lim_{x \to 1} (-x^2 + 2x - 2)$

$$\lim_{x \to 1} (-x^2 + 2x - 2) =$$

$$-1^2 + 2(1) - 2$$

$$= -1 + 2 - 2$$

$$= -1$$

A polynomial has no breaks and no gaps, so its limit is just its value there. Put $x = 1$ into every term: $-1^2 + 2(1) - 2$.

Work out each term on its own: $-1 + 2 - 2$.

Add them up. The limit is -1.