

Counting triangles — Grids up to 12 dots

Name: _____

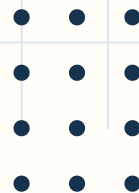
Date: _____

Count the triangles you can make from a grid of dots, and show why the straight lines come off.



- 1 An orchard is planted in a rectangle of 3 trees by 4 trees, 12 trees in all. A triangle is any three trees taken as its corners.

How many triangles can be made?



Why Every triangle is just 3 of the 12 trees chosen as corners, so start from 12 choose 3, $C(12,3) = 220$, built one factor at a time: $1 \times 12 = 12$, $12 \div 1 = 12$, $12 \times 11 = 132$, $132 \div 2 = 66$, $66 \times 10 = 660$, $660 \div 3 = 220$. But three trees on one straight line make no triangle. The straight-line triples here are 4 along the rows, 12 down the columns, 4 on slanted lines — 20 in all. Take them off: $220 - 20 = 200$. So 200 triangles, not the 220 you get by forgetting that some threes are in a line.

200

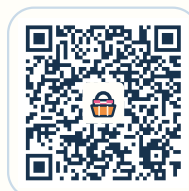
- 2 A grid of dots is 2 across and 6 down, so 12 dots in all. A triangle is any three of the dots joined up as corners.

How many triangles can be made?



Why

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Count the triangles you can make from a grid of dots, and show why the straight lines come off.

- 3 A geoboard has its pegs in a rectangle 2 pegs across and 3 pegs down — 6 pegs altogether. A triangle is any three pegs joined by rubber bands.

How many triangles can be made?



Why

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- 4 A grid of dots is 3 across and 3 down, so 9 dots in all. A triangle is any three of the dots joined up as corners.

How many triangles can be made?

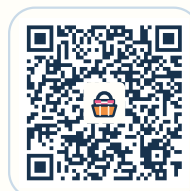


Why

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Counting triangles · Grids up to 12 dots — Answer key

If they are stuck

- A triangle is a choice of three dots. Count all the choices, then take off the threes that sit in a straight line.



1. Every triangle is just 3 of the 12 trees chosen as corners, so start from 12 choose 3, $C(12,3) = 220$, built one factor at a time: $1 \times 12 = 12$, $12 \div 1 = 12$, $12 \times 11 = 132$, $132 \div 2 = 66$, $66 \times 10 = 660$, $660 \div 3 = 220$. But three trees on one straight line make no triangle. The straight-line triples here are 4 along the rows, 12 down the columns, 4 on slanted lines — 20 in all. Take them off: $220 - 20 = 200$. So 200 triangles, not the 220 you get by forgetting that some threes are in a line.

3. Every triangle is just 3 of the 6 pegs chosen as corners, so start from 6 choose 3, $C(6,3) = 20$, built one factor at a time: $1 \times 6 = 6$, $6 \div 1 = 6$, $6 \times 5 = 30$, $30 \div 2 = 15$, $15 \times 4 = 60$, $60 \div 3 = 20$. But three pegs on one straight line make no triangle. The straight-line triples here are 2 down the columns — 2 in all. Take them off: $20 - 2 = 18$. So 18 triangles, not the 20 you get by forgetting that some threes are in a line.

2. Every triangle is just 3 of the 12 dots chosen as corners, so start from 12 choose 3, $C(12,3) = 220$, built one factor at a time: $1 \times 12 = 12$, $12 \div 1 = 12$, $12 \times 11 = 132$, $132 \div 2 = 66$, $66 \times 10 = 660$, $660 \div 3 = 220$. But three dots on one straight line make no triangle. The straight-line triples here are 40 down the columns — 40 in all. Take them off: $220 - 40 = 180$. So 180 triangles, not the 220 you get by forgetting that some threes are in a line.

4. Every triangle is just 3 of the 9 dots chosen as corners, so start from 9 choose 3, $C(9,3) = 84$, built one factor at a time: $1 \times 9 = 9$, $9 \div 1 = 9$, $9 \times 8 = 72$, $72 \div 2 = 36$, $36 \times 7 = 252$, $252 \div 3 = 84$. But three dots on one straight line make no triangle. The straight-line triples here are 3 along the rows, 3 down the columns, 2 on slanted lines — 8 in all. Take them off: $84 - 8 = 76$. So 76 triangles, not the 84 you get by forgetting that some threes are in a line.