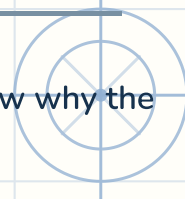


Integer triangles — Perimeter up to 30

Name: _____

Date: _____



Count the triangles with whole-number sides and the given perimeter, and show why the impossible ones are out.

- 1 19 matchsticks of equal length are laid end to end to make the three sides of a triangle, with none left over and none broken.

How many different triangles can be made? (The same triangle turned around counts once.)

Why Sides $a \leq b \leq c$ with $a + b + c = 19$ only make a triangle when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 17) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 9, 9), (2, 8, 9), (3, 7, 9), (3, 8, 8), (4, 6, 9), ..., (6, 6, 7). That is 10 different triangles.

10

- 2 A farmer has 27 metres of fence and wants a triangular pen whose three sides are each a whole number of metres.

How many differently shaped triangular pens can be built? (Turning or flipping a pen over does not make it a new one.)

Why

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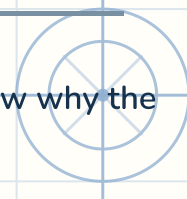
.....



Integer triangles — Perimeter up to 30

Name: _____

Date: _____



Count the triangles with whole-number sides and the given perimeter, and show why the impossible ones are out.

- 3 29 matchsticks of equal length are laid end to end to make the three sides of a triangle, with none left over and none broken.

How many different triangles can be made? (The same triangle turned around counts once.)

Why

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- 4 A farmer has 21 metres of fence and wants a triangular pen whose three sides are each a whole number of metres.

How many differently shaped triangular pens can be built? (Turning or flipping a pen over does not make it a new one.)

Why

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.....

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Integer triangles · Perimeter up to 30 — Answer key

If they are stuck

- Fix the longest side first, then ask which shorter pairs can actually reach across it.



1. Sides $a \leq b \leq c$ with $a + b + c = 19$ only make a triangle when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 17) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 9, 9), (2, 8, 9), (3, 7, 9), (3, 8, 8), (4, 6, 9), ..., (6, 6, 7). That is 10 different triangles.

3. Sides $a \leq b \leq c$ with $a + b + c = 29$ only make a triangle when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 27) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 14, 14), (2, 13, 14), (3, 12, 14), (3, 13, 13), (4, 11, 14), ..., (9, 10, 10). That is 21 different triangles.

2. Sides $a \leq b \leq c$ with $a + b + c = 27$ only make a triangle when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 25) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 13, 13), (2, 12, 13), (3, 11, 13), (3, 12, 12), (4, 10, 13), ..., (9, 9, 9). That is 19 different triangles.

4. Sides $a \leq b \leq c$ with $a + b + c = 21$ only make a triangle when $a + b > c$ — the two shorter sides have to out-reach the longest, or they never close up. So a split like (1, 1, 19) is out. Walking the shortest side up from 1 and keeping the splits that obey it: (1, 10, 10), (2, 9, 10), (3, 8, 10), (3, 9, 9), (4, 7, 10), ..., (7, 7, 7). That is 12 different triangles.