

## Locker doors — Up to 100 lockers

Name: \_\_\_\_\_

Date: \_\_\_\_\_



A row of lockers, and every student flips a share of them. Say how many end open, and why.

- 1 A school corridor has 68 lockers in a row, numbered 1 to 68, all shut. Student 1 opens every locker. Student 2 changes every second locker (2, 4, 6, ...). Student 3 changes every third, and so on up to student 68 — each student  $k$  changes every locker whose number  $k$  divides.

**In the end, how many lockers are left open?**

**Why** Each locker numbered  $L$  is changed once for each divisor of  $L$ . Divisors come in pairs ( $d$  with  $L \div d$ ), an even count, so a locker ends shut unless a divisor pairs with itself — that is, unless  $L$  is a perfect square. The lockers left open are the perfect squares up to 68: 1, 4, 9, 16, ..., 64. The largest is  $8 \times 8 = 64$ , so there are 8 of them and 8 lockers stay open.

8

- 2 A hall has 55 lamps in a line, numbered 1 to 55, all off. Switch 1 flips every lamp. Switch 2 flips every second lamp, switch 3 every third, and so on up to switch 55 — switch  $k$  flips every lamp whose number  $k$  divides.

**In the end, how many lamps are left on?**

**Why**

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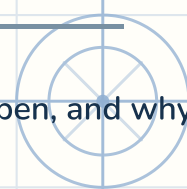
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## Locker doors — Up to 100 lockers

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A row of lockers, and every student flips a share of them. Say how many end open, and why.

- 3 A hall has 87 lamps in a line, numbered 1 to 87, all off. Switch 1 flips every lamp. Switch 2 flips every second lamp, switch 3 every third, and so on to switch 87 — switch  $k$  flips every lamp whose number  $k$  divides.

**In the end, how many lamps are left on?**

**Why**

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- 4 A long hallway has 56 doors, numbered 1 to 56, all closed. Person 1 opens every door. Person 2 changes every second door, person 3 every third, and so on to person 56 — person  $k$  changes every door whose number  $k$  divides.

**In the end, how many doors are left open?**

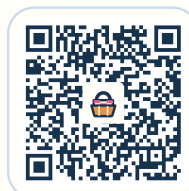
**Why**

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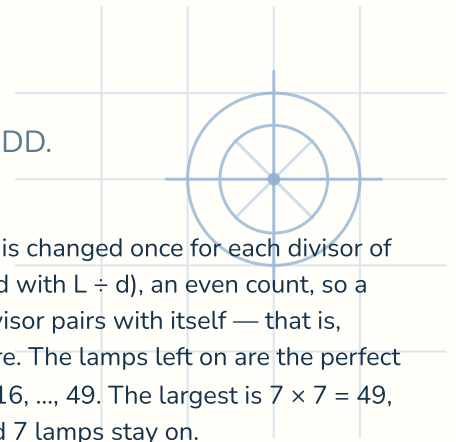
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# Locker doors · Up to 100 lockers — Answer key

## If they are stuck

- How many times does each one get changed? Ask when that count is ODD.



1. Each locker numbered  $L$  is changed once for each divisor of  $L$ . Divisors come in pairs ( $d$  with  $L \div d$ ), an even count, so a locker ends shut unless a divisor pairs with itself — that is, unless  $L$  is a perfect square. The lockers left open are the perfect squares up to 68: 1, 4, 9, 16, ..., 64. The largest is  $8 \times 8 = 64$ , so there are 8 of them and 8 lockers stay open.

3. Each lamp numbered  $L$  is changed once for each divisor of  $L$ . Divisors come in pairs ( $d$  with  $L \div d$ ), an even count, so a lamp ends off unless a divisor pairs with itself — that is, unless  $L$  is a perfect square. The lamps left on are the perfect squares up to 87: 1, 4, 9, 16, ..., 81. The largest is  $9 \times 9 = 81$ , so there are 9 of them and 9 lamps stay on.

2. Each lamp numbered  $L$  is changed once for each divisor of  $L$ . Divisors come in pairs ( $d$  with  $L \div d$ ), an even count, so a lamp ends off unless a divisor pairs with itself — that is, unless  $L$  is a perfect square. The lamps left on are the perfect squares up to 55: 1, 4, 9, 16, ..., 49. The largest is  $7 \times 7 = 49$ , so there are 7 of them and 7 lamps stay on.

4. Each door numbered  $L$  is changed once for each divisor of  $L$ . Divisors come in pairs ( $d$  with  $L \div d$ ), an even count, so a door ends closed unless a divisor pairs with itself — that is, unless  $L$  is a perfect square. The doors left open are the perfect squares up to 56: 1, 4, 9, 16, ..., 49. The largest is  $7 \times 7 = 49$ , so there are 7 of them and 7 doors stay open.