Name: ______________________ Date: __________

Count the triangles with whole-number sides and the given perimeter, and show why the impossible ones are out.

1. 19 matchsticks of equal length are laid end to end to make the three sides of a triangle, with none left over and none broken.

How many different triangles can be made? (The same triangle turned around counts once.)

Why Sides $a \le b \le c$ with $a + b + c = 19$ only make a triangle when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 17) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 9, 9), (2, 8, 9), (3, 7, 9), (3, 8, 8), (4, 6, 9), ..., (6, 6, 7). That is 10 different triangles.

10

2. A farmer has 27 metres of fence and wants a triangular pen whose three sides are each a whole number of metres.

How many differently shaped triangular pens can be built? (Turning or flipping a pen over does not make it a new one.)

Why

Name: ____________________ Date: __________

Count the triangles with whole-number sides and the given perimeter, and show why the impossible ones are out.

3 29 matchsticks of equal length are laid end to end to make the three sides of a triangle, with none left over and none broken.

How many different triangles can be made? (The same triangle turned around counts once.)

Why

___

4 A farmer has 21 metres of fence and wants a triangular pen whose three sides are each a whole number of metres.

How many differently shaped triangular pens can be built? (Turning or flipping a pen over does not make it a new one.)

Why

___

Integer triangles · Perimeter up to 30 — Answer key

If they are stuck

- Fix the longest side first, then ask which shorter pairs can actually reach across it.

1. Sides a ≤ b ≤ c with a + b + c = 19 only make a triangle when a + b > c — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 17) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 9, 9), (2, 8, 9), (3, 7, 9), (3, 8, 8), (4, 6, 9), ..., (6, 6, 7). That is 10 different triangles.

2. Sides a ≤ b ≤ c with a + b + c = 27 only make a pen when a + b > c — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 25) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 13, 13), (2, 12, 13), (3, 11, 13), (3, 12, 12), (4, 10, 13), ..., (9, 9, 9). That is 19 different triangles.

3. Sides a ≤ b ≤ c with a + b + c = 29 only make a triangle when a + b > c — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 27) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 14, 14), (2, 13, 14), (3, 12, 14), (3, 13, 13), (4, 11, 14), ..., (9, 10, 10). That is 21 different triangles.

4. Sides a ≤ b ≤ c with a + b + c = 21 only make a pen when a + b > c — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 19) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 10, 10), (2, 9, 10), (3, 8, 10), (3, 9, 9), (4, 7, 10), ..., (7, 7, 7). That is 12 different triangles.