Counting triangles — Grids up to 12 dots

Name: _____ Date: _____

> Count the triangles you can make from a grid of dots, and show why the straight lines come off.

1 An orchard is planted in a rectangle of 3 trees by 4 trees, 12 trees in all. A triangle is any three trees taken as its corners.

How many triangles can be made?

Why

_____

2 A grid of dots is 2 across and 6 down, so 12 dots in all. A triangle is any three of the dots joined up as corners.

How many triangles can be made?

Why

_____

Name: _____ Date: _____

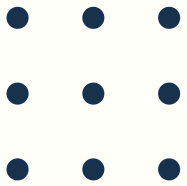
Count the triangles you can make from a grid of dots, and show why the straight lines come off.

3 A geoboard has its pegs in a rectangle 2 pegs across and 3 pegs down — 6 pegs altogether. A triangle is any three pegs joined by rubber bands.

How many triangles can be made?

Why

4 A grid of dots is 3 across and 3 down, so 9 dots in all. A triangle is any three of the dots joined up as corners.



How many triangles can be made?

Why

Counting triangles · Grids up to 12 dots — Answer key

If they are stuck

- A triangle is a choice of three dots. Count all the choices, then take off the threes that sit in a straight line.

1. Every triangle is just 3 of the 12 trees chosen as corners, so start from 12 choose 3, C(12,3) = 220, built one factor at a time: 1 × 12 = 12, 12 ÷ 1 = 12, 12 × 11 = 132, 132 ÷ 2 = 66, 66 × 10 = 660, 660 ÷ 3 = 220. But three trees on one straight line make no triangle. The straight-line triples here are 4 along the rows, 12 down the columns, 4 on slanted lines — 20 in all. Take them off: 220 − 20 = 200. So 200 triangles, not the 220 you get by forgetting that some threes are in a line.

2. Every triangle is just 3 of the 12 dots chosen as corners, so start from 12 choose 3, C(12,3) = 220, built one factor at a time: 1 × 12 = 12, 12 ÷ 1 = 12, 12 × 11 = 132, 132 ÷ 2 = 66, 66 × 10 = 660, 660 ÷ 3 = 220. But three dots on one straight line make no triangle. The straight-line triples here are 40 down the columns — 40 in all. Take them off: 220 − 40 = 180. So 180 triangles, not the 220 you get by forgetting that some threes are in a line.

3. Every triangle is just 3 of the 6 pegs chosen as corners, so start from 6 choose 3, C(6,3) = 20, built one factor at a time: 1 × 6 = 6, 6 ÷ 1 = 6, 6 × 5 = 30, 30 ÷ 2 = 15, 15 × 4 = 60, 60 ÷ 3 = 20. But three pegs on one straight line make no triangle. The straight-line triples here are 2 down the columns — 2 in all. Take them off: 20 − 2 = 18. So 18 triangles, not the 20 you get by forgetting that some threes are in a line.

4. Every triangle is just 3 of the 9 dots chosen as corners, so start from 9 choose 3, C(9,3) = 84, built one factor at a time: 1 × 9 = 9, 9 ÷ 1 = 9, 9 × 8 = 72, 72 ÷ 2 = 36, 36 × 7 = 252, 252 ÷ 3 = 84. But three dots on one straight line make no triangle. The straight-line triples here are 3 along the rows, 3 down the columns, 2 on slanted lines — 8 in all. Take them off: 84 − 8 = 76. So 76 triangles, not the 84 you get by forgetting that some threes are in a line.