Square gaps — Numbers up to 30

Name: _____ Date: _____

Two facts each time, and neither is enough alone. Show how you narrowed it.

1 A carpenter has two square boards. The larger has 133 square centimeters more surface than the smaller, and each side is a whole number of centimeters between 4 and 15.

How long is the side of each board?

Why A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 133. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $1 \times 133 \rightarrow 66$ and 67; $7 \times 19 \rightarrow 6$ and 13. Only 6 and 13 are both between 4 and 15. Check: $13 \times 13 = 169$, $6 \times 6 = 36$, and $169 - 36 = 133$.

6 and 13

2 Two square patches of floor are laid with the same tiles. The bigger patch takes 207 more tiles than the smaller, and both patches are between 6 and 18 tiles along each side.

How many tiles long is each patch?

Why

_____ and _____

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Two facts each time, and neither is enough alone. Show how you narrowed it.

3 A carpenter has two square boards. The larger has 224 square centimeters more surface than the smaller, and each side is a whole number of centimeters between 8 and 20.

How long is the side of each board?

Why

_____ and _____

4 Two whole numbers are both between 2 and 11. Squaring the bigger one gives 39 more than squaring the smaller one.

What are the two numbers?

Why

_____ and _____

Square gaps · Numbers up to 30 — Answer key

If they are stuck

- What does $a^2 - b^2$ factor into? Read that backwards.

1. A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 133. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $1 \times 133 \rightarrow 66$ and 67; $7 \times 19 \rightarrow 6$ and 13. Only 6 and 13 are both between 4 and 15. Check: $13 \times 13 = 169$, $6 \times 6 = 36$, and $169 - 36 = 133$.

2. A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 207. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $1 \times 207 \rightarrow 103$ and 104; $3 \times 69 \rightarrow 33$ and 36; $9 \times 23 \rightarrow 7$ and 16. Only 7 and 16 are both between 6 and 18. Check: $16 \times 16 = 256$, $7 \times 7 = 49$, and $256 - 49 = 207$.

3. A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 224. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $2 \times 112 \rightarrow 55$ and 57; $4 \times 56 \rightarrow 26$ and 30; $8 \times 28 \rightarrow 10$ and 18; $14 \times 16 \rightarrow 1$ and 15. Only 10 and 18 are both between 8 and 20. Check: $18 \times 18 = 324$, $10 \times 10 = 100$, and $324 - 100 = 224$.

4. A difference of two squares always factorises: $a^2 - b^2 = (a + b)(a - b)$, so the two numbers' sum and difference multiply to 39. Both factors must be even or both odd, or halving gives fractions. Halving what is left: $1 \times 39 \rightarrow 19$ and 20; $3 \times 13 \rightarrow 5$ and 8. Only 5 and 8 are both between 2 and 11. Check: $8 \times 8 = 64$, $5 \times 5 = 25$, and $64 - 25 = 39$.