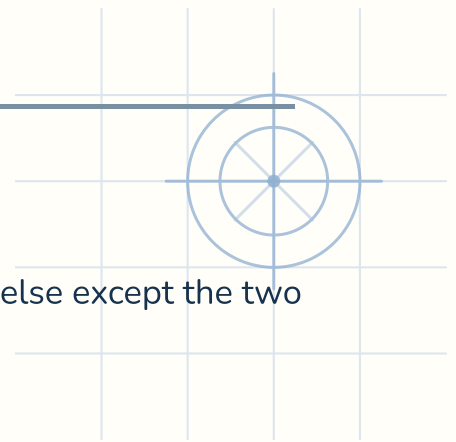


Diagonal counting — Up to 20 corners

Name: _____

Date: _____



Count them without drawing them. Show your working.

- 1 10 people sit in a ring. Everyone shakes hands with everyone else except the two sitting next to them.

How many handshakes are there?

Why Every pair of the 10 could be joined. Each person reaches 9 others, so $10 \times 9 = 90$ — and each pair is counted from both ends, so $90 \div 2 = 45$ lines in all. But 10 of them join neighbors, so take them off: $45 - 10 = 35$ handshakes. More than one way works here and it never counts the neighbor pairs: each person joins all but itself and its two neighbors, so $10 \times 7 = 70$, and $70 \div 2 = 35$.

35

- 2 14 towns are arranged in a ring, and a road already joins each town to the two beside it. New roads are built so that every pair of towns is joined directly.

How many new roads are built?

Why

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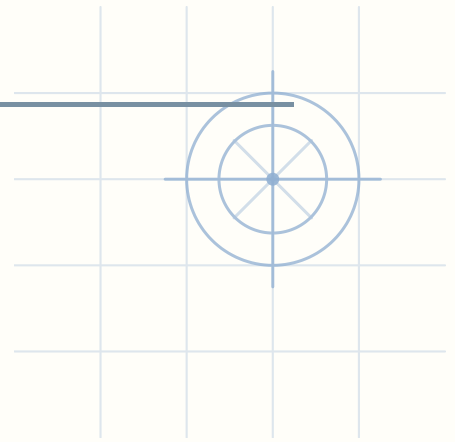
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Diagonal counting — Up to 20 corners

Name: _____

Date: _____



Count them without drawing them. Show your working.

- 3 A polygon has 12 sides.
How many diagonals does it have?

Why

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- 4 13 people sit in a ring. Everyone shakes hands with everyone else except the two sitting next to them.

How many handshakes are there?

Why

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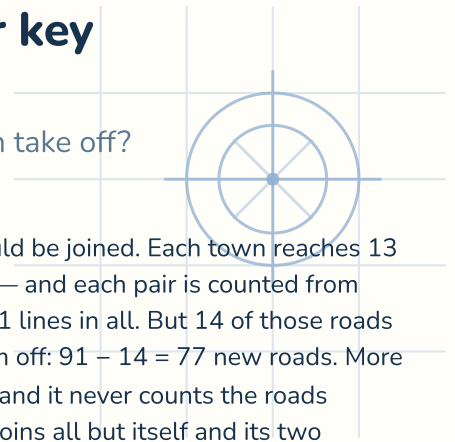
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Diagonal counting · Up to 20 corners — Answer key

If they are stuck

- Join every pair first, including the ones already joined. What do you then take off?



1. Every pair of the 10 could be joined. Each person reaches 9 others, so $10 \times 9 = 90$ — and each pair is counted from both ends, so $90 \div 2 = 45$ lines in all. But 10 of them join neighbors, so take them off: $45 - 10 = 35$ handshakes. More than one way works here and it never counts the neighbor pairs: each person joins all but itself and its two neighbors, so $10 \times 7 = 70$, and $70 \div 2 = 35$.

3. Every pair of the 12 could be joined. Each corner reaches 11 others, so $12 \times 11 = 132$ — and each pair is counted from both ends, so $132 \div 2 = 66$ lines in all. But 12 of them are its sides, so take them off: $66 - 12 = 54$ diagonals. More than one way works here and it never counts the sides: each corner joins all but itself and its two neighbors, so $12 \times 9 = 108$, and $108 \div 2 = 54$.

2. Every pair of the 14 could be joined. Each town reaches 13 others, so $14 \times 13 = 182$ — and each pair is counted from both ends, so $182 \div 2 = 91$ lines in all. But 14 of those roads already exist, so take them off: $91 - 14 = 77$ new roads. More than one way works here and it never counts the roads already there: each town joins all but itself and its two neighbors, so $14 \times 11 = 154$, and $154 \div 2 = 77$.

4. Every pair of the 13 could be joined. Each person reaches 12 others, so $13 \times 12 = 156$ — and each pair is counted from both ends, so $156 \div 2 = 78$ lines in all. But 13 of them join neighbors, so take them off: $78 - 13 = 65$ handshakes. More than one way works here and it never counts the neighbor pairs: each person joins all but itself and its two neighbors, so $13 \times 10 = 130$, and $130 \div 2 = 65$.