Cubes a diagonal pierces — **Edges up to 8**

Name: ______________________ Date: ____________

A rod is pushed corner to corner through a block of unit cubes. Count the ones it passes through, and show why a + b + c is too many.

1. A solid block is built from unit cubes, 3 along one edge, 7 along the next and 8 along the last. A straight rod is pushed from one corner right through to the opposite corner.

   How many of the little cubes does it pass through?

   Why

   ___

2. A cuboid of sugar is 2 small cubes wide, 3 deep and 4 tall. A thin skewer is run from a bottom corner straight to the far top corner.

   How many of the small cubes does it pass through?

   Why

   ___

Name: ____________________ Date: __________

A rod is pushed corner to corner through a block of unit cubes. Count the ones it passes through, and show why a + b + c is too many.

3. A 2 × 6 × 6 box is packed exactly with unit cubes. A laser is aimed from one corner to the corner diagonally opposite, passing straight through the block.

How many of the unit cubes does it pass through?

Why

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4. A 2 × 7 × 8 box is packed exactly with unit cubes. A laser is aimed from one corner to the corner diagonally opposite, passing straight through the block.

How many of the unit cubes does it pass through?

Why

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Cubes a diagonal pierces · Edges up to 8 — Answer key

If they are stuck

- Count a + b + c first. Then ask where the rod crosses two walls at once — those got counted twice.

1. Corner to corner, the rod enters a new little cube at every wall it crosses — 18 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens gcd(3,7) = 1, gcd(7,8) = 1 and gcd(8,3) = 1 times, 3 in all, each an over-count of one. Taking them off: 18 − 3 = 15. All three walls meet together gcd(3,7,8) = 1 times, and that was subtracted once too often, so add it back: 15 + 1 = 16.

2. Corner to corner, the rod enters a new small cube at every wall it crosses — 9 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens gcd(2,3) = 1, gcd(3,4) = 1 and gcd(4,2) = 2 times, 4 in all, each an over-count of one. Taking them off: 9 − 4 = 5. All three walls meet together gcd(2,3,4) = 1 times, and that was subtracted once too often, so add it back: 5 + 1 = 6.

3. Corner to corner, the rod enters a new unit cube at every wall it crosses — 14 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens gcd(2,6) = 2, gcd(6,6) = 6 and gcd(6,2) = 2 times, 10 in all, each an over-count of one. Taking them off: 14 − 10 = 4. All three walls meet together gcd(2,6,6) = 2 times, and that was subtracted once too often, so add it back: 4 + 2 = 6.

4. Corner to corner, the rod enters a new unit cube at every wall it crosses — 17 of them if none lined up, which is the count to beat. But two walls meeting on one line are crossed together: that happens gcd(2,7) = 1, gcd(7,8) = 1 and gcd(8,2) = 2 times, 4 in all, each an over-count of one. Taking them off: 17 − 4 = 13. All three walls meet together gcd(2,7,8) = 1 times, and that was subtracted once too often, so add it back: 13 + 1 = 14.