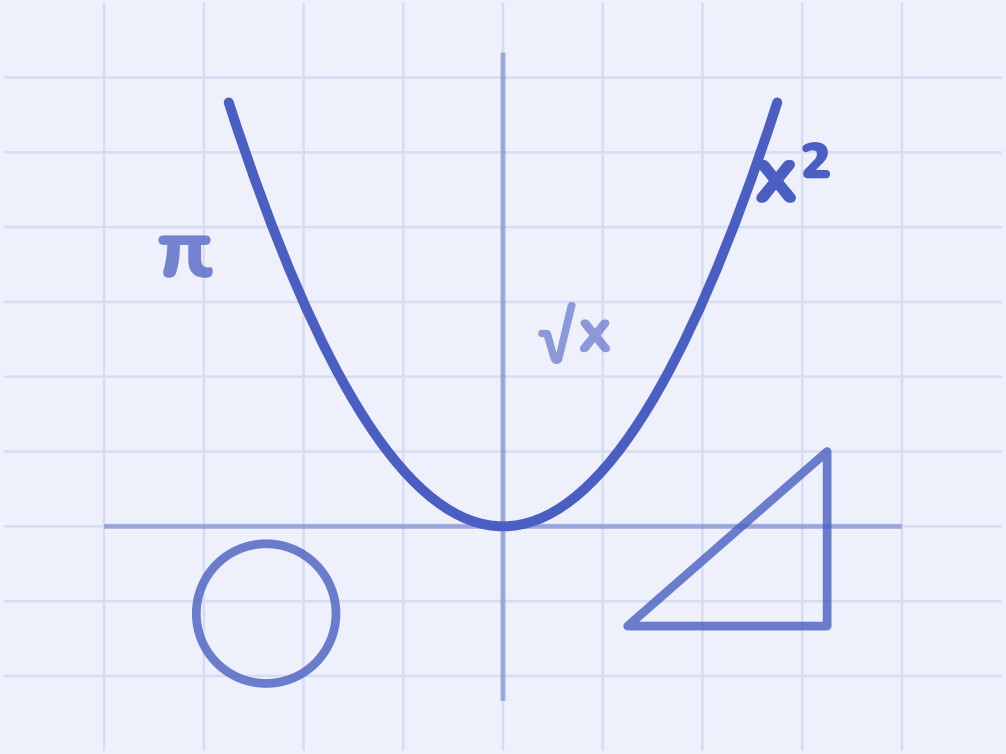
MATH PICNIC · WORKBOOK

Precalculus Math Workbook

10 lessons · Ages 16–18 · answer key at the back



Name _____ Date _____

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1-1

Radians and Degrees

WORKED EXAMPLE

$\frac{5\pi}{3}$ in degrees = **300°**

1. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

1 $\frac{\pi}{4}$ in degrees = _____

2 180° in radians = _____

3 225° in radians = _____

4 135° in radians = _____

5 $\frac{\pi}{2}$ in degrees = _____

6 $\frac{5\pi}{4}$ in degrees = _____

7 π in degrees = _____

8 150° in radians = _____

9 120° in radians = _____

10 315° in radians = _____

11 $\frac{2\pi}{3}$ in degrees = _____

12 $\frac{5\pi}{6}$ in degrees = _____

13 330° in radians = _____

14 30° in radians = _____

15 $\frac{3\pi}{4}$ in degrees = _____

16 $\frac{4\pi}{3}$ in degrees = _____

Exact Values on the Unit Circle

WORKED EXAMPLE

$$\sin 45° = \frac{\sqrt{2}}{2}$$

2. Exact values on the unit circle

Give each exact value. Leave surds and fractions exact — no calculator.

1 $\cos 60° =$ _____

2 $\cos 90° =$ _____

3 $\cos 30° =$ _____

4 $\sin 30° =$ _____

5 $\cos 0° =$ _____

6 $\sin 60° =$ _____

7 $\sin 0° =$ _____

8 $\tan 60° =$ _____

9 $\sin 90° =$ _____

10 $\cos 45° =$ _____

11 $\tan 30° =$ _____

12 $\tan 0° =$ _____

Simplifying with Trigonometric Identities

3. Trigonometric identities

Simplify each expression. Every answer is a whole number.

1 $2 + 3(\sin^2 A + \cos^2 A) =$ ____

2 $5 - 5\sin^2 A - 5\cos^2 A =$ ____

3 $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) =$ ____

4 $\sin^2 A + \cos^2 A + 6 =$ ____

5 $12(1 - \sin^2 A) \div (1 - \sin^2 A) =$ ____

6 $\sin^2 A + \cos^2 A =$ ____

7 $10 - \sin^2 A - \cos^2 A =$ ____

8 $4(\sin^2 A + \cos^2 A) =$ ____

9 $8(\sin^2 A + \cos^2 A)^3 =$ ____

10 $10(1 - \cos^2 A) \div \sin^2 A =$ ____

11 $6(1 - \sin^2 A) \div \cos^2 A =$ ____

12 $3\sin^2 A \div (1 - \cos^2 A) =$ ____

The Equation of a Circle

WORKED EXAMPLE

$(x - 5)^2 + (y + 4)^2 = 16$

center (5, −4) r = 4

4. The equation of a circle

Write the center and radius of each circle.

1 $(x - 3)^2 + (y - 5)^2 = 36$

2 $(x + 1)^2 + (y - 2)^2 = 16$

3 $(x + 1)^2 + (y - 6)^2 = 9$

4 $(x - 4)^2 + (y - 2)^2 = 36$

5 $(x + 2)^2 + (y + 1)^2 = 16$

6 $x^2 + (y + 1)^2 = 16$

7 $(x - 1)^2 + (y + 3)^2 = 9$

8 $(x - 1)^2 + (y - 2)^2 = 36$

9 $(x + 5)^2 + (y - 2)^2 = 25$

10 $(x + 1)^2 + (y + 1)^2 = 16$

11 $(x + 2)^2 + (y - 3)^2 = 4$

12 $(x - 1)^2 + (y - 1)^2 = 64$

5. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

13 $\frac{3\pi}{4}$ in degrees = _____

14 $\frac{4\pi}{3}$ in degrees = _____

15 $\frac{\pi}{3}$ in degrees = _____

16 $\frac{\pi}{2}$ in degrees = _____

17 240° in radians = _____

18 60° in radians = _____

A Circle from Its General Form

WORKED EXAMPLE

$x^2 + y^2 - 10x + 12y + 57 = 0$

$(x^2 - 10x) + (y^2 + 12y) = -57$
$(x^2 - 10x + 25) + (y^2 + 12y + 36) = 4$
$(x - 5)^2 + (y + 6)^2 = 4$
center $(5, -6)$ $r = 2$

6. The equation of a circle

Complete the square to find the center and radius of each circle.

1 **$x^2 + y^2 - 12x - 10y + 25 = 0$**

2 **$x^2 + y^2 + 2x - 10y + = 0$**

3 **$x^2 + y^2 - 4x - 8y - 61 = 0$**

4 **$x^2 + y^2 + 10x + 6y - 47 = 0$**

5 **$x^2 + y^2 + 4x = 0$**

6 **$x^2 + y^2 + 8x - 4y - 5 = 0$**

7 **$x^2 + y^2 - 10x - 6y - 66 = 0$**

8 **$x^2 + y^2 + 8x - 12y - 48 = 0$**

9 **$x^2 + y^2 - 8x + 6y + 21 = 0$**

10 **$x^2 + y^2 - 10x + 8y + 25 = 0$**

7. Exact values on the unit circle

Give each exact value. Leave surds and fractions exact — no calculator.

11 $\sin 90° =$ _____

12 $\tan 0° =$ _____

13 $\cos 90° =$ _____

14 $\tan 60° =$ _____

15 $\sin 0° =$ _____

16 $\sin 60° =$ _____

The Equation of a Parabola

WORKED EXAMPLE

focus (−4, 4) **directrix y = 0**

vertex = (−4, 2)
p = 4 − 2 = 2
$(x + 4)^2 = 8(y - 2)$

8. The equation of a parabola

Work between a parabola's equation and its focus and directrix.

1 **focus (3, 3)** **directrix y = 5**

2 $(x + 1)^2 = 8(y + 3)$

3 **focus (1, 3)** **directrix y = 1**

4 $(x - 1)^2 = -4(y - 1)$

5 **focus (−1, 0)** **directrix y = 2**

6 $(x - 1)^2 = 8y$

7 $x^2 = -4(y + 4)$

8 **focus (4, 7)** **directrix y = −1**

9 **focus (−4, −2)** **directrix y = 4**

10 **focus (0, −2)** **directrix y = 0**

9. Trigonometric identities

Simplify each expression. Every answer is a whole number.

11 $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) =$ ____

12 $3\sin^2 A \div (1 - \cos^2 A) =$ ____

13 $2 + 3(\sin^2 A + \cos^2 A) =$ ____

14 $12(1 - \sin^2 A) \div (1 - \sin^2 A) =$ ____

15 $4(\sin^2 A + \cos^2 A) =$ ____

16 $10 - \sin^2 A - \cos^2 A =$ ____

Limits by Substitution

WORKED EXAMPLE

$$\lim_{x\to -1} \frac{-x-3}{x+2}$$

$$\lim_{x\to -1} \frac{-x-3}{x+2} = \frac{-(-1)-3}{(-1)+2}$$

$$= -2$$

10. Limits

Find each limit. Check the bottom first: if it is not 0 there, substitute.

1 $\lim_{x\to 1} (-x^2 - 2x - 1)$

2 $\lim_{x\to 3} (x^2 + 2x - 2)$

3 $\lim_{x\to 1} (-x^2 - 3x - 1)$

4 $\lim_{x\to -3} \frac{-x-1}{x-2}$

5 $\lim_{x\to -2} (-x^2 + x - 2)$

6 $\lim_{x\to -1} (x^2 - 3x - 3)$

7 $\lim_{x\to 3} \frac{x-2}{x+2}$

8 $\lim_{x\to -3} (x^2 - 3x - 1)$

9 $\lim_{x\to -1} \frac{-x}{x+2}$

10 $\lim_{x\to 2} \frac{-x-2}{x+3}$

11 $\lim_{x\to 2} (-x^2 + 2x - 2)$

12 $\lim_{x\to -1} (-x^2 - 2x - 1)$

11. The equation of a circle

Write the center and radius of each circle.

13 $(x+3)^2 + (y-3)^2 = 4$

14 $x^2 + (y+3)^2 = 64$

15 $x^2 + (y+4)^2 = 9$

16 $(x+6)^2 + (y+1)^2 = 25$

17 $(x-2)^2 + (y+1)^2 = 25$

18 $(x-3)^2 + (y-2)^2 = 36$

Limits by Factoring

WORKED EXAMPLE

$$\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6}$$

$x = 2$: $\frac{0}{0}$

$$\lim_{x\to 2}\frac{(x-2)(x-1)}{(x-2)(x-3)}$$

$$= \lim_{x\to 2}\frac{x-1}{x-3}$$

$$= \frac{1}{-1}$$

$$= -1$$

12. Limits

Substituting makes the top and bottom both 0: factor first, then cancel.

1 $\lim_{x\to 2}\frac{x^2+x-6}{x-2}$

2 $\lim_{x\to 1}\frac{x^2-3x+2}{x^2-4x+3}$

3 $\lim_{x\to 2}\frac{x^2-5x+6}{x^2-2x}$

4 $\lim_{x\to 1}\frac{x^2-1}{x^2-x}$

5 $\lim_{x\to 3}\frac{x^2-3x}{x^2-9}$

6 $\lim_{x\to 2}\frac{x^2-5x+6}{x^2+x-6}$

7 $\lim_{x\to 2}\frac{x^2-2x}{x^2-3x+2}$

8 $\lim_{x\to -1}\frac{x^2+3x+2}{x^2-2x-3}$

9 $\lim_{x\to -2}\frac{x^2+5x+6}{x^2-x-6}$

10 $\lim_{x\to -1}\frac{x^2-x-2}{x^2-1}$

13. The equation of a circle

Complete the square to find the center and radius of each circle.

11 $x^2 + y^2 - 6x - 8y + 21 = 0$

12 $x^2 + y^2 - 6x + 12y - 36 = 0$

13 $x^2 + y^2 + 4x + 8y + 11 = 0$

14 $x^2 + y^2 - 10x - 6y + 25 = 0$

15 $x^2 + y^2 + 12x - 10y + 25 = 0$

Limits at Infinity

14. Limits

Find each limit. Divide every term by the highest power of x on the bottom.

1 $\lim_{x\to\infty} \frac{-2x - 3}{x + 1}$

2 $\lim_{x\to\infty} \frac{x^2 - 2x - 2}{3x^2 + 2}$

3 $\lim_{x\to\infty} \frac{-x^2 - 2x - 2}{3x^2 + 2}$

4 $\lim_{x\to\infty} \frac{-3x + 1}{2x + 1}$

5 $\lim_{x\to\infty} \frac{3x + 3}{x^2 - 2}$

6 $\lim_{x\to\infty} \frac{2x}{3x + 3}$

7 $\lim_{x\to\infty} \frac{-x}{2x^2 - 1}$

8 $\lim_{x\to\infty} \frac{3x + 1}{x - 2}$

9 $\lim_{x\to\infty} \frac{-3x^2 + 2x - 2}{x^2 + 2}$

10 $\lim_{x\to\infty} \frac{x}{2x + 1}$

11 $\lim_{x\to\infty} \frac{-3x^2 + x + 3}{3x^2 - 3}$

12 $\lim_{x\to\infty} \frac{x - 1}{x^2 + 1}$

15. The equation of a parabola

Work between a parabola's equation and its focus and directrix.

13 **focus (1, 3)** directrix $y = 1$

14 **focus (−3, 2)** directrix $y = 6$

15 $(x - 1)^2 = 16(y + 1)$

16 **focus (4, −7)** directrix $y = -1$

17 **focus (0, 2)** directrix $y = -4$

Choosing How to Find a Limit

WORKED EXAMPLE

$$\lim_{x\to 3}(x^2+x+3)$$
$$= 3^2+3+3$$
$$= 9+3+3$$
$$= 15$$

16. Limits

Find each limit. Substitute first; what you get decides the next move.

1 $\lim_{x\to 3}\dfrac{x^2-9}{x-3}$

2 $\lim_{x\to -3}\dfrac{x+3}{x-1}$

3 $\lim_{x\to \infty}\dfrac{-2x}{x^2-3}$

4 $\lim_{x\to 3}\dfrac{x^2-x-6}{x^2-3x}$

5 $\lim_{x\to \infty}\dfrac{-x+3}{2x^2-3}$

6 $\lim_{x\to -2}\dfrac{x^2-x-6}{x^2+3x+2}$

7 $\lim_{x\to \infty}\dfrac{-3x-1}{3x^2+2}$

8 $\lim_{x\to 3}(x^2-3x-3)$

9 $\lim_{x\to \infty}\dfrac{-3x+1}{3x-1}$

10 $\lim_{x\to 2}\dfrac{x^2-3x+2}{x^2+x-6}$

17. Degrees and radians

Convert between degrees and radians. Leave radians in terms of π.

11 $\frac{3\pi}{2}$ in degrees = _____

12 $\frac{\pi}{3}$ in degrees = _____

13 $\frac{7\pi}{6}$ in degrees = _____

14 180° in radians = _____

15 240° in radians = _____

16 330° in radians = _____

Answer Key (1 of 3)

Radians and Degrees

Degrees and radians

1. $\pi/4$ in degrees = 45°
2. 180° in radians = π
3. 225° in radians = $5\pi/4$
4. 135° in radians = $3\pi/4$
5. $\pi/2$ in degrees = 90°
6. $5\pi/4$ in degrees = 225°
7. π in degrees = 180°
8. 150° in radians = $5\pi/6$
9. 120° in radians = $2\pi/3$
10. 315° in radians = $7\pi/4$
11. $2\pi/3$ in degrees = 120°
12. $5\pi/6$ in degrees = 150°
13. 330° in radians = $11\pi/6$
14. 30° in radians = $\pi/6$
15. $3\pi/4$ in degrees = 135°
16. $4\pi/3$ in degrees = 240°

Exact Values on the Unit Circle

Exact values on the unit circle

1. $\cos 60° = \frac{1}{2}$
2. $\cos 90° = 0$
3. $\cos 30° = \frac{\sqrt{3}}{2}$
4. $\sin 30° = \frac{1}{2}$
5. $\cos 0° = 1$
6. $\sin 60° = \frac{\sqrt{3}}{2}$
7. $\sin 0° = 0$
8. $\tan 60° = \sqrt{3}$
9. $\sin 90° = 1$
10. $\cos 45° = \frac{\sqrt{2}}{2}$
11. $\tan 30° = \frac{\sqrt{3}}{3}$
12. $\tan 0° = 0$

Simplifying with Trigonometric Identities

Trigonometric identities

1. $2 + 3(\sin^2 A + \cos^2 A) = 5$
2. $5 - 5\sin^2 A - 5\cos^2 A = 0$
3. $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) = 2$
4. $\sin^2 A + \cos^2 A + 6 = 7$
5. $12(1 - \sin^2 A) \div (1 - \sin^2 A) = 12$
6. $\sin^2 A + \cos^2 A = 1$
7. $10 - \sin^2 A - \cos^2 A = 9$
8. $4(\sin^2 A + \cos^2 A) = 4$
9. $8(\sin^2 A + \cos^2 A)^3 = 8$
10. $10(1 - \cos^2 A) \div \sin^2 A = 10$
11. $6(1 - \sin^2 A) \div \cos^2 A = 6$
12. $3\sin^2 A \div (1 - \cos^2 A) = 3$

The Equation of a Circle

The equation of a circle

1. $(x - 3)^2 + (y - 5)^2 = 36$ → center (3, 5), radius 6
2. $(x + 1)^2 + (y - 2)^2 = 16$ → center (−1, 2), radius 4
3. $(x + 1)^2 + (y - 6)^2 = 9$ → center (−1, 6), radius 3
4. $(x - 4)^2 + (y - 2)^2 = 36$ → center (4, 2), radius 6
5. $(x + 2)^2 + (y + 1)^2 = 16$ → center (−2, −1), radius 4
6. $x^2 + (y + 1)^2 = 16$ → center (0, −1), radius 4
7. $(x - 1)^2 + (y + 3)^2 = 9$ → center (1, −3), radius 3
8. $(x - 1)^2 + (y - 2)^2 = 36$ → center (1, 2), radius 6
9. $(x + 5)^2 + (y - 2)^2 = 25$ → center (−5, 2), radius 5
10. $(x + 1)^2 + (y + 1)^2 = 16$ → center (−1, −1), radius 4
11. $(x + 2)^2 + (y - 3)^2 = 4$ → center (−2, 3), radius 2
12. $(x - 1)^2 + (y - 1)^2 = 64$ → center (1, 1), radius 8

Degrees and radians

13. $3\pi/4$ in degrees = 135°
14. $4\pi/3$ in degrees = 240°
15. $\pi/3$ in degrees = 60°
16. $\pi/2$ in degrees = 90°
17. 240° in radians = $4\pi/3$
18. 60° in radians = $\pi/3$

A Circle from Its General Form

The equation of a circle

1. $x^2 + y^2 - 12x - 10y + 25 = 0$ → center (6, 5), radius 6
2. $x^2 + y^2 + 2x - 10y + = 0$ → center (−1, 5), radius 5
3. $x^2 + y^2 - 4x - 8y - 61 = 0$ → center (2, 4), radius 9
4. $x^2 + y^2 + 10x + 6y - 47 = 0$ → center (−5, −3), radius 9
5. $x^2 + y^2 + 4x = 0$ → center (−2, 0), radius 2
6. $x^2 + y^2 + 8x - 4y - 5 = 0$ → center (−4, 2), radius 5
7. $x^2 + y^2 - 10x - 6y - 66 = 0$ → center (5, 3), radius 10
8. $x^2 + y^2 + 8x - 12y - 48 = 0$ → center (−4, 6), radius 10
9. $x^2 + y^2 - 8x + 6y + 21 = 0$ → center (4, −3), radius 2
10. $x^2 + y^2 - 10x + 8y + 25 = 0$ → center (5, −4), radius 4

Exact values on the unit circle

11. $\sin 90° = 1$
12. $\tan 0° = 0$
13. $\cos 90° = 0$
14. $\tan 60° = \sqrt{3}$
15. $\sin 0° = 0$
16. $\sin 60° = \frac{\sqrt{3}}{2}$

The Equation of a Parabola

The equation of a parabola

1. focus (3, 3), $y = 5$ → $(x - 3)^2 = -4(y - 4)$
2. $(x + 1)^2 = 8(y + 3)$ → focus (−1, −1), directrix $y = -5$
3. focus (1, 3), $y = 1$ → $(x - 1)^2 = 4(y - 2)$
4. $(x - 1)^2 = -4(y - 1)$ → focus (1, 0), directrix $y = 2$
5. focus (−1, 0), $y = 2$ → $(x + 1)^2 = -4(y - 1)$
6. $(x - 1)^2 = 8y$ → focus (1, 2), directrix $y = -2$

Answer Key (2 of 3)

The Equation of a Parabola (continued)

The equation of a parabola (continued)

7. $x^2 = -4(y + 4)$ → focus $(0, -5)$, directrix $y = -3$
8. focus $(4, 7)$, $y = -1$ → $(x - 4)^2 = 16(y - 3)$
9. focus $(-4, -2)$, $y = 4$ → $(x + 4)^2 = -12(y - 1)$
10. focus $(0, -2)$, $y = 0$ → $x^2 = -4(y + 1)$

Trigonometric identities

11. $(\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) = 2$
12. $3\sin^2 A \div (1 - \cos^2 A) = 3$
13. $2 + 3(\sin^2 A + \cos^2 A) = 5$
14. $12(1 - \sin^2 A) \div (1 - \sin^2 A) = 12$
15. $4(\sin^2 A + \cos^2 A) = 4$
16. $10 - \sin^2 A - \cos^2 A = 9$

Limits by Substitution

Limits

1. $\lim_{x \to 1} (-x^2 - 2x - 1) = -4$
2. $\lim_{x \to 3} (x^2 + 2x - 2) = 13$
3. $\lim_{x \to 1} (-x^2 - 3x - 1) = -5$
4. $\lim_{x \to -3} \frac{-x - 1}{x - 2} = -\frac{2}{5}$
5. $\lim_{x \to -2} (-x^2 + x - 2) = -8$
6. $\lim_{x \to -1} (x^2 - 3x - 3) = 1$
7. $\lim_{x \to 3} \frac{x - 2}{x + 2} = \frac{1}{5}$
8. $\lim_{x \to -3} (x^2 - 3x - 1) = 17$
9. $\lim_{x \to -1} \frac{-x}{x + 2} = 1$
10. $\lim_{x \to 2} \frac{-x - 2}{x + 3} = -\frac{4}{5}$
11. $\lim_{x \to 2} (-x^2 + 2x - 2) = -2$
12. $\lim_{x \to -1} (-x^2 - 2x - 1) = 0$

The equation of a circle

13. $(x + 3)^2 + (y - 3)^2 = 4$ → center $(-3, 3)$, radius 2
14. $x^2 + (y + 3)^2 = 64$ → center $(0, -3)$, radius 8
15. $x^2 + (y + 4)^2 = 9$ → center $(0, -4)$, radius 3
16. $(x + 6)^2 + (y + 1)^2 = 25$ → center $(-6, -1)$, radius 5
17. $(x - 2)^2 + (y + 1)^2 = 25$ → center $(2, -1)$, radius 5
18. $(x - 3)^2 + (y - 2)^2 = 36$ → center $(3, 2)$, radius 6

Limits by Factoring

Limits

1. $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} = 5$
2. $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3} = \frac{1}{2}$
3. $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 2x} = -\frac{1}{2}$
4. $\lim_{x \to 1} \frac{x^2 - 1}{x^2 - x} = 2$
5. $\lim_{x \to 3} \frac{x^2 - 3x}{x^2 - 9} = \frac{1}{2}$
6. $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 + x - 6} = -\frac{1}{5}$
7. $\lim_{x \to 2} \frac{x^2 - 2x}{x^2 - 3x + 2} = 2$
8. $\lim_{x \to -1} \frac{x^2 + 3x + 2}{x^2 - 2x - 3} = -\frac{1}{4}$
9. $\lim_{x \to -2} \frac{x^2 + 5x + 6}{x^2 - x - 6} = -\frac{1}{5}$

Limits by Factoring (continued)

Limits (continued)

10. $\lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 1} = \frac{3}{2}$

The equation of a circle

11. $x^2 + y^2 - 6x - 8y + 21 = 0$ → center $(3, 4)$, radius 2
12. $x^2 + y^2 - 6x + 12y - 36 = 0$ → center $(3, -6)$, radius 9
13. $x^2 + y^2 + 4x + 8y + 11 = 0$ → center $(-2, -4)$, radius 3
14. $x^2 + y^2 - 10x - 6y + 25 = 0$ → center $(5, 3)$, radius 3
15. $x^2 + y^2 + 12x - 10y + 25 = 0$ → center $(-6, 5)$, radius 6

Limits at Infinity

Limits

1. $\lim_{x \to \infty} \frac{-2x - 3}{x + 1} = -2$
2. $\lim_{x \to \infty} \frac{x^2 - 2x - 2}{3x^2 + 2} = \frac{1}{3}$
3. $\lim_{x \to \infty} \frac{-x^2 - 2x - 2}{3x^2 + 2} = -\frac{1}{3}$
4. $\lim_{x \to \infty} \frac{-3x + 1}{2x + 1} = -\frac{3}{2}$
5. $\lim_{x \to \infty} \frac{3x + 3}{x^2 - 2} = 0$
6. $\lim_{x \to \infty} \frac{2x}{3x + 3} = \frac{2}{3}$
7. $\lim_{x \to \infty} \frac{-x}{2x^2 - 1} = 0$
8. $\lim_{x \to \infty} \frac{3x + 1}{x - 2} = 3$
9. $\lim_{x \to \infty} \frac{-3x^2 + 2x - 2}{x^2 + 2} = -3$
10. $\lim_{x \to \infty} \frac{x}{2x + 1} = \frac{1}{2}$
11. $\lim_{x \to \infty} \frac{-3x^2 + x + 3}{3x^2 - 3} = -1$
12. $\lim_{x \to \infty} \frac{x - 1}{x^2 + 1} = 0$

The equation of a parabola

13. focus $(1, 3)$, $y = 1$ → $(x - 1)^2 = 4(y - 2)$
14. focus $(-3, 2)$, $y = 6$ → $(x + 3)^2 = -8(y - 4)$
15. $(x - 1)^2 = 16(y + 1)$ → focus $(1, 3)$, directrix $y = -5$
16. focus $(4, -7)$, $y = -1$ → $(x - 4)^2 = -12(y + 4)$
17. focus $(0, 2)$, $y = -4$ → $x^2 = 12(y + 1)$

Choosing How to Find a Limit

Limits

1. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$
2. $\lim_{x \to -3} \frac{x + 3}{x - 1} = 0$
3. $\lim_{x \to \infty} \frac{-2x}{x^2 - 3} = 0$
4. $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 3x} = \frac{5}{3}$
5. $\lim_{x \to \infty} \frac{-x + 3}{2x^2 - 3} = 0$
6. $\lim_{x \to -2} \frac{x^2 - x - 6}{x^2 + 3x + 2} = 5$
7. $\lim_{x \to \infty} \frac{-3x - 1}{3x^2 + 2} = 0$
8. $\lim_{x \to 3} (x^2 - 3x - 3) = -3$
9. $\lim_{x \to \infty} \frac{-3x + 1}{3x - 1} = -1$
10. $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 + x - 6} = \frac{1}{5}$

Degrees and radians

11. $3\pi/2$ in degrees = 270°

Answer Key (3 of 3)

Choosing How to Find a Limit (continued)

Degrees and radians (continued)

12. π/3 in degrees = 60°
13. 7π/6 in degrees = 210°
14. 180° in radians = π
15. 240° in radians = 4π/3
16. 330° in radians = 11π/6

Solutions (1 of 3)

Radians and Degrees

Degrees and radians

1. $\frac{\pi}{4}$ in degrees = ____

$$\frac{\pi}{4} \times \boxed{180/\pi}$$

$$\frac{\pi}{4} \text{ in degrees} = \boxed{45°}$$

π radians is 180°, so multiply radians by 180/π to get degrees.

The π cancels, leaving 180 ÷ 4 = 45°.

2. 180° in radians = ____

$$180° \times \boxed{\frac{\pi}{180}}$$

$$180° \text{ in radians} = \boxed{\pi}$$

A straight angle is 180° and also π radians, so multiply degrees by $\frac{\pi}{180}$.

$180 \times \frac{\pi}{180} = \frac{180\pi}{180}$, which is π.

Simplifying with Trigonometric Identities

Trigonometric identities

1. $2 + 3(\sin^2 A + \cos^2 A)$ = ____

$$\sin^2 A + \cos^2 A = \boxed{1}$$

$$(\sin^2 A + \cos^2 A) \rightarrow \boxed{1}$$

$$2 + 3(\sin^2 A + \cos^2 A) = \boxed{5}$$

Every one of these collapses on the same fact: $\sin^2 A + \cos^2 A = 1$, for any angle at all. Find it in the expression and the rest is arithmetic.

Here it is in plain sight. Replace the whole $\sin^2 A + \cos^2 A$ with 1 and leave everything around it alone.

What is left is a number: 5.

2. $5 - 5\sin^2 A - 5\cos^2 A$ = ____

$$\sin^2 A + \cos^2 A = \boxed{1}$$

$$(\sin^2 A + \cos^2 A) \rightarrow \boxed{1}$$

$$5 - 5\sin^2 A - 5\cos^2 A = \boxed{0}$$

Every one of these collapses on the same fact: $\sin^2 A + \cos^2 A = 1$, for any angle at all. Find it in the expression and the rest is arithmetic.

Here it is in plain sight. Replace the whole $\sin^2 A + \cos^2 A$ with 1 and leave everything around it alone.

What is left is a number: 0.

Solutions (2 of 3)

The Equation of a Circle

The equation of a circle

1. $(x - 3)^2 + (y - 5)^2 = 36$

$$(x - 3)^2 + (y - 5)^2 = 36$$

center $(3, 5)$ $r = 6$

In $(x - 3)^2 + (y - 5)^2 = r^2$, the center is the numbers in the brackets with their signs REVERSED, and the right-hand side is r^2, not r.

So the center is (3, 5) and the radius is $\sqrt{36} = 6$.

2. $(x + 1)^2 + (y - 2)^2 = 16$

$$(x + 1)^2 + (y - 2)^2 = 16$$

center $(-1, 2)$ $r = 4$

In $(x + 1)^2 + (y - 2)^2 = r^2$, the center is the numbers in the brackets with their signs REVERSED, and the right-hand side is r^2, not r.

So the center is (−1, 2) and the radius is $\sqrt{16} = 4$.

The Equation of a Parabola

The equation of a parabola

1. focus $(3, 3)$ directrix $y = 5$

vertex = $(3, 4)$

$p = 3 - 4 = -1$

$(x - 3)^2 = -4(y - 4)$

Every point on a parabola is the same distance from the focus as from the directrix, so the vertex sits halfway between them: halfway from 3 to 5 is 4. The vertex is (3, 4).

p is the gap from the vertex to the focus, and it is SIGNED: the focus is below the vertex, so $p = -1$ and the parabola opens downwards.

Put the vertex and p into $(x - h)^2 = 4p(y - k)$. With $4p = -4$, that is the equation.

Solutions (3 of 3)

The Equation of a Parabola (continued)

The equation of a parabola (continued)

2. $(x + 1)^2 = 8(y + 3)$

vertex = (−1, −3)

$4p = 8 \rightarrow$ **p = 2**

focus (−1, −1) · **y = −5**

Read the vertex off the brackets, remembering that the signs flip: $(x + 1)^2 = 8(y + 3)$ has its vertex at (−1, −3).

The number multiplying the bracket is 4p, so $p = 8 \div 4 = 2$. It is positive, so the parabola opens upwards.

The focus is p from the vertex, and the directrix is the same distance the other way.

Limits by Substitution

Limits

1. $\lim_{x \to 1} (-x^2 - 2x - 1)$

$$\lim_{x \to 1} (-x^2 - 2x - 1) =$$

$$-1^2 - 2(1) - 1$$

$$= -1 - 2 - 1$$

$$= \mathbf{-4}$$

Read the vertex off the brackets, remembering that the signs flip: $(x + 1)^2 = 8(y + 3)$ has its vertex at (−1, −3).

The number multiplying the bracket is 4p, so $p = 8 \div 4 = 2$. It is positive, so the parabola opens upwards.

The focus is p from the vertex, and the directrix is the same distance the other way.

2. $\lim_{x \to 3} (x^2 + 2x - 2)$

$$\lim_{x \to 3} (x^2 + 2x - 2) =$$

$$3^2 + 2(3) - 2$$

$$= 9 + 6 - 2$$

$$= \mathbf{13}$$

A polynomial has no breaks and no gaps, so its limit is just its value there. Put x = 1 into every term: $-1^2 - 2(1) - 1$.

Work out each term on its own: $-1 - 2 - 1$.

Add them up. The limit is −4.

A polynomial has no breaks and no gaps, so its limit is just its value there. Put x = 3 into every term: $3^2 + 2(3) - 2$.

Work out each term on its own: $9 + 6 - 2$.

Add them up. The limit is 13.