Locker doors — Up to 100 lockers

Name: ____________________ Date: ____________

A row of lockers, and every student flips a share of them. Say how many end open, and why.

1. A school corridor has 68 lockers in a row, numbered 1 to 68, all shut. Student 1 opens every locker. Student 2 changes every second locker (2, 4, 6, ...). Student 3 changes every third, and so on up to student 68 — each student k changes every locker whose number k divides.

   In the end, how many lockers are left open?

   Why

   ______

2. A hall has 55 lamps in a line, numbered 1 to 55, all off. Switch 1 flips every lamp. Switch 2 flips every second lamp, switch 3 every third, and so on up to switch 55 — switch k flips every lamp whose number k divides.

   In the end, how many lamps are left on?

   Why

   ______

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A row of lockers, and every student flips a share of them. Say how many end open, and why.

3. A hall has 87 lamps in a line, numbered 1 to 87, all off. Switch 1 flips every lamp. Switch 2 flips every second lamp, switch 3 every third, and so on up to switch 87 — switch k flips every lamp whose number k divides.

In the end, how many lamps are left on?

Why

4. A long hallway has 56 doors, numbered 1 to 56, all closed. Person 1 opens every door. Person 2 changes every second door, person 3 every third, and so on to person 56 — person k changes every door whose number k divides.

In the end, how many doors are left open?

Why

Locker doors · Up to 100 lockers — Answer key

If they are stuck

- How many times does each one get changed? Ask when that count is ODD.

1. Each locker numbered L is changed once for each divisor of L. Divisors come in pairs (d with L ÷ d), an even count, so a locker ends shut unless a divisor pairs with itself — that is, unless L is a perfect square. The lockers left open are the perfect squares up to 68: 1, 4, 9, 16, ..., 64. The largest is 8 × 8 = 64, so there are 8 of them and 8 lockers stay open.

2. Each lamp numbered L is changed once for each divisor of L. Divisors come in pairs (d with L ÷ d), an even count, so a lamp ends off unless a divisor pairs with itself — that is, unless L is a perfect square. The lamps left on are the perfect squares up to 55: 1, 4, 9, 16, ..., 49. The largest is 7 × 7 = 49, so there are 7 of them and 7 lamps stay on.

3. Each lamp numbered L is changed once for each divisor of L. Divisors come in pairs (d with L ÷ d), an even count, so a lamp ends off unless a divisor pairs with itself — that is, unless L is a perfect square. The lamps left on are the perfect squares up to 87: 1, 4, 9, 16, ..., 81. The largest is 9 × 9 = 81, so there are 9 of them and 9 lamps stay on.

4. Each door numbered L is changed once for each divisor of L. Divisors come in pairs (d with L ÷ d), an even count, so a door ends closed unless a divisor pairs with itself — that is, unless L is a perfect square. The doors left open are the perfect squares up to 56: 1, 4, 9, 16, ..., 49. The largest is 7 × 7 = 49, so there are 7 of them and 7 doors stay open.