Diagonal counting — Up to 20 corners

Name: ______________________ Date: ______________

Count them without drawing them. Show your working.

1 10 people sit in a ring. Everyone shakes hands with everyone else except the two sitting next to them.

How many handshakes are there?

Why Every pair of the 10 could be joined. Each person reaches 9 others, so 10 × 9 = 90 — and each pair is counted from both ends, so 90 ÷ 2 = 45 lines in all. But 10 of them join neighbors, so take them off: 45 – 10 = 35 handshakes. More than one way works here and it never counts the neighbor pairs: each person joins all but itself and its two neighbors, so 10 × 7 = 70, and 70 ÷ 2 = 35.

35

2 14 towns are arranged in a ring, and a road already joins each town to the two beside it. New roads are built so that every pair of towns is joined directly.

How many new roads are built?

Why

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Count them without drawing them. Show your working.

3 A polygon has 12 sides.

How many diagonals does it have?

Why

____

4 13 people sit in a ring. Everyone shakes hands with everyone else except the two sitting next to them.

How many handshakes are there?

Why

____

Diagonal counting · Up to 20 corners — Answer key

If they are stuck

- Join every pair first, including the ones already joined. What do you then take off?

1. Every pair of the 10 could be joined. Each person reaches 9 others, so 10 × 9 = 90 — and each pair is counted from both ends, so 90 ÷ 2 = 45 lines in all. But 10 of them join neighbors, so take them off: 45 – 10 = 35 handshakes. More than one way works here and it never counts the neighbor pairs: each person joins all but itself and its two neighbors, so 10 × 7 = 70, and 70 ÷ 2 = 35.

2. Every pair of the 14 could be joined. Each town reaches 13 others, so 14 × 13 = 182 — and each pair is counted from both ends, so 182 ÷ 2 = 91 lines in all. But 14 of those roads already exist, so take them off: 91 – 14 = 77 new roads. More than one way works here and it never counts the roads already there: each town joins all but itself and its two neighbors, so 14 × 11 = 154, and 154 ÷ 2 = 77.

3. Every pair of the 12 could be joined. Each corner reaches 11 others, so 12 × 11 = 132 — and each pair is counted from both ends, so 132 ÷ 2 = 66 lines in all. But 12 of them are its sides, so take them off: 66 – 12 = 54 diagonals. More than one way works here and it never counts the sides: each corner joins all but itself and its two neighbors, so 12 × 9 = 108, and 108 ÷ 2 = 54.

4. Every pair of the 13 could be joined. Each person reaches 12 others, so 13 × 12 = 156 — and each pair is counted from both ends, so 156 ÷ 2 = 78 lines in all. But 13 of them join neighbors, so take them off: 78 – 13 = 65 handshakes. More than one way works here and it never counts the neighbor pairs: each person joins all but itself and its two neighbors, so 13 × 10 = 130, and 130 ÷ 2 = 65.