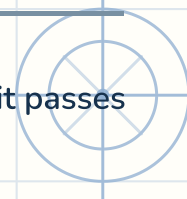


Cubes a diagonal pierces — Edges up to 8

Name: _____

Date: _____



A rod is pushed corner to corner through a block of unit cubes. Count the ones it passes through, and show why $a + b + c$ is too many.

- 1 A solid block is built from unit cubes, 3 along one edge, 7 along the next and 8 along the last. A straight rod is pushed from one corner right through to the opposite corner.

How many of the little cubes does it pass through?

Why Corner to corner, the rod enters a new little cube at every wall it crosses — 18 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens $\gcd(3,7) = 1$, $\gcd(7,8) = 1$ and $\gcd(8,3) = 1$ times, 3 in all, each an over-count of one. Taking them off: $18 - 3 = 15$. All three walls meet together $\gcd(3,7,8) = 1$ times, and that was subtracted once too often, so add it back: $15 + 1 = 16$.

16

- 2 A cuboid of sugar is 2 small cubes wide, 3 deep and 4 tall. A thin skewer is run from a bottom corner straight to the far top corner.

How many of the small cubes does it pass through?

Why

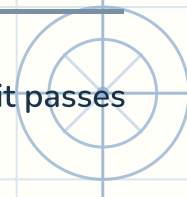
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Cubes a diagonal pierces — Edges up to 8

Name: _____

Date: _____



A rod is pushed corner to corner through a block of unit cubes. Count the ones it passes through, and show why $a + b + c$ is too many.

- 3 A $2 \times 6 \times 6$ box is packed exactly with unit cubes. A laser is aimed from one corner to the corner diagonally opposite, passing straight through the block.

How many of the unit cubes does it pass through?

Why

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- 4 A $2 \times 7 \times 8$ box is packed exactly with unit cubes. A laser is aimed from one corner to the corner diagonally opposite, passing straight through the block.

How many of the unit cubes does it pass through?

Why

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Cubes a diagonal pierces · Edges up to 8 — Answer key

If they are stuck

- Count $a + b + c$ first. Then ask where the rod crosses two walls at once — those got counted twice.



1. Corner to corner, the rod enters a new little cube at every wall it crosses — 18 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens $\gcd(3,7) = 1$, $\gcd(7,8) = 1$ and $\gcd(8,3) = 1$ times, 3 in all, each an over-count of one. Taking them off: $18 - 3 = 15$. All three walls meet together $\gcd(3,7,8) = 1$ times, and that was subtracted once too often, so add it back: $15 + 1 = 16$.

3. Corner to corner, the rod enters a new unit cube at every wall it crosses — 14 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens $\gcd(2,6) = 2$, $\gcd(6,6) = 6$ and $\gcd(6,2) = 2$ times, 10 in all, each an over-count of one. Taking them off: $14 - 10 = 4$. All three walls meet together $\gcd(2,6,6) = 2$ times, and that was subtracted once too often, so add it back: $4 + 2 = 6$.

2. Corner to corner, the rod enters a new small cube at every wall it crosses — 9 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens $\gcd(2,3) = 1$, $\gcd(3,4) = 1$ and $\gcd(4,2) = 2$ times, 4 in all, each an over-count of one. Taking them off: $9 - 4 = 5$. All three walls meet together $\gcd(2,3,4) = 1$ times, and that was subtracted once too often, so add it back: $5 + 1 = 6$.

4. Corner to corner, the rod enters a new unit cube at every wall it crosses — 17 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens $\gcd(2,7) = 1$, $\gcd(7,8) = 1$ and $\gcd(8,2) = 2$ times, 4 in all, each an over-count of one. Taking them off: $17 - 4 = 13$. All three walls meet together $\gcd(2,7,8) = 1$ times, and that was subtracted once too often, so add it back: $13 + 1 = 14$.