Invariant puzzles — Up to 16 numbers

Name: _____ Date: _____

Find what every move leaves alone. Show why the order cannot matter.

1 The numbers 3 to 13 are written on a board. Rub out any two of them and write their sum plus 2 in their place. Keep going until one is left.

What is the last number?

Why

2 The numbers 1 to 10 are written on cards. Two cards are taken away and replaced by one showing their sum plus 3. Repeat until one card is left.

What number is on the last card?

Why

Name: _____ Date: _____

Find what every move leaves alone. Show why the order cannot matter.

3 The numbers 4 to 19 are written on a board. Rub out any two of them and write their sum minus 1 in their place. Keep going until one is left.

What is the last number?

Why

4 The numbers 3 to 11 are written on a board. Rub out any two of them and write their sum minus 3 in their place. Keep going until one is left.

What is the last number?

Why

Invariant puzzles · Up to 16 numbers — Answer key

If they are stuck

- Play two moves. The numbers all change — is there anything that does not?

1. Each move rubs out two numbers and writes one in their place, so the count falls by 1 each time: 11 numbers means 10 moves. Each move also raises the TOTAL by exactly 2, so 10 × 2 = 20 is gained altogether. The numbers add to 88, so whatever order anyone uses, the last number is 88 + 20 = 108.

2. Each move takes away two cards and puts one back, so the count falls by 1 each time: 10 numbers means 9 moves. Each move also raises the TOTAL by exactly 3, so 9 × 3 = 27 is gained altogether. The numbers add to 55, so whatever order anyone uses, the number on the last card is 55 + 27 = 82.

3. Each move rubs out two numbers and writes one in their place, so the count falls by 1 each time: 16 numbers means 15 moves. Each move also drops the TOTAL by exactly 1, so 15 × 1 = 15 is lost altogether. The numbers add to 184, so whatever order anyone uses, the last number is 184 − 15 = 169.

4. Each move rubs out two numbers and writes one in their place, so the count falls by 1 each time: 9 numbers means 8 moves. Each move also drops the TOTAL by exactly 3, so 8 × 3 = 24 is lost altogether. The numbers add to 63, so whatever order anyone uses, the last number is 63 − 24 = 39.