Domino tilings — Strips up to 8 long

Name: ______________________ Date: ____________

Count the ways to tile a 2-by-n strip with dominoes, and show why it is not a power of two.

1. A hallway floor is 2 tiles wide and 4 tiles long, and it is covered exactly by 2-by-1 tiles with none overlapping or sticking out.

In how many different ways can it be covered?

Why Look at the LAST column of the 2-by-4 hall. Either one domino stands upright and fills it, leaving a strip 3 long, or two dominoes lie flat across the last two columns, leaving a strip 2 long. So the ways for length 4 are the ways for 3 plus the ways for 2. Counting up: there is 1 way to tile a strip 1 long and 2 ways for one 2 long, and then each length adds the two before it: $2 + 1 = 3$, $3 + 2 = 5$. So 5 ways. It is not 8 — the last column is not a free choice, because a flat domino uses two columns at once.

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2. A patio is 2 slabs wide and 8 slabs long, and it is paved with 1-by-2 paving stones laid flat.

In how many different ways can it be covered?

Why

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Name: ______________________ Date: ______________

Count the ways to tile a 2-by-n strip with dominoes, and show why it is not a power of two.

3 A board is 2 squares tall and 6 squares wide, and it is covered completely by 1-by-2 dominoes.

In how many different ways can it be covered?

Why

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4 A patio is 2 slabs wide and 3 slabs long, and it is paved with 1-by-2 paving stones laid flat.

In how many different ways can it be covered?

Why

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Domino tilings · Strips up to 8 long — Answer key

> **If they are stuck**
>
> - Look at the last column: one upright domino, or two flat ones. So each length is the two shorter lengths added — not a doubling.

1. Look at the LAST column of the 2-by-4 hall. Either one domino stands upright and fills it, leaving a strip 3 long, or two dominoes lie flat across the last two columns, leaving a strip 2 long. So the ways for length 4 are the ways for 3 plus the ways for 2. Counting up: there is 1 way to tile a strip 1 long and 2 ways for one 2 long, and then each length adds the two before it: 2 + 1 = 3, 3 + 2 = 5. So 5 ways. It is not 8 — the last column is not a free choice, because a flat domino uses two columns at once.

2. Look at the LAST column of the 2-by-8 patio. Either one domino stands upright and fills it, leaving a strip 7 long, or two dominoes lie flat across the last two columns, leaving a strip 6 long. So the ways for length 8 are the ways for 7 plus the ways for 6. Counting up: there is 1 way to tile a strip 1 long and 2 ways for one 2 long, and then each length adds the two before it: 2 + 1 = 3, 3 + 2 = 5, 5 + 3 = 8, 8 + 5 = 13, 13 + 8 = 21, 21 + 13 = 34. So 34 ways. It is not 128 — the last column is not a free choice, because a flat domino uses two columns at once.

3. Look at the LAST column of the 2-by-6 board. Either one domino stands upright and fills it, leaving a strip 5 long, or two dominoes lie flat across the last two columns, leaving a strip 4 long. So the ways for length 6 are the ways for 5 plus the ways for 4. Counting up: there is 1 way to tile a strip 1 long and 2 ways for one 2 long, and then each length adds the two before it: 2 + 1 = 3, 3 + 2 = 5, 5 + 3 = 8, 8 + 5 = 13. So 13 ways. It is not 32 — the last column is not a free choice, because a flat domino uses two columns at once.

4. Look at the LAST column of the 2-by-3 patio. Either one domino stands upright and fills it, leaving a strip 2 long, or two dominoes lie flat across the last two columns, leaving a strip 1 long. So the ways for length 3 are the ways for 2 plus the ways for 1. Counting up: there is 1 way to tile a strip 1 long and 2 ways for one 2 long, and then each length adds the two before it: 2 + 1 = 3. So 3 ways. It is not 4 — the last column is not a free choice, because a flat domino uses two columns at once.